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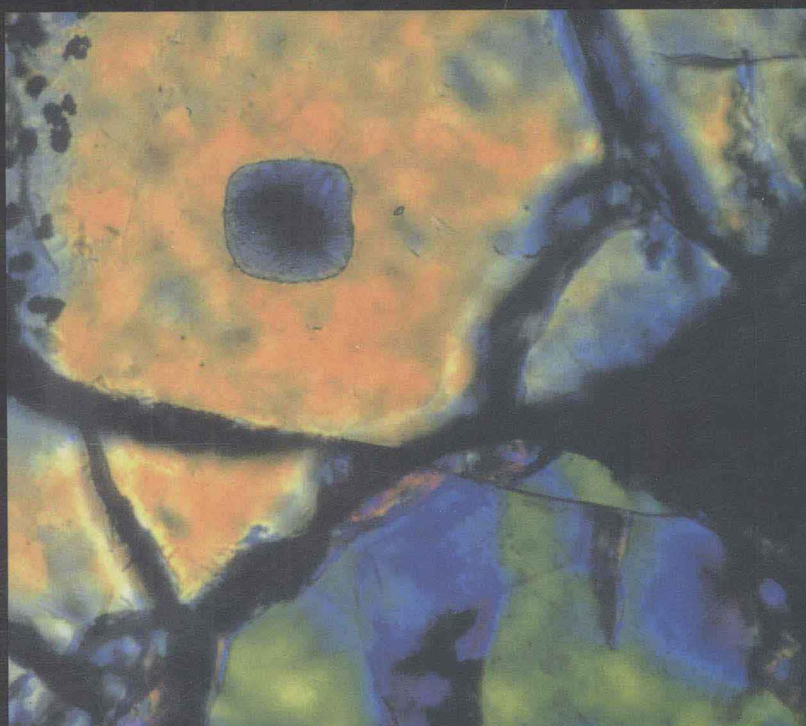
Paul Strange

Relativistic Quantum Mechanics

with applications in condensed matter and atomic physics

相对论量子力学



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RELATIVISTIC QUANTUM MECHANICS

WITH APPLICATIONS IN CONDENSED MATTER AND ATOMIC PHYSICS

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CAMBRIDGE
UNIVERSITY PRESS

图书在版编目（C I P）数据

相对论量子力学=Relativistic Quantum Mechanics：英文／（英）斯诸哲（Strange,P.）著.—北京：世界图书出版公司北京公司，2008.8
ISBN 978-7-5062-9258-0

Ⅰ.相… Ⅱ.斯… Ⅲ.相对论－量子力学－研究生－教材－英文 Ⅳ.0413.1

中国版本图书馆CIP数据核字（2008）第112707号

书　　名:	Relativistic Quantum Mechanics
作　　者:	Paul Strange
中 译 名:	相对论量子力学
责任编辑:	高蓉 刘慧
出 版 者:	世界图书出版公司北京公司
印 刷 者:	三河国英印务有限公司
发　　行:	世界图书出版公司北京公司（北京朝内大街 137 号　100010）
联系电话:	010-64015659
电子信箱:	kjsk@vip.sina.com
开　　本:	16开
印　　张:	39
版　　次:	2008 年 8月第 1 次印刷
版权登记:	图字:01-2008-3011
书　　号:	978-7-5062-9258-0 / O・629　　定　价: 118.00 元

Preface

I always thought I would write a book and this is it. In the end, though, I hardly wrote it at all, it evolved from my research notes, from essays I wrote for postgraduates starting work with me, and from lecture handouts I distribute to students taking the relativistic quantum mechanics option in the Physics department at Keele University. Therefore the early chapters of this book discuss pure relativistic quantum mechanics and the later chapters discuss applications of relevance in condensed matter physics. This book, then, is written with an audience ranging from advanced students to professional researchers in mind. I wrote it because anyone aiming to do research in relativistic quantum theory applied to condensed matter has to pull together information from a wide range of sources using different conventions, notation and units, which can lead to a lot of confusion (I speak from experience). Most relativistic quantum mechanics books, it seems to me, are directed towards quantum field theory and particle physics, not condensed matter physics, and many start off at too advanced a level for present day physics graduates from a British university. Therefore, I have tried to start at a sufficiently elementary level, and have used the SI system of units throughout.

When I started preparing this book I thought I might be able to write everything I knew in around fifty pages. It soon became apparent that that was not the case. Indeed it now appears to me that the principal decisions to be taken in writing a book are about what to omit. I have written this much quantum mechanics and not used the word Lagrangian. This saddens me, but surely must make me unique in the history of relativistic quantum theory. I have not discussed the very interesting quantum mechanics describing the neutrino and its helicity, another topic that invariably appears in other relativistic quantum mechanics texts. However, as we are leaning towards condensed matter physics in this book, there are sections on topics such as magneto-optical effects and magnetic anisotropy which don't appear in other books despite being intrinsically relativistic and quantum mechanical in nature. In the end, what is included and what is omitted is just a question of taste, and it is up to the reader to decide whether such decisions were good or bad.

This book is very mathematical, containing something like two thousand equations. I make no apology for that. I think the way the mathematics works is the great beauty of the subject. Throughout the book I try to make the mathematics clear, but I do not try to avoid it. Paraphrasing Niels Bohr I believe that "If you can't do the maths, you don't understand it." If you don't like maths, you are reading the wrong book.

There are a lot of people I would like to thank for their help with, and influence on, my understanding of quantum mechanics, particularly the relativistic version of the theory. They are Dr E. Arola, Dr P.J. Durham, Professor H. Ebert, Professor W.M. Fairbairn, Professor J.M.F. Gunn, Prof B.L. Györffy, Dr R.B. Jones, Dr P.M. Lee, Dr J.B. Staunton, Professor J.G. Valatin, and Dr W. Yeung.

Several of the examples and problems in this book stem from projects done by undergraduate students during their time at Keele, and from the work of my Ph.D students. Thanks are also due to them, C. Blewitt, H.J. Gotsis, O. Gratton, A.C. Jenkins, P.M. Mobit, and E. Pugh, and to the funding agencies who supported them (Keele University physics department, the EPSRC, and the Nuffield foundation).

There are several other people I would like to thank for their general influence, encouragement and friendship. They are Dr T. Ellis, Professor M.J. Gillan, Dr M.E. Hagen, Dr P.W. Haycock, Mr J. Hodgeson and Mr B.G. Locke-Scobie. I would also like to thank R. Neal and L. Nightingale of Cambridge University Press for their encouragement of, and patience with, me. Finally, my parents do not have a scientific background, nonetheless they have always supported me in my education and have taken a keen interest in the writing of this book. Thanks are also due to them, R.J. and V.A. Strange.

I hope you enjoy this book, although I am not sure 'enjoy' is the right word to describe the feeling one has when reading a quantum mechanics textbook. Perhaps it would be better to say that I hope you find this book informative and instructive. What I would really like would be for you to be inspired to look deeper into the subject, as I was by my undergraduate lectures many years ago. Many people think quantum mechanics is not relevant to everyday life, but it has certainly influenced my life for the better! I hope it will do the same for you.

Contents

1
The Theory of Special Relativity

Relativistic quantum mechanics is the unification into a consistent theory of Einstein's theory of relativity and the quantum mechanics of physicists such as Bohr, Schrödinger, and Heisenberg. Evidently, to appreciate relativistic quantum theory it is necessary to have a good understanding of these component theories. Apart from this chapter we assume the reader has this understanding. However, here we are going to recall some of the important points of the classical theory of special relativity. There is good reason for doing this. As you will discover all too soon, relativistic quantum mechanics is a very mathematical subject and my experience has been that the complexity of the mathematics often obscures the physics being described. To facilitate the interpretation of the mathematics here, appropriate limits are taken wherever possible, to obtain expressions with which the reader should be familiar. Clearly, when this is done it is useful to have the limiting expressions handy. Presenting them in this chapter means they can be referred to easily.

Taking the above argument to its logical conclusion means we should include a chapter on non-relativistic quantum mechanics as well. However, that is too vast a subject to include in a single chapter. Furthermore, there already exists a plethora of good books on the subject. Therefore, where it is appropriate, the reader will be referred to one of these (Baym 1967, Dicke and Wittke 1974, Gasiorowicz 1974, Landau and Lifschitz 1977, Merzbacher 1970, and McMurry 1993).

This chapter is included for revision purposes and for reference later on, therefore some topics are included without much justification and without proof. The reader should either accept these statements or refer to books on the classical theory of special relativity. In the first section of this chapter we state the fundamental assumptions of the special theory of relativity. Then we discuss the Lorentz transformations of time and space. Next we come to discuss velocities, momentum and energy. Then we go on to think about relativity and the electromagnetic field. Finally, we look at the Compton effect where relativity and quantum theory are brought together for the first time in most physics courses.

1.1 The Lorentz Transformations

Newton's laws are known to be invariant under a Galilean transformation from one reference frame to another. However, Maxwell's equations are not invariant under such a transformation. This led Michelson and Morley (1887) to attempt their famous experiment which tried to exploit the non-invariance of Maxwell's equations to determine the absolute velocity of the earth. Here, I do not propose to go through the Michelson–Morley experiment (Shankland *et al.* 1955). However, its failure to detect the movement of the earth through the ether is the experimental foundation of the theory of relativity and led to a revolution in our view of time and space. Within the theory of relativity both Newton's laws and the Maxwell equations remain the same when we transform from one frame to another. This theory can be encapsulated in two well-known postulates, the first of which can be written down simply as

(1) All inertial frames are equivalent.

By this we mean that in an isolated system (e.g. a spaceship with no windows moving at a constant velocity $\mathbf{v}$ (with respect to distant stars or something)) there is no experiment that can be done that will determine $\mathbf{v}$. According to Feynman (1962) this principle has been verified experimentally (although a bunch of scientists standing around in a spaceship not knowing how to measure their own velocity is not a sufficient verification). Here, we are implicitly assuming that space is isotropic and uniform. The second postulate is

(2) There exists a maximum speed, c. If a particle is measured to have speed c in one inertial frame, a measurement in any other inertial frame will also give the value c (provided the measurement is done correctly). That is, the speed of light is independent of the speed of the source and the observer.

The whole vast consequences of the theory of relativity follow directly from these two statements (French 1968, Kittel *et al.* 1973). It is necessary to find transformation laws from one frame of reference to another that are consistent with these postulates (Einstein 1905). Consider a Cartesian frame S in which there is a source of light at the origin. At time $t = 0$ a spherical wavefront of light is emitted. The distance of the wavefront

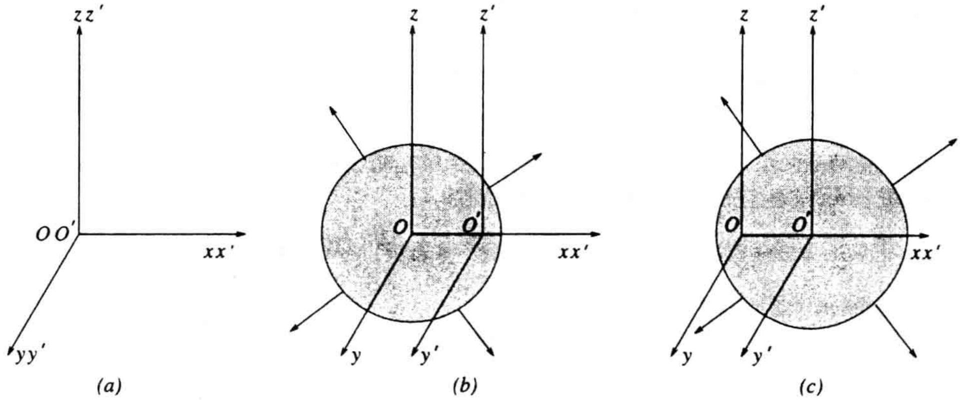

Fig. 1.1. (*a*) At $t = t' = 0$ the two frames are coincident and observers O and O' are at the origin. At this time a spherical wavefront is emitted. (*b*) At a time $t > 0$ as viewed by an observer stationary in the unprimed frame. Observer O is at the centre of the wavefront. (*c*) At a time $t' > 0$ as viewed by an observer stationary in the primed frame. Observer O' is at the centre of the wavefront. Note that in (*b*) and (*c*) it is not possible for the observer not at the centre of the wavefront to be outside the wavefront.

from the origin at any subsequent time t is given by

$$x^2 + y^2 + z^2 = c^2t^2 \tag{1.1}$$

Now consider a second frame S' moving in the x-direction with velocity v relative to S. Let us set up S' such that its origin coincides with the origin of S at $t' = t = 0$ when the wavefront is emitted. Now the equation giving the distance of the wavefront from the origin of S' at a subsequent time t' as measured in S' is

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \tag{1.2}$$

So, at all times $t, t' > 0$, observers at the origin of both frames would believe themselves to be at the centre of the wavefront. However, each observer would see the other as being displaced from the centre. This is illustrated in figure 1.1. It can easily be seen that a Galilean transformation relating the coordinates in equations (1.1) and (1.2) does not give consistent results. A set of coordinate transformations that are consistent with (1.1) and (1.2) is

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - (xv/c^2)}{\sqrt{1 - v^2/c^2}} \tag{1.3a}$$

and the inverse transformations are

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \quad y = y', \quad z = z', \quad t = \frac{t' + (x'v/c^2)}{\sqrt{1 - v^2/c^2}} \tag{1.3b}$$

These equations are known as the Lorentz transformations. Under these transformations the interval s defined by

$$s^2 = (ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2 \tag{1.4}$$

is a constant in all frames.

It is conventional to adopt the notation

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \qquad \beta = v/c \tag{1.5}$$

Equations (1.3) lead to some startling conclusions. Firstly consider measurements of length. If we measure the length of a rod by looking at the position of its ends relative to a ruler, then if in the S frame the rod is at rest we can measure the ends at x_1 and x_2 and infer that its length is $L = x_2 - x_1$. Now consider the situation in the primed frame. The observer will measure the ends as being at points x'_1 and x'_2 and hence $L' = x'_2 - x'_1$. We want to know the relation between these two lengths. The rod is moving at velocity $-v$ in the x-direction relative to the observer in S'. To find the length this observer must have measured the position of the ends simultaneously (at t'_0) in his frame. So, considering the first of equations (1.3b) we have

$$x_1 = \frac{x'_1 + vt'_0}{\sqrt{1 - v^2/c^2}}, \qquad x_2 = \frac{x'_2 + vt'_0}{\sqrt{1 - v^2/c^2}} \tag{1.6}$$

Subtracting these equations leads directly to

$$L' = x'_2 - x'_1 = \sqrt{(1 - v^2/c^2)}(x_2 - x_1) = \sqrt{(1 - v^2/c^2)}L \tag{1.7}$$

This is the famous Lorentz–Fitzgerald contraction and is illustrated in figure 1.2. It shows that observers in different inertial frames of reference will measure lengths differently. The length of any object takes on its maximum value in its rest frame. Let us emphasize that nothing physical has happened to the rod. Measuring the length of the rod from one reference frame is a different experiment to measuring the length from another reference frame, and the different experiments give different answers. The process of measuring correctly gives a different result in different inertial frames of reference.

The above description of Lorentz–Fitzgerald contraction depended crucially on the fact that the observer in S' performed his measurements of

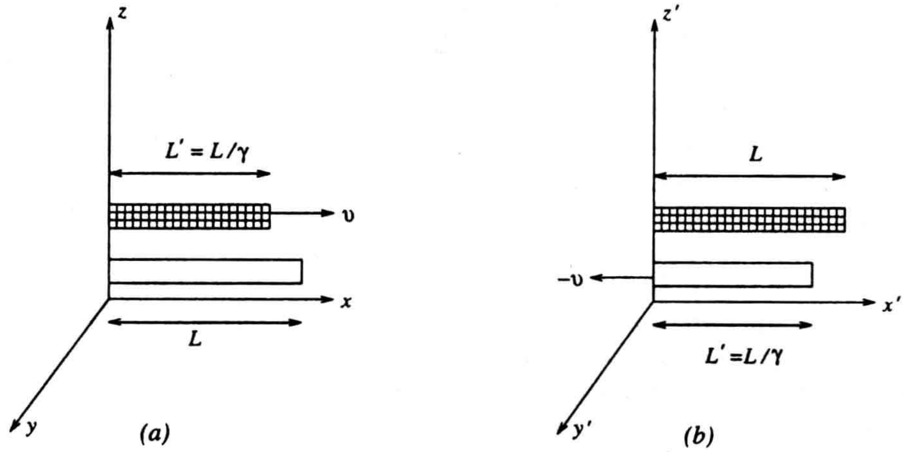

Fig. 1.2. Here are two rods that are identical in their rest frames with length L. In (a) we are in the rest frame of the lower rod. The upper rod is moving in the positive x-direction with velocity v and is Lorentz–Fitzgerald contracted so that its length is measured as L/γ. In (b) we are in the rest frame of the upper rod and the lower rod is moving in the negative x-direction with velocity $-v$. In this frame of reference it is the lower rod that appears to be Lorentz–Fitzgerald contracted.

the position of the end points simultaneously. It is important to note that simultaneous in S' does not mean simultaneous in S. So the fact that the light from the ends of the rod arrived at the observer in S' at the same time does not mean it left the ends of the rod at the same time. This is trivial to verify from the time transformations in equations (1.3).

Next we consider intervals of time. Imagine a clock and an observer in frame S at rest with respect to the clock. The observer can measure a time interval easily enough as the time between two readings on the clock

$$\tau = t_2 - t_1 \tag{1.8}$$

Now we can use the Lorentz time transformations to find the times t'_2 and t'_1 as measured by an observer in S' again moving with velocity v in the x-direction relative to the observer in S:

$$t'_1 = \frac{t_1 - (x_1 v/c^2)}{\sqrt{1 - v^2/c^2}}, \qquad t'_2 = \frac{t_2 - (x_2 v/c^2)}{\sqrt{1 - v^2/c^2}} \tag{1.9}$$

We can subtract one of these from the other to discover how to transform time intervals from one frame to another:

$$t'_2 - t'_1 = \frac{t_2 - t_1}{\sqrt{1 - v^2/c^2}} = \gamma(t_2 - t_1) \tag{1.10}$$

where we have set $x_2 - x_1 = 0$. This is obviously true as the clock is defined as staying at the same coordinate in S. What we have found here is the time dilation formula. The time interval measured in S' is longer than the time interval measured in S. Another way of stating the same thing is to say that moving clocks appear to run more slowly than stationary clocks. This, of course, is completely counter-intuitive and takes some getting used to. However, it has been well established experimentally, particularly from measurements of the lifetime of elementary particles. It is also responsible for one of the most famous of all problems in physics, the twin paradox.

Next, let me describe a thought experiment that one can do, which demystifies time dilation to some extent, and shows explicitly that it arises from the constancy of the speed of light. Consider a train in its rest frame S as shown in the top diagram in figure 1.3 (with a rather idealized train). Light is emitted from a transmitter/receiver on the floor of the train in a vertical direction at time zero. It is reflected from a mirror on the ceiling and the time of its arrival back at the receiver is noted. The ceiling is at a height L above the floor, so the time taken for the light to make the return journey is

$$t = \frac{2L}{c} \tag{1.11}$$

Now suppose there is an observer in frame S', i.e. sitting by the track as the train goes past while the experiment is being done, and there is a series of synchronized clocks in this frame. This is shown in the lower part of figure 1.3. The observer in S' can also time the light pulse. Using Pythagoras's theorem it is easy to see from the figure that when the light travels a distance L in S, it travels a distance $(L^2 + (\frac{1}{2}vt')^2)^{1/2}$ in S', and it goes the same distance for the reflected path. So the total distance travelled as viewed by the observer in S' is

$$d = 2(L^2 + (\tfrac{1}{2}vt')^2)^{1/2} \tag{1.12}$$

But the velocity of light is the same in all frames. So

$$d^2 = c^2t'^2 = 4L^2 + v^2t'^2 \tag{1.13}$$

Rearranging this

$$t' = \frac{2L}{(c^2 - v^2)^{1/2}} = \frac{2L}{c}\gamma = \gamma t \tag{1.14}$$

Thus if the clock in the train tells us the light's journey time was t, the clocks by the side of the track tell us it was $\gamma t > t$. So, to the observer at the side of the track, the clock in S will appear to be running slowly. Equation (1.14) is exactly the same as equation (1.10) which was obtained directly from the Lorentz transformations.

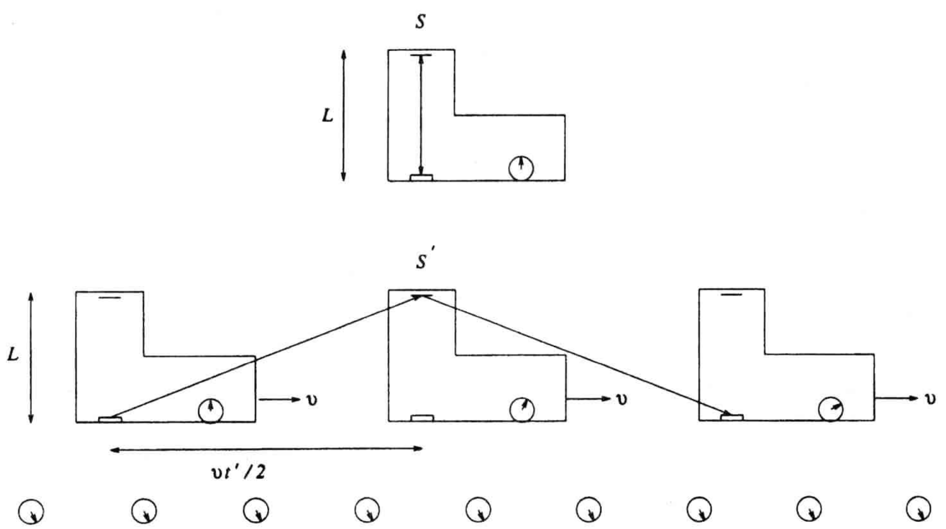

Fig. 1.3. Thought experiment illustrating time dilation, as discussed in the text. The upper figure shows the experiment in the rest frame of the train and the lower figure shows it in the frame of an observer by the side of the track.

Equations (1.3) are easy to derive from the postulates, and easy to apply. However, their meaning is not so clear. In fact they can be interpreted in several ways. Depending on the circumstances, I tend to think of them in two ways. Firstly, a rather woolly and obvious statement. At low velocities non-relativistic mechanics is OK because the time taken for light to get from the object to the detector (your eye) is infinitesimal compared with the time taken for the object to move, so the velocity of light does not affect your perception. However, when the object is moving at an appreciable fraction of the speed of light, the time taken for the light to reach your eye does have an appreciable effect on your perception. Secondly, a rather grander statement. Let us consider space and time as different components of the same thing, as is implied by equations (1.1) and (1.3). Any observer (Observer 1) can split space–time into space and time unambiguously, and will know what he or she means by space and time separately. Any observer (Observer 2) moving with a non-zero velocity with respect to Observer 1 will be able to do the same. However, Observer 2 will not split up time and space in the same way as Observer 1. Observers in different inertial frames separate time and space in different ways!

1.2 Relativistic Velocities

Once we have the Lorentz transformations for position and time, it is an easy matter to construct the velocity transformation equations. As before,