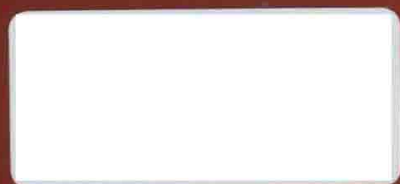


Applied and Numerical Harmonic Analysis



$$(\gamma) = \int f(x) e^{-2\pi i x \gamma} dx$$

Radu Balan, Matthew J. Begué,  
John J. Benedetto, Wojciech Czaja,  
Kasso A. Okoudjou, Editors

# Excursions in Harmonic Analysis, Volume 3

The February Fourier Talks at the  
Norbert Wiener Center

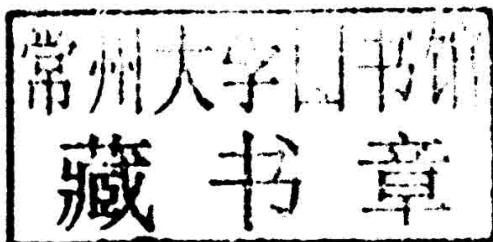
 Birkhäuser

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Wojciech Czaja • Kasso A. Okoudjou

Editors

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*Dedicated to*

*Henry J. Landau,*

*who has set the standard for excellence and creativity in  
harmonic analysis and its applications.*



# ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

<i>Antenna theory</i>	<i>Prediction theory</i>
<i>Biomedical signal processing</i>	<i>Radar applications</i>
<i>Digital signal processing</i>	<i>Sampling theory</i>
<i>Fast algorithms</i>	<i>Spectral estimation</i>
<i>Gabor theory and applications</i>	<i>Speech processing</i>
<i>Image processing</i>	<i>Time-frequency and time-scale analysis</i>
<i>Numerical partial differential equations</i>	<i>Wavelet theory</i>

The above point of view for the ANHA book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of “function.” Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor’s set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener’s Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for deal-

ing with signal reconstruction in communications theory. We are back to the *raison d'être* of the *ANHA* series!

University of Maryland  
College Park

*John J. Benedetto*  
Series Editor



# Preface

The chapters in these Volumes 3 and 4 have at least one author who spoke at the February Fourier Talks during the period 2002–2013 or at the workshop on Phaseless Reconstruction that immediately followed the 2013 February Fourier Talks. Volumes 1 and 2 were limited to the period 2006–2011.

## The February Fourier Talks (FFT)

The *FFT* were initiated in 2002 and 2003 as small meetings on harmonic analysis and applications, held at the University of Maryland, College Park. There were no *FFTs* in 2004 and 2005. The Norbert Wiener Center (NWC) for Harmonic Analysis and Applications was founded in 2004 in the Department of Mathematics at the university, and, since 2006, the *FFT* has been organized by the NWC. The *FFT* has developed into a major annual conference that brings together applied and pure harmonic analysts along with scientists and engineers from universities, industry, and government for an intense and enriching 2-day meeting.

The goals of the *FFT* are the following:

- To offer a forum for applied and pure harmonic analysts to present their latest cutting-edge research to scientists working not only in the academic community but also in industry and government agencies;
- To give harmonic analysts the opportunity to hear from government and industry scientists about the latest problems in need of mathematical formulation and solution;
- To provide government and industry scientists with exposure to the latest research in harmonic analysis;
- To introduce young mathematicians and scientists to applied and pure harmonic analysis;
- To build bridges between pure harmonic analysis and applications thereof.

These goals stem from our belief that many of the problems arising in engineering today are directly related to the process of making pure mathematics applicable. The Norbert Wiener Center sees the *FFT* as the ideal venue to enhance this process in a constructive and creative way. Furthermore, we believe that our vision is shared by the scientific community, as shown by the steady growth of the *FFT* over the years.

The *FFT* is formatted as a two-day single-track meeting consisting of 30-min talks as well as the following:

- Norbert Wiener Distinguished Lecturer Series;
- General Interest Keynote Address;
- Norbert Wiener Colloquium;
- Graduate and Postdoctoral Poster Session.

The talks are given by experts in applied and pure harmonic analysis, including academic researchers and invited scientists from industry and government agencies.

The Norbert Wiener Distinguished Lecture caps the technical talks of the first day. It is given by a senior harmonic analyst, whose vision and depth through the years have had profound impact on our field. In contrast to the highly technical day sessions, the Keynote Address is aimed at a general public audience and highlights the role of mathematics, in general, and harmonic analysis, in particular. Furthermore, this address can be seen as an opportunity for practitioners in a specific area to present mathematical problems that they encounter in their work. The concluding lecture of each *FFT*, our Norbert Wiener Colloquium, features a mathematical talk by a renowned applied or pure harmonic analyst. The objective of the Norbert Wiener Colloquium is to give an overview of a particular problem or a new challenge in the field. We include here a list of speakers for these three lectures.

Distinguished Lecturer	Keynote Address	Colloquium
• Ronald Coifman	• Peter Carr	• Rama Chellappa
• Ingrid Daubechies	• Barry Cipra	• Margaret Cheney
• Ronald DeVore	• James Coddington	• Charles Fefferman
• Richard Kadison	• Nathan Crone	• Robert Fefferman
• Peter Lax	• Mario Livio	• Gerald Folland
• Elias Stein	• William Noel	• Christopher Heil
• Gilbert Strang	• Steven Schiff	• Peter Jones
	• Mark Stopfer	• Thomas Strohmer
	• Frederick Williams	• Victor Wickerhauser

In 2013, the February Fourier Talks were followed by a workshop on phaseless reconstruction, also hosted by the Norbert Wiener Center and intellectually in the spirit of the *FFT*.

## The Norbert Wiener Center

The Norbert Wiener Center for Harmonic Analysis and Applications provides a national focus for the broad area of mathematical engineering. Applied harmonic analysis and its theoretical underpinnings form the technological basis for this area. It can be confidently asserted that mathematical engineering will be to today's mathematics departments what mathematical physics was to those of a century ago. At that time, mathematical physics provided the impetus for tremendous advances within mathematics departments, with particular impact in fields such as differential equations, operator theory, and numerical analysis. Tools developed in these fields were essential in the advances of applied physics, e.g., the development of the solid state devices, which now enable our information economy.

Mathematical engineering impels the study of fundamental harmonic analysis issues in the theories and applications of topics such as signal and image processing, machine learning, data mining, waveform design, and dimension reduction into mathematics departments. The results will advance the technologies of this millennium.

The golden age of mathematical engineering is upon us. The Norbert Wiener Center reflects the importance of integrating new mathematical technologies and algorithms in the context of current industrial and academic needs and problems.

The Norbert Wiener Center has three goals:

- Research activities in harmonic analysis and applications;
- Education—undergraduate to postdoctoral;
- Interaction within the international harmonic analysis community.

We believe that educating the next generation of harmonic analysts, with a strong understanding of the foundations of the field and a grasp of the problems arising in applications, is important for a high level and productive industrial, government, and academic workforce.

The Norbert Wiener Center website: **[www.norbertwiener.umd.edu](http://www.norbertwiener.umd.edu)**.

## The Structure of the Volumes

To some extent the four parts for each of these volumes are artificial placeholders for all the diverse chapters. It is an organizational convenience that reflects major areas in harmonic analysis and its applications, and it is also a means to highlight significant modern thrusts in harmonic analysis. Each part includes an introduction that describes the chapters therein.

Volume 1

- I Sampling Theory
- II Remote Sensing
- III Mathematics of Data Processing
- IV Applications of Data Processing

Volume 2

- V Measure Theory
- VI Filtering
- VII Operator Theory
- VIII Biomathematics

Volume 3

- IX Special Topics in Harmonic Analysis
- X Applications and Algorithms in the Physical Sciences
- XI Gabor Theory
- XII RADAR and Communications: Design, Theory, and Applications

Volume 4

- XIII Theoretical Harmonic Analysis
- XIV Sparsity
- XV Signal Processing and Sampling
- XVI Spectral Analysis and Correlation

# Acknowledgements

The editors of Volumes 3 and 4 gratefully acknowledge additional editorial assistance by Dr. Alfredo Nava-Tudela as well as the support of Danielle Walker, Associate Editor for Springer-Birkhäuser in New York.

The Norbert Wiener Center also gratefully acknowledges the indispensable support of the following groups: Birkhäuser and Springer Publishers, the IEEE Baltimore Section, MiMoCloud, Inc., Radyn, Inc., the SIAM Washington-Baltimore Section, and SR2 Group, LLC. One of the successes of the February Fourier Talks has been the dynamic participation of graduate student and postdoctoral engineers, mathematicians, and scientists. We have been fortunate to be able to provide travel and living expenses to this group due to continuing, significant grants from the National Science Foundation, which, along with the aforementioned organizations and companies, believes in and supports our vision of the FFT.



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