NONLINEAR PHYSICAL SCIENCE

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Marat Akhmet
Mehmet Onur Fen

Replication of Chaos in Neural Networks, Economics and Physics

神经网络、经济学和物理中的混沌复制

HIGHER EDUCATION PRESS

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NONLINEAR PHYSICAL SCIENCE

非线性物理科学

NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences

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To our beloved families

Preface

The main novelty of this book is the consideration of chaos as an *input* for differential and hybrid equations. More precisely, we insert chaos on the right-hand side of the equations and investigate the results of perturbation. Moreover, we investigate many possible consequences of the input–output analysis in systems with many compartments. This is what makes our book on chaos unique among all others.

Let us give some arguments toward the importance of the input–output analysis of chaos for both theory and applications:

1. In the theory of dynamical systems, a large number of results use the input–output analysis. For example, there are many theorems that can be loosely formulated as follows: if a perturbation is periodic (bounded, almost periodic), then there is a unique periodic (bounded, almost periodic) solution. Generally speaking, our results can be formulated in the following way: if a perturbation is chaotic, then there exists a chaos in the set of solutions. Thus, one can say that our main proposal is to return investigation of chaos into the mainstream of classical differential/difference equations theory and, consequently, a huge number of rigorous mathematical methods, numerical instruments, and applications that rely on the input–output analysis will be involved for the investigation of chaotic processes.
2. Despite the fact that many distinguished specialists in the chaos theory and mathematics have been involved in the investigation, there are still many challenging problems related to the origin of the chaos theory. For instance, we do not have a rigorously approved chaos in Lorenz systems, Duffing equations, and other systems. Moreover, there is no universal method to detect chaos in multidimensional systems. Hopefully, the input–output analysis will give new opportunities for the analyses of the basic models and help to unify the knowledge of chaos. We believe that the exploitation of the mechanism in the considered models can give mathematical clarity there.

3. The input–output analysis can become a strong instrument in applications to real-world problems through the modeling of chaos expansion. We hope that unpredictability of weather, economical unpredictability, and irregularity as a global phenomenon will be reflected in mathematical investigations more comprehensively through this machinery. This is true not only for atmospheric or economic processes, but also for any large systems in biology, neural networks, and computer sciences. Utilization of the input–output analysis in cryptography and deciphering may also give effective results. The input–output analysis is very popular, for instance, in mechanics, chemistry, biology, cryptography, etc. Consequently, one can suppose that what we have suggested has to be realized for real-world problems of various natures.
4. We describe the expansion of chaos on the basis of the input–output mechanism using the concept of *morphogenesis* to emphasize that the expansion keeps the geometrical properties of chaos. Furthermore, it is not surprising that the replication of chaos, introduced in the book, relates to concepts of science with broad applications: *self-organization, synergetics, chaos-order relations, thermodynamics, biological patterns*.

The book is attractive in the mathematical sense, since we have introduced rigorous description of chaos for systems with continuous time for the first time. This may give a push for the functional analysis of chaos to involve the operator theory results, etc. Hopefully, our approach will give a basis for deeper comprehension and the possibility to unite different appearances of chaos. In this framework, we also hope that the results can be developed for partial differential equations, integro-differential equations, functional differential equations, evolution systems, etc.

A part of the book is devoted to problems of economics. We have analyzed chaos extension in economic models. Unpredictability in economics as sensitivity in dynamical models is considered, and on that basis, global extension of unpredictability is discussed.

The presence of chaos in neural networks is indispensable, and as applications of our results, replication of chaos by neural networks is presented in a separate chapter in this book.

We pay great attention to expansion of chaos through Lorenz models in meteorology. A special mathematical analysis has been made, since only dissipativeness property of a system is used to prove the chaos presence in perturbed systems.

Entrainment of limit cycles by chaos is discovered numerically through specially designed unidirectional coupling of two glow discharge-semiconductor systems. The result demonstrates that the input–output machinery is working effectively for partial differential equations. Chaotic control is through the external circuit equation and governs the electrical potential on the boundary. The expandability of the theory to collectives of glow discharge systems is discussed, and this increases the potential of applications of the results.

The content of the book is a good background for applications in mechanics, biology, molecular biology, physiology, pharmacology, secure communications, neural networks, and other real-world problems involving complex behavior of models. Since chaos is present everywhere, we can say that our results are applicable in any field, where differential and difference equations are utilized as models.

The authors would like to express their gratitude to those who contributed to the preparation of this book, Zhanar Akhmetova and Ismail Rafatov for the joined results, the Series Editor Prof. Albert Luo and Editor of HEP Liping Wang for their interest in the monograph and patience during the publication of the book.

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Chapter 1
Introduction

The theory of dynamical systems starts with H. Poincaré, who studied nonlinear differential equations by introducing qualitative techniques to discuss the global properties of solutions [1]. His discovery of the homoclinic orbits figures prominently in the studies of chaotic dynamical systems. Poincaré first encountered the presence of homoclinic orbits in the three-body problem of celestial mechanics [2]. A Poincaré homoclinic orbit is an orbit of intersection of the stable and unstable manifolds of a saddle periodic orbit. It is called structurally stable if the intersection is transverse, and structurally unstable or a homoclinic tangency if the invariant manifolds are tangent along the orbit [3]. In any neighborhood of a structurally stable Poincaré homoclinic orbit, there exist nontrivial hyperbolic sets containing a countable number of saddle periodic orbits and continuum of non-periodic Poisson stable orbits [3–5]. For this reason, the presence of a structurally stable Poincaré homoclinic orbit can be considered as a criterion for the presence of complex dynamics [3].

The first mathematically rigorous definition of chaos is introduced by Li and Yorke [6] for one-dimensional difference equations. According to [6], a continuous map $F : J \to J$, where $J \subset \mathbb{R}$ is an interval, exhibits chaos if: (i) For every natural number p, there exists a $p-$periodic point of F in J; (ii) There is an uncountable set $S \subset J$ containing no periodic points such that for every $s_1, s_2 \in S$ with $s_1 \neq s_2$ we have $\limsup_{k\to\infty} \left|F^k(s_1) - F^k(s_2)\right| > 0$ and $\liminf_{k\to\infty} \left|F^k(s_1) - F^k(s_2)\right| = 0$; (iii) For every $s \in S$ and periodic point $\sigma \in J$ we have $\limsup_{k\to\infty} \left|F^k(s) - F^k(\sigma)\right| > 0$. In the paper [6], it was proved that if a map on an interval has a point of period three, then it is chaotic.

Generalizations of Li-Yorke chaos to high-dimensional difference equations were provided in [7–10]. According to Marotto [10], if a repelling fixed point of a differentiable map has an associated homoclinic orbit that is transversal in some sense, then the map must exhibit chaotic behavior. More precisely, if a multidimensional differentiable map has a snap-back repeller, then it is chaotic. In the paper [9], Marotto's Theorem was used to prove rigorously the existence of Li-Yorke chaos in a spatiotemporal chaotic system. Furthermore, the notion of Li-Yorke sensitivity, which links the Li-Yorke chaos with the notion of sensitivity, was studied in [7], and generalizations of Li-Yorke chaos to mappings in Banach spaces and complete metric spaces were considered in [8].

Another mathematical definition of chaos for discrete-time dynamics was introduced by Devaney [1]. It is mentioned in [1] that a map $F : J \to J$, where $J \subset \mathbb{R}$ is an interval, has sensitive dependence on initial conditions if there exists $\delta > 0$ such that for any $x \in J$ and any neighborhood N of x there exists $y \in J$ and a positive integer k such that $\left|F^k(x) - F^k(y)\right| > \delta$. On the other hand, F is said to be topologically transitive if for any pair of open sets $U, V \subset J$ there exists a positive integer k such that $F^k(U) \cap V \neq \emptyset$. According to Devaney, a map $F : J \to J$ is chaotic on J if: (i) F has sensitive dependence on initial conditions; (ii) F is topologically transitive; (iii) Periodic points of F are dense in J. In other words, a chaotic map possesses three ingredients: unpredictability, indecomposability, and an element of regularity.

Symbolic dynamics, whose earliest examples were constructed by Hadamard [11] and Morse [12], is one of the oldest techniques for the study of chaos. Symbolic dynamical systems are systems whose phase space consists of one-sided or two-sided infinite sequences of symbols chosen from a finite alphabet. Such dynamics arises in a variety of situations such as in horseshoe maps and the logistic map. The set of allowed sequences is invariant under the shift map, which is the most important ingredient in symbolic dynamics [1, 13–17]. Moreover, it is known that the symbolic dynamics admits the chaos in the sense of both Devaney and Li-Yorke [1, 18–21].

The Smale Horseshoe map is first studied by Smale [22] and it is an example of a diffeomorphism which is structurally stable and possesses a chaotic invariant set [1, 15, 17]. The horseshoe arises whenever one has transverse homoclinic orbits, as in the case of the Duffing equation [23]. People used the symbolic dynamics to discover chaos, but we suppose that it can serve as an "embryo" for the morphogenesis of chaos.

From the mathematical point of view, chaotic systems are characterized by local instability and uniform boundedness of the trajectories. Since local instability of a linear system implies unboundedness of its solutions, chaotic system should be necessarily nonlinear [24]. Chaos in dynamical systems is commonly associated with the notion of a strange attractor, which is an attractive limit set with a complicated structure of orbit behavior. This term was introduced by Ruelle and Takens [25] in the sense where the word strange means the limit set has a fractal structure [3]. The dynamics of chaotic systems are sensitive to small perturbations of initial conditions. This means that if we take two close but different points in the phase space and follow their evolution, then we see that the two phase trajectories starting from these points eventually diverge [1, 26]. The sensitive dependence on the initial condition is used both to stabilize the chaotic behavior in periodic orbits and to direct trajectories to a desired state [27].

It was Lorenz [28] who discovered that the dynamics of an infinite-dimensional system being reduced to three-dimensional equation can be next analyzed in its chaotic appearances by application of the simple unimodal one-dimensional map. Smale [22] explained that the geometry of the horseshoe map is underneath of the

Van der Pol equation's complex dynamics which was investigated by Cartwright and Littlewood [29] and later by Levinson [30]. Nowadays, the Smale horseshoes with its chaotic dynamics is one of the basic instruments when one tries to recognize a chaos in a process. Guckenheimer and Williams [31] gave a geometric description of the flow of Lorenz attractor to show the structural stability of codimension 2. In addition to this, it was found out that the topology of the Lorenz attractor is considerably more complicated than the topology of the horseshoe [23]. Moreover, Levi [32] used a geometric approach for a simplified version of the Van der pol equation to show existence of horseshoes embedded within the Van der Pol map and how the horseshoes fit in the phase plane.

It is natural *to discover a chaos* [6, 10, 25, 28, 33–42] and proceed by producing basic definitions and creating the theory. On the other hand, one can *shape* an irregular process by inserting chaotic elements in a system which has regular dynamics (let us say comprising an asymptotically stable equilibrium, a global attractor, etc.). This approach to the problem also deserves consideration as it may allow for a more rigorous treatment of the phenomenon, and helps develop new methods of investigation. Our results are of this type.

In this book, we use the idea that chaos can be utilized as input in systems of equations. To explain the input–output procedure realized in our book, let us introduce examples of systems called as *the base-system, the replicator*, and *the generator*, which will be intensively used in the manuscript. Consider the following system of differential equations,

$$\frac{dz}{dt} = B(z). \tag{1.1}$$

The system (1.1) is called *the base-system*. We assume that the system admits a regular property. For example, there is a globally asymptotically stable equilibrium of (1.1). Next we apply to the system a perturbation, $I(t)$, which will be called an *input* and obtain the following system,

$$\frac{dy}{dt} = B(y) + I(t), \tag{1.2}$$

which will be called as *the replicator*.

Suppose that the input I admits a certain property, let us say, it is a bounded function. We assume then that there exists a unique solution, $y(t)$, of the last equation, the replicator, with the same property of boundedness. This solution is considered as an *output*. The process for obtaining the solution $y(t)$ of the replicator system by applying perturbation $I(t)$ to the base-system (1.1) is called the *input–output mechanism*, and sometimes we shall call it the *machinery*. It is known that for certain base-systems, if the input is periodic, almost periodic, bounded, then there exists an output, which is also periodic, almost periodic, bounded, respectively. In our book, we consider inputs of the new nature: chaotic sets and chaotic functions. The motions which are in the chaotic attractor of the Lorenz system considered altogether provide