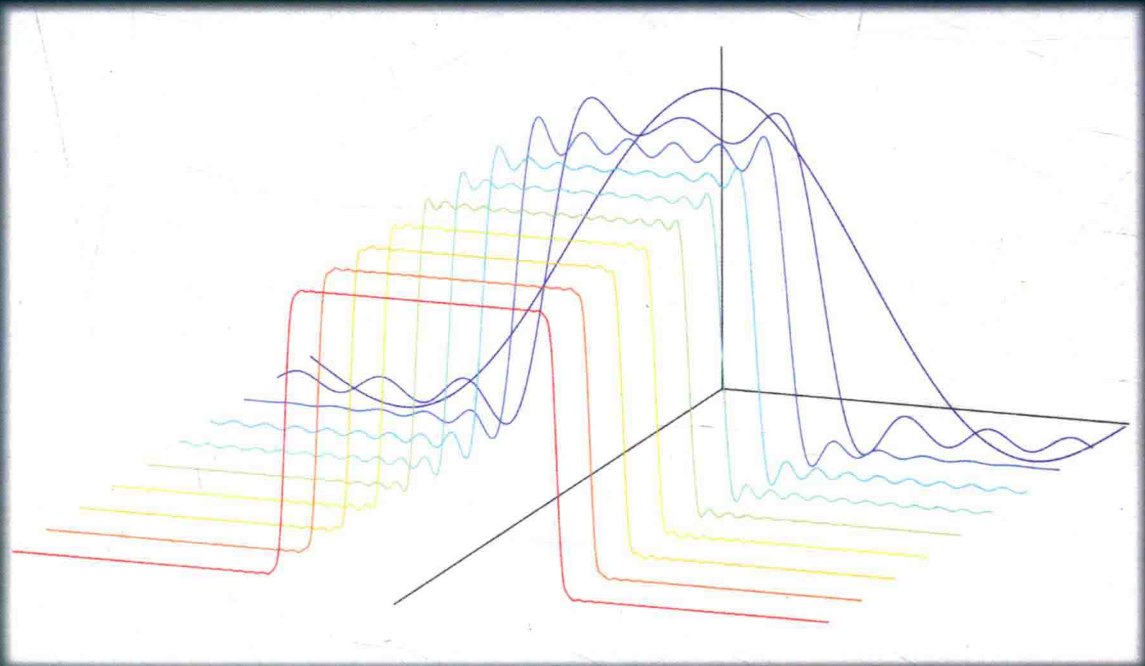
TEXTBOOKS IN MATHEMATICS

A Course in Differential Equations with Boundary-Value Problems

SECOND EDITION



Stephen A. Wirkus
Randall J. Swift
Ryan S. Szypowski

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To our families

Erika Tatiana, Alan, Abdi, and Avani,

Kelly, Kaelin, Robyn, Erin, and Ryley,

and

Jillian

for bringing us more joy than math and showing us the true concept

and meaning of

∞ infinity ∞

with their tireless

patience, love, and understanding.

About the Authors

Stephen A. Wirkus completed his Ph.D. at Cornell University under the direction of Richard Rand. He began guiding undergraduate research projects while in graduate school and came to California State Polytechnic University, Pomona (Cal Poly Pomona) in 2000 after being a Visiting Professor at Cornell for a year. He co-founded the Applied Mathematical Sciences Summer Institute (AMSSI), an undergraduate research program jointly hosted by Loyola Marymount University, that ran from 2005 through 2007. He came to Arizona State University in 2007 as a tenured associate professor and won the 2013 Professor of the Year Award at ASU as well as the 2011 NSF AGEP Mentor of the Year award. He was a Visiting MLK Professor at the Massachusetts Institute of Technology in 2013–2014. He has guided over 80 undergraduate students in research and has served as chair for four M.S. students, and two Ph.D. students. He has over 30 publications and technical reports with over 40 students and has received grants from the NSF and NSA for guiding undergraduate research.

Randall J. Swift completed his Ph.D. at the University of California, Riverside under the direction of M. M. Rao. He began his career at Western Kentucky University and taught there for nearly a decade before moving to Cal Poly Pomona in 2001 as a tenured associate professor. He is active in research and teaching, having authored more than 80 journal articles, three research monographs and three textbooks in addition to serving as chair for 25 M.S. students. Now a professor, he received the 2011–12 Ralph W. Ames Distinguished Research Award from the College of Science at Cal Poly Pomona. The award honors Swift for his outstanding research in both pure and applied mathematics, and his contributions to the mathematics field as a speaker, journal editor, and principal investigator on numerous grants. He was also a visiting professor in 2007–2008 at the Australian National University in Canberra, Australia as well as having taught at the Claremont Colleges.

Ryan S. Szypowski completed his Ph.D. at the University of California, San Diego under the direction of Michael Holst. He was hired at Cal Poly Pomona in 2011 and became a tenured Associate Professor in 2016. His main research is in the area of computational methods for approximation of solutions to partial differential equations, but studies broad problems from applied mathematics and science with his students and colleagues. He had an active NSF research grant from 2012 to 2015 to study adaptive techniques in finite element exterior calculus and has won numerous smaller grants to support his work with students. In 2015, he won the Department of Mathematics and Statistics Excellence in Teaching Award.

Preface

This book is based on lectures given by the first author at Cal Poly Pomona, Arizona State University (ASU), and the Massachusetts Institute of Technology (MIT), the second author at Western Kentucky University (WKU) and Cal Poly Pomona, and the third author at Cal Poly Pomona. The first eight chapters and two appendices are identical to those in *A Course in Ordinary Differential Equations, 2nd Edition.* The text can be used for a traditional one-semester sophomore-level course in ordinary differential equations (such as WKU's MATH 331), a one-semester sophomore-level course in differential equations in which partial differential equations replace Laplace transforms, or a two-semester sophomore- or junior-level course in differential equations. There is ample material for a two-quarter sequence (such as Cal Poly Pomona's MAT 216-431 or MAT 431-432), as well as sufficient linear algebra in the text so that it can be used for a one-quarter course that combines differential equations and linear algebra (such as Cal Poly Pomona's Math 224), or a one-semester course in differential equations that brings in linear algebra in a significant way (such as ASU's MAT 275 or MIT's 18.03). Most significantly, computer labs are given in MATLAB®,[1] Maple™, and *Mathematica* at the end of each chapter so the book may be used for a course to introduce and equip the student with a knowledge of the given software (such as ASU's MAT 275). Near the end of this Preface, we give some sample course outlines that will help show the independence of various sections and chapters. The focus of the text is on applications and methods of solution, both analytical and numerical, with emphasis on methods used in the typical engineering, physics, or mathematics student's field of study. We have tried to provide sufficient problems of a mathematical nature at the end of each section so that even the pure math major will be sufficiently challenged.

Key Features

This second edition of the book keeps many of the key features from the first edition:

- MATLAB, Maple, and *Mathematica* are incorporated at the end of each chapter, helping students with pages of tedious algebra and many of the differential equations topics; the goal of the software is still to show students how to make informed use of the relevant software in the field; all three software packages have parallel code and exercises;
- There are numerous problems of varying difficulty for both the applied and pure math major, as well as problems for the nonmathematician (engineers, etc.);
- An appendix that gives the reader a "crash course" in the three software packages; no prior knowledge is assumed;
- Chapter reviews at the end of each chapter to help the students review;
- Projects at the end of each chapter that go into detail about certain topics and sometimes introduce new topics that the students are now ready to see;
- Answers to most of the odd problems in the back of the book;

[1]MATLAB is a registered trademark of The MathWorks, Inc. For product information, please contact:
The Mathworks, Inc.
3 Apple Hill Drive
Natick, MA, 01760-2098 USA
Tel: 508-647-7000
Fax: 508-647-7001
E-mail: info@mathworks.com
Web: www.mathworks.com

- An appendix on linear algebra to supplement the treatment within the text, should it be appropriate for the reader/course;
- A full solutions manual for the qualified instructor.

It also incorporates new features, many of which have been suggested by professors and students who have taught/learned from the first edition:

- The computer codes are moved to the end of each chapter as **Computer Labs** to facilitate reading of the book by students and professors who either choose not to use the technology or who do not have access to it immediately;
- The latest software versions are used; significant changes have occurred in certain aspects of MATLAB, Maple, and *Mathematica* since the first edition in 2006, and the relevant changes are incorporated;
- Much of the linear algebra discussion has been moved to Chapter 5 (from Chapter 3), which deals with linear systems;
- Sections have been added on complex variables (Chapter 3), the exponential response formula for solving nonhomogeneous equations (Chapter 4), forced vibrations (Chapter 4) as well as a subsection on nondimensionalization (Chapter 2), and a combining of the sections on Euler and Runge-Kutta methods (Chapter 2);
- Many rewritten sections highlight applications and modeling within many fields;
- Exercises flow from easiest to hardest;
- Color graphs to help the reader better understand crucial concepts in ordinary differential equations;
- Updated and extended projects at the end of each chapter to reflect changes within the chapters.

Approaches to Teaching Differential Equations

The second edition of this book has evolved with our understanding of how to teach the material in the best possible way. Some notable examples from the above list:

1. The structure of the course in covering linear systems in their entirety before covering applications to nonlinear systems (phase plane, etc.) was a direct result of numerous conversations with MIT's Professor Haynes Miller (who frequently teaches MIT's 18.03) as was the incorporation of the new sections on essential topics from complex variables, exponential response, and complex replacement (developed by Haynes) for solving nonhomogeneous differential equations, and the s-domain and poles as an important use of Laplace transforms by engineers.
2. Combining the computer codes into Computer Labs at the end of each section rather than having snippets of code embedded throughout the text was a direct result of a switch in ASU's method of teaching this course. Setting aside six class periods for such labs is the way differential equations is now taught at ASU.
3. The presentation of essential linear algebra topics to aid in the understanding of differential equations was helped by discussions with MIT's Professor Gil Strang as well as seeing some of his lectures firsthand.

Most differential equations we have encountered in practice have needed analytical approximations or numerical approximations to gain insight into their behavior. We don't feel that students use technology wisely if they simply ask the computer to solve a given problem. We thus focus on what we consider to be the basics necessary for adequately preparing a student for study in her or his respective fields, including mathematics. We present the syntax from MATLAB, Maple, and *Mathematica* in Computer Labs at the end of each chapter. We feel that this provides the readers a better understanding of the theory and allows them to gain more insight into real-world problems they are likely to encounter. The vast majority of our students also have *no previous experience with* MATLAB, Maple, or *Mathematica* and we start from the basics and teach informed use of the relevant mathematical software. The student whom we "typically encounter" has had one year of calculus and is usually a major in a field other than pure mathematics.

Our book is traditional in its approach and coverage of basic topics in ordinary differential equations. However, we cover a number of "modern" topics such as direction fields, phase lines, the Runge–Kutta method, and nondimensionalization in Chapter 2 and epidemiological and ecological models in Chapter 6. As mentioned earlier, we also bring elements from linear algebra, such as eigenvectors, bases, and transformations, in order to best equip the reader of the book with a solid understanding of the material. Besides the Computer Labs there are also Projects at the end of each chapter that give useful insight into past and future topics covered in the book. The topics covered in these projects include a mix of traditional, modeling, numerical, and linear algebra aspects of ordinary differential equations. It is the intent that students who study this book and work *most* of the problems contained in these pages will be very prepared to continue their studies in engineering and mathematics.

Some Sample Course Outlines

While we could not begin to prescribe how this book may best be used for each school, we include some *possible* sections covered for various course outlines. There is sufficient material for a two-quarter or two-semester course sequence involving ordinary differential equations and partial differential equations with or without an emphasis on linear algebra that would utilize most of the book. We stress that if you intend to incorporate MATLAB, Maple, or *Mathematica* into your course, it is crucial to assign Exercises 1–4 (plus a few others) from Appendix A and the Chapter 1 Computer Lab early in the course. Appendix A requires only a knowledge of college algebra and some calculus (Taylor series), while Chapter 1 Computer Lab requires knowledge of calculus as it is applied to differential equations. Thus both can be assigned within the first 2 weeks of the course (and likely together).

Traditional semester ODE course:

Chap. 1	Chap. 2	Chap. 3	Chap. 4	Chap. 5	Chap. 7	Chap. 8
1.1–1.6	2.1–2.2	3.1–3.3	4.1, 4.3	5.1	7.1–7.4	8.1–8.5
		3.5–3.6	4.5–4.6	5.4–5.8		

Semester ODE course with modeling or application emphasis:

Chap. 1	Chap. 2	Chap. 3	Chap. 4	Chap. 5	Chap. 6	Chap. 7
1.1–1.4	2.1–2.6	3.1–3.2	4.1–4.2	5.1, 5.4	6.1–6.5	7.1–7.5
		3.4–3.7	4.4–4.7	5.5, 5.7		

Semester DE course with PDEs instead of Laplace Transforms:

Chap. 1	Chap. 2	Chap. 3	Chap. 4	Chap. 5	Chap. 6	Chap. 10
1.1–1.6	2.1–2.2	3.1–3.3	4.1, 4.3	5.1	6.1	10.1–10.6
		3.5–3.6	4.5–4.6	5.4–5.8		

Semester ODE course with linear algebra emphasis and no separate computer labs:

Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6	Ch. 7	App. B
1.1–1.4	2.1–2.2	3.1–3.2	4.1–4.2	5.1–5.5	6.1	7.1–7.7	B.1–B.4
	2.5	3.4–3.7	4.4, 4.7	5.7–5.8			

Semester DE course with linear algebra emphasis and no separate computer labs:

Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6	Ch. 10	App. B
1.1–1.4	2.1–2.2	3.1–3.2	4.1–4.2	5.1–5.5	6.1	10.1–10.6	B.1–B.4
	2.5	3.4–3.7	4.4, 4.7	5.7–5.8			

Semester ODE course with linear algebra emphasis and 6 computer labs:

Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 7	Comp. Labs
1.1–1.4	2.1–2.2	3.1–3.2	4.1–4.2	5.1–5.5	7.1–7.6	A & 1, 2,
	2.5	3.4–3.7	4.4, 4.7			3, 4, 5 & B, 7

Quarter ODE course with linear algebra emphasis:

Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	App. B
1.1–1.4	2.1–2.2	3.1–3.2	4.1–4.2	5.1–5.5	B.1–B.4
	2.5	3.4–3.7	4.7		

Acknowledgments

Students, with their questions both in class and during office hours, helped shaped this second edition as did those professors who used the first edition and/or provided constructive feedback to us, including Erika Camacho, Andrew Knyazev, Luis Melara, Jenny Switkes, Karen Vaughn, Steven Weintraub, and many others. Various chapters were read by Jonathan Burkow, Alexandra Churikova, Maytee Cruz-Aponte, Clay Goggil, Chris Graham, Christine Sowa, and Kathryn Stefanko, and their feedback has been of great help. Mike Pappas, in particular, was a big help in proofreading near-final drafts of several chapters. Valerie Cheathon provided a valuable check of all the codes as did Joshua Grosso (MATLAB) and Alan Wirkus-Camacho (Maple and Mathematica). Scott Wilde, again, provided invaluable help in revising the solutions manual.

As texts based upon lecture notes seemingly develop, many of the examples, exercises, and projects have been collected over many years for various courses taught by the authors. Some were taken from others' textbooks and papers. We have tried to give proper credit throughout this text; however, it was not always possible to properly acknowledge the original sources. It is our hope that we repay this explicit debt to earlier writers by contributing our (and their) ideas to further student understanding of differential equations.

We particularly wish to thank our project editor, Karen Simon, the production coordinator for the ODE version of the book, Jessica Vakili, as well as Michele Dimont, Amy Blalock, Hayley Ruggieri, and Sherry Thomas. Bob Stern and Bob Ross, our editors at Chapman & Hall/CRC Press, both deserve a big thanks for believing in this project and for helpful guidance, advice, and patience. We sincerely thank all these individuals; without their assistance, this text would not have succeeded.

URL for typos and errata:

`http://www.public.asu.edu/~swirkus/ACourseInDEs`

Finally, we would appreciate any comments that you might have regarding this book.

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From the first edition:

We owe a very special thanks to Erika Camacho (Arizona State University) for her help in writing the MATLAB and Maple code for this book and for detailed suggestions on numerous sections. John Fay and Gary Etgen reviewed earlier drafts of this text and provided helpful feedback. Scott Wilde provided valuable assistance in writing and preparing the solutions manual for the book. We owe a big thanks to our former students David Monarres, for help in preparing portions of this book, and Walter Sosa and Moore Chung, for their help in preparing solutions. We would also like to acknowledge our Cal Poly Pomona colleagues Michael Green, Jack Hofer, Tracy McDonald, Jim McKinney, Siew-Ching Pye, Dick Robertson, Paul Salomaa, Jenny Switkes, Karen Vaughn, and Mason Porter (Caltech/Oxford) for their willingness to use draft versions of this text in their courses and their important suggestions, which improved the overall readability of the text. The faculty and students of AMSSI and MTBI also deserve a special thanks for comments on early drafts of the computer code. Mary Jane Hill assisted us with certain aspects of the text and helped in typesetting some of the chapters of the initial drafts of the book; her effort is greatly appreciated. The production and support staff at Chapman & Hall/CRC Press have been very helpful. We particularly wish to thank our project coordinator, Theresa Del Forn and project editor, Prudence Board. Our editor Bob Stern deserves a special thanks for believing in this project and for his guidance, advice, and patience. We sincerely thank all these individuals; without their assistance this text would not have succeeded.

A few remarks for students and professors:

This book will succeed if any fears and reservations about learning one of the three computer algebra systems used in this book are put aside. Computers are not here to supplant us, but rather they are here to help illustrate and illuminate concepts and insights that we have. Nothing is foolproof and we stress the importance of *informed use of the relevant mathematical software.* Numerical answers, although quite accurate most of the time, should always be examined carefully because computers are as smart as the programmer allows them to be. There should never be a blind trust in an answer.

It is essential that the technology that you choose—MATLAB, Maple, or Mathematica—be introduced early in the class, just as it is introduced early in the book. While certain mathematical software packages may be better suited for studying differential equations, none have the versatility that the above three programs have to give insight into other areas of mathematics. The two keys to learning the programs are (1) learning the syntax and (2) learning to use the help menus to figure out some of the commands. Setting aside one class, for example, to give a brief tutorial on one of these software packages in the computer lab is a very worthwhile investment. It is by no means necessary and the typical student will be able to learn the material on his/her own by carefully following Appendix A. For reinforcement, it is crucial to include at least one or two technology problems with each homework assignment. The conscientious student will be well prepared to use the same software package in any upper division course in *any* branch of the mathematical sciences and its applications.

It is not necessary to bring computer demonstrations into the classroom. Both authors have taught their courses successfully without classroom demonstrations; handouts sometimes are useful, especially from the appendices. The students, for better or worse, are generally far less afraid of technology than one might expect. If students are sent to the computer lab with an assignment to do and aided with Appendix A, the vast majority will come back with satisfactory answers. Yes, you may bang your head against your desk in frustration at times, but just ask the person next to you for help and also seek the help menus and you will be able to learn MATLAB, Maple, and *Mathematica* quite well.

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