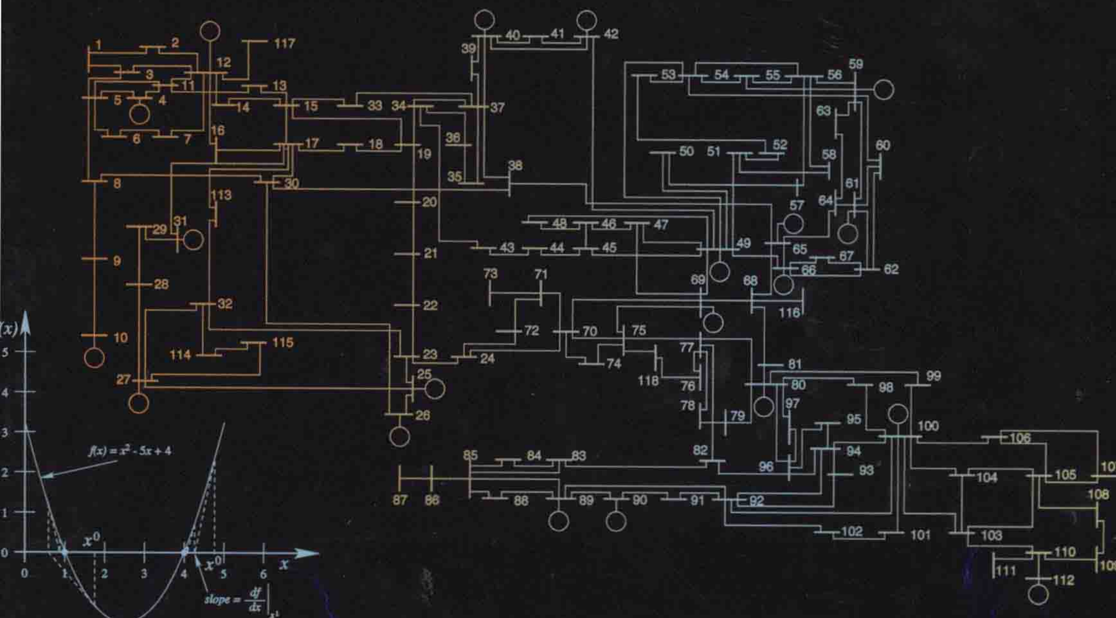
Third Edition

Computational Methods for Electric Power Systems



Mariesa L. Crow

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Computational Methods for Electric Power Systems

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To
Jim, David, and Jacob

Preface to the third edition

This book is intended for a graduate level course. The material is first presented in a general algorithmic manner followed by power system applications. Users do not necessarily have to have a power systems background to find this book useful, but many of the comprehensive exercises do require a working knowledge of power system problems and notation.

This new edition has been updated to include new material. Specifically, this new edition has added the following material:

- Updated examples on sparse LU factorization
- Preconditioners for linear iterative methods
- Broyden's method
- Jacobian free Newton–Krylov methods
- Double-shift method for computing complex eigenvalues
- Eigensystem Realization Algorithm

and additional problems and examples.

A course structure would typically include the following chapters in sequence: Chapters 1, 2, and 3. Chapter 2 provides a basic background in linear system solution (both direct and iterative) followed by a discussion of nonlinear system solution in Chapter 3. Chapter 2 can be directly followed by Chapter 4, which covers sparse storage and computation and follows directly from LU factorization. Chapters 5, 6, and 7 can be covered in any order after Chapter 3 depending on the interest of the reader.

Many of the methods presented in this book have commercial software packages that will accomplish their solution far more rigorously with many failsafe attributes included (such as accounting for ill conditioning, etc.). It is not my intent to make students experts in each topic, but rather to develop an appreciation for the methods behind the packages. Many commercial packages provide default settings or choices of parameters for the user; through better understanding of the methods driving the solution, informed users can make better choices and have a better understanding of the situations in which the methods may fail. If this book provides any reader with more confidence in using commercial packages, I have succeeded in my intent.

Mariesa L. Crow
Rolla, Missouri

Lecture notes are available from the CRC Web site:
http://www.crcpress.com/product/isbn/9781498711593

Contents

1

Introduction

In today's deregulated environment, the nation's electric power network is being forced to operate in a manner for which it was not intentionally designed. Therefore, system analysis is very important to predict and continually update the operating status of the network. This includes estimating the current power flows and bus voltages (power flow analysis and state estimation), determining the stability limits of the system (continuation power flow, numerical integration for transient stability, and eigenvalue analysis), and minimizing costs (optimal power flow). This book provides an introductory study of the various computational methods that form the basis of many analytical studies in power systems and other engineering and science fields. This book provides the analytical background of the algorithms used in numerous commercial packages. By understanding the theory behind many of the algorithms, the reader/user can better use the software and make more informed decisions (i.e., choice of integration method and step size in simulation packages).

Due to the sheer size of the power grid, hand-based calculations are nearly impossible, and computers offer the only truly viable means for system analysis. The power industry is one of the largest users of computer technology and one of the first industries to embrace the potential of computer analysis when mainframes first became available. Although the first algorithms for power system analysis were developed in the 1940s, it wasn't until the 1960s that computer usage became widespread within the power industry. Many of the analytical techniques and algorithms used today for the simulation and analysis of large systems were originally developed for power system applications.

As power systems increasingly operate under stressed conditions, computer simulation will play a large role in control and security assessment. Commercial packages routinely fail or give erroneous results when used to simulate stressed systems. Understanding of the underlying numerical algorithms is imperative to correctly interpret the results of commercial packages. For example, will the system really exhibit the simulated behavior or is the simulation simply an artifact of a numerical inaccuracy? The educated user can make better judgments about how to compensate for numerical shortcomings in such packages, either by better choice of simulation parameters or by posing the problem in a more numerically tractable manner. This book will provide the background for a number of widely used numerical algorithms that underlie many commercial packages for power system analysis and design.

This book is intended to be used as a text in conjunction with a semester-long graduate level course in computational algorithms. While the majority of examples in this text are based on power system applications, the theory is presented in a general manner so as to be applicable to a wide range of engineering systems. Although some knowledge of power system engineering may be required to fully appreciate the subtleties of some of the illustrations, such knowledge is not a prerequisite for understanding the algorithms themselves. The text and examples are used to provide an introduction to a wide range of numerical methods without being an exhaustive reference. Many of the algorithms presented in this book have been the subject of numerous modifications and are still the object of on-going research. As this text is intended to provide a foundation, many of these new advances are not explicitly covered, but are rather given as references for the interested reader. The examples in this text are intended to be simple and thorough enough to be reproduced easily. Most "real world" problems are much larger in size and scope, but the methodologies presented in this text should sufficiently prepare the reader to cope with any difficulties he/she may encounter.

Most of the examples in this text were produced using code written in MATLAB®. Although this was the platform used by the author, in practice, any computer language may be used for implementation. There is no practical reason for a preference for any particular platform or language.

2

The Solution of Linear Systems

In many branches of engineering and science it is desirable to be able to mathematically determine the state of a system based on a set of physical relationships. These physical relationships may be determined from characteristics such as circuit topology, mass, weight, or force, to name a few. For example, the injected currents, network topology, and branch impedances govern the voltages at each node of a circuit. In many cases, the relationship between the known, or input, quantities and the unknown, or output, states is a linear relationship. Therefore, a linear system may be generically modeled as

$$Ax = b \tag{2.1}$$

where b is the $n \times 1$ vector of known quantities, x is the $n \times 1$ unknown state vector, and A is the $n \times n$ matrix that relates x to b. For the time being, it will be assumed that the matrix A is invertible, or non-singular; thus each vector b will yield a unique corresponding vector x. Thus the matrix A^{-1} exists and

$$x^* = A^{-1}b \tag{2.2}$$

is the unique solution to Equation (2.1).

The natural approach to solving Equation (2.1) is to directly calculate the inverse of A and multiply it by the vector b. One method to calculate A^{-1} is to use *Cramer's* rule:

$$A^{-1}(i,j) = \frac{1}{det(A)} (A_{ij})^T \quad \text{for } i = 1, \ldots, n, j = 1, \ldots, n \tag{2.3}$$

where $A^{-1}(i,j)$ is the ijth entry of A^{-1} and A_{ij} is the cofactor of each entry a_{ij} of A. This method requires the calculation of $(n+1)$ determinants, which results in $2(n+1)!$ multiplications to find A^{-1}! For large values of n, the calculation requirement grows too rapidly for computational tractability; thus alternative approaches have been developed.

Basically, there are two approaches to solving Equation (2.1):

- *Direct methods*, or elimination methods, find the exact solution (within the accuracy of the computer) through a finite number of arithmetic operations. The solution x of a direct method would be completely accurate were it not for computer roundoff errors.

- *Iterative methods*, on the other hand, generate a sequence of (hopefully) progressively improving approximations to the solution based on the application of the same computational procedure at each step. The iteration is terminated when an approximate solution is obtained having some prespecified accuracy or when it is determined that the iterates are not improving.

The choice of solution methodology usually relies on the structure of the system under consideration. Certain systems lend themselves more amenably to one type of solution method versus the other. In general, direct methods are best for full matrices, whereas iterative methods are better for matrices that are large and sparse. But, as with most generalizations, there are notable exceptions to this rule of thumb.

2.1 Gaussian Elimination

An alternate method for solving Equation (2.1) is to solve for x without calculating A^{-1} explicitly. This approach is a *direct method* of linear system solution, since x is found directly. One common direct method is the method of *Gaussian elimination*. The basic idea behind Gaussian elimination is to use the first equation to eliminate the first unknown from the remaining equations. This process is repeated sequentially for the second unknown, the third unknown, etc., until the elimination process is completed. The nth unknown is then calculated directly from the input vector b. The unknowns are then recursively substituted back into the equations until all unknowns have been calculated.

Gaussian elimination is the process by which the augmented $n \times (n+1)$ matrix

$$[A \mid b]$$

is converted to the $n \times (n+1)$ matrix

$$[I \mid b^*]$$

through a series of elementary row operations, where

$$\begin{aligned} Ax &= b \\ A^{-1}Ax &= A^{-1}b \\ Ix &= A^{-1}b = b^* \\ x^* &= b^* \end{aligned}$$

Thus, if a series of elementary row operations exist that can transform the matrix A into the identity matrix I, then the application of the same set of