Real Analysis

Modern Techniques and Their Applications

实分析

第 2 版

Second Edition

Gerald B. Folland

Pure and Applied Mathematics:
A Wiley-Interscience Series Series of Texts, Monographs, and Tracts

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世界图书出版公司
www.wpcbj.com.cn

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A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.
New York / Chichester / Weinheim / Brisbane / Singapore / Toronto

图书在版编目（C I P）数据

实分析=Real Analysis: Modern Techniques and Their Applications: 英文 / （美）福伦德（Folland, G. B.）著.—北京：世界图书出版公司北京公司，2007.5
ISBN 978-7-5062-8275-8

I. 实… II. 福… III. 实分析—英文 IV. 0174.1

中国版本图书馆CIP数据核字（2007）第060863号

书　　名:	Real Analysis: Modern Techniques and Their Applications 2nd ed.
作　　者:	Gerald B. Folland
中 译 名:	实分析 第 2 版
责任编辑:	焦小浣
出 版 者:	世界图书出版公司北京公司
印 刷 者:	北京世图印刷厂
发　　行:	世界图书出版公司北京公司 (北京朝内大街 137 号 100010)
联系电话:	010-64015659, 64038348
电子信箱:	kjsk@vip.sina.com
开　　本:	24 开
印　　张:	17
版　　次:	2007 年 5 月第 1 次印刷
版权登记:	图字:01-2007-1466
书　　号:	978-7-5062-8275-8/ O · 578　　定　价: 38.00 元

To the memory of my mother and father
Helen B. Folland
and
Harold F. Folland

影印版前言

Gerald B. Folland 现任美国华盛顿大学西雅图分校数学系教授。早年师从分析大师 E. M. Stein，在调和分析、复分析、微分方程等领域都有着杰出的研究成果。他的著作《相空间中的分析》、《抽象调和分析》、《实分析》等一直是国内外数学专业以及相关专业研究生的重要参考书籍。这些书的内容丰富且可读性很强。

作为理工类研究生的专业课目，实分析一直是我国研究生教育的一个重要内容。它的内容既要深刻体现作为研究基础的数学特别是分析数学的主要思想，同时又要使学生能够在很短的时间内对这些思想加以掌握并可以运用到自己的研究和学习中去，因此，使用一本好的教材是十分必要的。本书是 Folland 教授的名著《实分析》的第 2 版，第 1 版长期以来一直是国内外众多大学实分析课程的参考教材，深受国内外同行的好评。与第 1 版相比，作者在第 2 版中作了以下几方面的修改：

重新编排了 n 维 Lebesgue 空间并对它的内容进行了扩充。

给出了 Tychonoff 定理的一个简单证明。

将傅里叶分析的内容分解为两章并加入了傅里叶级数收敛的 Dirichlet—Jordan 定理。

增加了关于分布函数和微分方程的章节。

增加了 Hausdorff 维数和自相似的内容，同时去掉了已经过时的关于 Cantor 集的 Hausdorff 维数的计算。

作为一部优秀的教材，本书不仅涵盖了分析学的基本内容和技巧，还介绍了一些从事其他领域的研究工作所必需的基础知识。此外，教材中的大量习题，能够进一步拓展思维，从而易于读者更加深入地了解这些内容背后的真实想法。在每一章的注记中，作者对正文里没有提及或没有深入展开的内容都列出了详实的参考文献，通过这些注记，读者可以对自己感兴趣的问题展开进一步的研究。

《实分析》第 2 版的影印出版，必将推动我国实分析课程教学的发展。我们期望，将来能够有更多像《实分析》这样的国外名著在国内出版，这会对我国的数学研究以及数学教育事业的进步起到巨大的推动作用。

苗长兴
北京应用物理与计算数学研究所
2007 年 5 月

前　言

The name "real analysis" is something of an anachronism. Originally applied to the theory of functions of a real variable, it has come to encompass several subjects of a more general and abstract nature that underlie much of modern analysis. These general theories and their applications are the subject of this book, which is intended primarily as a text for a graduate-level analysis course. Chapters 1 through 7 are devoted to the core material from measure and integration theory, point set topology, and functional analysis that is a part of most graduate curricula in mathematics, together with a few related but less standard items with which I think all analysts should be acquainted. The last four chapters contain a variety of topics that are meant to introduce some of the other branches of analysis and to illustrate the uses of the preceding material. I believe these topics are all interesting and important, but their selection in preference to others is largely a matter of personal predilection.

The things one needs to know in order to read this book are as follows:

1. First and foremost, the classical theory of functions of a real variable: limits and continuity, differentiation and (Riemann) integration, infinite series, uniform convergence, and the notion of a metric space.

2. The arithmetic of complex numbers and the basic properties of the complex exponential function $e^{x+iy} = e^x(\cos y + i \sin y)$. (More advanced results from complex function theory are used only in the proof of the Riesz-Thorin theorem and in a few exercises and remarks.)

3. Some elementary set theory.

4. A bit of linear algebra — actually, not much beyond the definitions of vector spaces, linear mappings, and determinants.

All of the necessary material in (1) and (2) can be found in W. Rudin's classic *Principles of Mathematical Analysis* (3rd ed., McGraw-Hill, 1976) or its descendants such as R. S. Strichatrz's *The Way of Analysis* (Jones and Bartlett, 1995) or S. G. Krantz's *Real Analysis and Foundations* (CRC Press, 1991). A summary of the relevant facts about sets and metric spaces is provided here in Chapter 0. The reader should begin this book by examining §0.1 and §0.5 to become familiar with my notation and terminology; the rest of Chapter 0 can then be referred to as needed.

Each chapter concludes with a section entitled "Notes and References." These sections contain miscellaneous remarks, acknowledgments of sources, indications of results not discussed in the text, references for further reading, and historical notes. The latter are quite sketchy, although references to more detailed sources are provided; they are intended mainly to give an idea of how the subject grew out of its classical origins. I found it entertaining and instructive to read some of the original papers, and I hope to encourage others to do the same.

A sizable portion of this book is devoted to exercises. They are mostly in the form of assertions to be proved, and they range from trivial to difficult; hints and intermediate steps are provided for the more complicated ones. Every reader should peruse them, although only the most ambitious will try to work them all out. They serve several purposes: amplification of results and completion of proofs in the text, discussion of examples and counterexamples, applications of theorems, and development of further ideas. Instructors will probably wish to do some of the exercises in class; to maximize flexibility and minimize verbosity, I have followed the principle of "When in doubt, leave it as an exercise," especially with regard to examples. Exercises occur at the end of each section, but they are numbered consecutively within each chapter. In referring to them, "Exercise n" means the nth exercise in the present chapter unless another section is explicitly mentioned.

The topics in the book are arranged so as to allow some flexibility of presentation. For example, Chapters 4 and 5 do not depend on Chapters 1–3 except for a few examples and exercises. On the other hand, if one wishes to proceed quickly to L^p theory, one can skip from §3.3 to §§5.1–2 and thence to Chapter 6. Chapters 10 and 11 are independent of Chapters 8 and 9 except that the ideas in §8.6 are used in Chapter 10.

The new features of this edition are as follows:

- The material on the n-dimensional Lebesgue integral (§§2.6–7) has been rearranged and expanded.

- Tychonoff's theorem (§4.6) is proved by an elegant argument recently discovered by Paul Chernoff.

- The chapter on Fourier analysis has been split into two chapters (8 and 9). The material on Fourier series and integrals (§§8.3–5) has been rearranged and now contains the Dirichlet-Jordan theorem on convergence of Fourier series.

The material on distributions (§§9.1–2) has been extensively rewritten and expanded.

- A section on self-similarity and Hausdorff dimension (§11.3) has been added, replacing the outdated calculation of the Hausdorff dimension of Cantor sets in the old §10.2.
- Innumerable small changes have been made in the hope of improving the exposition.

The writer of a text on such a well-developed subject as real analysis must necessarily be indebted to his predecessors. I kept a large supply of books on hand while writing this one; they are too numerous to list here, but most of them can be found in the bibliography. I am also happy to acknowledge the influence of two of my teachers: the late Lynn Loomis, from whose lectures I first learned this subject, and Elias Stein, who has done much to shape my point of view. Finally, I am grateful to a number of people — especially Steven Krantz, Kenneth Ross, and William Faris — whose comments and corrigenda concerning the first edition have helped me to prepare the new one.

GERALD B. FOLLAND

Seattle, Washington

This book is printed on acid-free paper. ♾

Published simultaneously in Canada.

Library of Congress Cataloging-in-Publication Data:

Folland, Gerald B.
Real analysis : modern techniques and their applications / Gerald B. Folland. — 2nd ed.
p. cm. — (Pure and applied mathematics)
"A Wiley-Interscience publication."
Includes bibliographical references and index.
ISBN 0-471-31716-0 (cloth : alk. paper)
1. Mathematical analysis. 2. Functions of real variables.
I. Title. II. Series: Pure and applied mathematics (John Wiley & Sons : Unnumbered)
QA300.F67 1999
515—dc21 98-37260

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