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Steven E. Shreve

Stochastic Calculus for Finance I

The Binomial Asset Pricing
Model

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Introduction

Background

By awarding Harry Markowitz, William Sharpe, and Merton Miller the 1990 Nobel Prize in Economics, the Nobel Prize Committee brought to worldwide attention the fact that the previous forty years had seen the emergence of a new scientific discipline, the "theory of finance." This theory attempts to understand how financial markets work, how to make them more efficient, and how they should be regulated. It explains and enhances the important role these markets play in capital allocation and risk reduction to facilitate economic activity. Without losing its application to practical aspects of trading and regulation, the theory of finance has become increasingly mathematical, to the point that problems in finance are now driving research in mathematics.

Harry Markowitz's 1952 Ph.D. thesis *Portfolio Selection* laid the groundwork for the mathematical theory of finance. Markowitz developed a notion of mean return and covariances for common stocks that allowed him to quantify the concept of "diversification" in a market. He showed how to compute the mean return and variance for a given portfolio and argued that investors should hold only those portfolios whose variance is minimal among all portfolios with a given mean return. Although the language of finance now involves stochastic (Itô) calculus, management of risk in a quantifiable manner is the underlying theme of the modern theory and practice of quantitative finance.

In 1969, Robert Merton introduced stochastic calculus into the study of finance. Merton was motivated by the desire to understand how prices are set in financial markets, which is the classical economics question of "equilibrium," and in later papers he used the machinery of stochastic calculus to begin investigation of this issue.

At the same time as Merton's work and with Merton's assistance, Fischer Black and Myron Scholes were developing their celebrated option pricing formula. This work won the 1997 Nobel Prize in Economics. It provided a satisfying solution to an important practical problem, that of finding a fair price for a European call option (i.e., the right to buy one share of a given

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stock at a specified price and time). In the period 1979–1983, Harrison, Kreps, and Pliska used the general theory of continuous-time stochastic processes to put the Black-Scholes option-pricing formula on a solid theoretical basis, and, as a result, showed how to price numerous other “derivative” securities.

Many of the theoretical developments in finance have found immediate application in financial markets. To understand how they are applied, we digress for a moment on the role of financial institutions. A principal function of a nation’s financial institutions is to act as a risk-reducing intermediary among customers engaged in production. For example, the insurance industry pools premiums of many customers and must pay off only the few who actually incur losses. But risk arises in situations for which pooled-premium insurance is unavailable. For instance, as a hedge against higher fuel costs, an airline may want to buy a security whose value will rise if oil prices rise. But who wants to sell such a security? The role of a financial institution is to design such a security, determine a “fair” price for it, and sell it to airlines. The security thus sold is usually “derivative” (i.e., its value is based on the value of other, identified securities). “Fair” in this context means that the financial institution earns just enough from selling the security to enable it to trade in other securities whose relation with oil prices is such that, if oil prices do indeed rise, the firm can pay off its increased obligation to the airlines. An “efficient” market is one in which risk-hedging securities are widely available at “fair” prices.

The Black-Scholes option pricing formula provided, for the first time, a theoretical method of fairly pricing a risk-hedging security. If an investment bank offers a derivative security at a price that is higher than “fair,” it may be underbid. If it offers the security at less than the “fair” price, it runs the risk of substantial loss. This makes the bank reluctant to offer many of the derivative securities that would contribute to market efficiency. In particular, the bank only wants to offer derivative securities whose “fair” price can be determined in advance. Furthermore, if the bank sells such a security, it must then address the hedging problem: how should it manage the risk associated with its new position? The mathematical theory growing out of the Black-Scholes option pricing formula provides solutions for both the pricing and hedging problems. It thus has enabled the creation of a host of specialized derivative securities. This theory is the subject of this text.

Relationship between Volumes I and II

Volume II treats the continuous-time theory of stochastic calculus within the context of finance applications. The presentation of this theory is the *raison d’être* of this work. Volume II includes a self-contained treatment of the probability theory needed for stochastic calculus, including Brownian motion and its properties.

Volume I presents many of the same finance applications, but within the simpler context of the discrete-time binomial model. It prepares the reader for Volume II by treating several fundamental concepts, including martingales, Markov processes, change of measure and risk-neutral pricing in this less technical setting. However, Volume II has a self-contained treatment of these topics, and strictly speaking, it is not necessary to read Volume I before reading Volume II. It is helpful in that the difficult concepts of Volume II are first seen in a simpler context in Volume I.

In the Carnegie Mellon Master's program in Computational Finance, the course based on Volume I is a prerequisite for the courses based on Volume II. However, graduate students in computer science, finance, mathematics, physics and statistics frequently take the courses based on Volume II without first taking the course based on Volume I.

The reader who begins with Volume II may use Volume I as a reference. As several concepts are presented in Volume II, reference is made to the analogous concepts in Volume I. The reader can at that point choose to read only Volume II or to refer to Volume I for a discussion of the concept at hand in a more transparent setting.

Summary of Volume I

Volume I presents the binomial asset pricing model. Although this model is interesting in its own right, and is often the paradigm of practice, here it is used primarily as a vehicle for introducing in a simple setting the concepts needed for the continuous-time theory of Volume II.

Chapter 1, *The Binomial No-Arbitrage Pricing Model*, presents the no-arbitrage method of option pricing in a binomial model. The mathematics is simple, but the profound concept of risk-neutral pricing introduced here is not. Chapter 2, *Probability Theory on Coin Toss Space*, formalizes the results of Chapter 1, using the notions of martingales and Markov processes. This chapter culminates with the risk-neutral pricing formula for European derivative securities. The tools used to derive this formula are not really required for the derivation in the binomial model, but we need these concepts in Volume II and therefore develop them in the simpler discrete-time setting of Volume I. Chapter 3, *State Prices*, discusses the change of measure associated with risk-neutral pricing of European derivative securities, again as a warm-up exercise for change of measure in continuous-time models. An interesting application developed here is to solve the problem of optimal (in the sense of expected utility maximization) investment in a binomial model. The ideas of Chapters 1 to 3 are essential to understanding the methodology of modern quantitative finance. They are developed again in Chapters 4 and 5 of Volume II.

The remaining three chapters of Volume I treat more specialized concepts. Chapter 4, *American Derivative Securities*, considers derivative securities whose owner can choose the exercise time. This topic is revisited in

a continuous-time context in Chapter 8 of Volume II. Chapter 5, *Random Walk*, explains the reflection principle for random walk. The analogous reflection principle for Brownian motion plays a prominent role in the derivation of pricing formulas for exotic options in Chapter 7 of Volume II. Finally, Chapter 6, *Interest-Rate-Dependent Assets*, considers models with random interest rates, examining the difference between forward and futures prices and introducing the concept of a forward measure. Forward and futures prices reappear at the end of Chapter 5 of Volume II. Forward measures for continuous-time models are developed in Chapter 9 of Volume II and used to create forward LIBOR models for interest rate movements in Chapter 10 of Volume II.

Summary of Volume II

Chapter 1, *General Probability Theory*, and Chapter 2, *Information and Conditioning*, of Volume II lay the measure-theoretic foundation for probability theory required for a treatment of continuous-time models. Chapter 1 presents probability spaces, Lebesgue integrals, and change of measure. Independence, conditional expectations, and properties of conditional expectations are introduced in Chapter 2. These chapters are used extensively throughout the text, but some readers, especially those with exposure to probability theory, may choose to skip this material at the outset, referring to it as needed.

Chapter 3, *Brownian Motion*, introduces Brownian motion and its properties. The most important of these for stochastic calculus is quadratic variation, presented in Section 3.4. All of this material is needed in order to proceed, except Sections 3.6 and 3.7, which are used only in Chapter 7, *Exotic Options* and Chapter 8, *Early Exercise*.

The core of Volume II is Chapter 4, *Stochastic Calculus*. Here the Itô integral is constructed and Itô's formula (called the Itô-Doeblin formula in this text) is developed. Several consequences of the Itô-Doeblin formula are worked out. One of these is the characterization of Brownian motion in terms of its quadratic variation (Lévy's theorem) and another is the Black-Scholes equation for a European call price (called the Black-Scholes-Merton equation in this text). The only material which the reader may omit is Section 4.7, *Brownian Bridge*. This topic is included because of its importance in Monte Carlo simulation, but it is not used elsewhere in the text.

Chapter 5, *Risk-Neutral Pricing*, states and proves Girsanov's Theorem, which underlies change of measure. This permits a systematic treatment of risk-neutral pricing and the Fundamental Theorems of Asset Pricing (Section 5.4). Section 5.5, *Dividend-Paying Stocks*, is not used elsewhere in the text. Section 5.6, *Forwards and Futures*, appears later in Section 9.4 and in some exercises.

Chapter 6, *Connections with Partial Differential Equations*, develops the connection between stochastic calculus and partial differential equations. This is used frequently in later chapters.

With the exceptions noted above, the material in Chapters 1–6 is fundamental for quantitative finance is essential for reading the later chapters. After Chapter 6, the reader has choices.

Chapter 7, *Exotic Options*, is not used in subsequent chapters, nor is Chapter 8, *Early Exercise*. Chapter 9, *Change of Numéraire*, plays an important role in Section 10.4, *Forward LIBOR model*, but is not otherwise used. Chapter 10, *Term Structure Models*, and Chapter 11, *Introduction to Jump Processes*, are not used elsewhere in the text.

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The Binomial No-Arbitrage Pricing Model

1.1 One-Period Binomial Model

The *binomial asset-pricing model* provides a powerful tool to understand *arbitrage pricing theory* and probability. In this chapter, we introduce this tool for the first purpose, and we take up the second in Chapter 2. In this section, we consider the simplest binomial model, the one with only one period. This is generalized to the more realistic multiperiod binomial model in the next section.

For the general one-period model of Figure 1.1.1, we call the beginning of the period *time zero* and the end of the period *time one*. At time zero, we have a stock whose price per share we denote by S_0 , a positive quantity known at time zero. At time one, the price per share of this stock will be one of two positive values, which we denote $S_1(H)$ and $S_1(T)$, the H and T standing for *head* and *tail*, respectively. Thus, we are imagining that a coin is tossed, and the outcome of the coin toss determines the price at time one. We do not assume this coin is fair (i.e., the probability of head need not be one-half). We assume only that the probability of head, which we call p , is positive, and the probability of tail, which is $q = 1 - p$, is also positive.

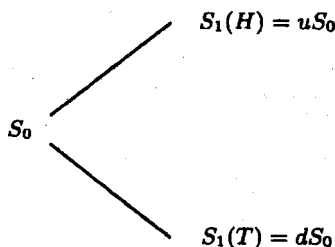


Fig. 1.1.1. General one-period binomial model.

The outcome of the coin toss, and hence the value which the stock price will take at time one, is known at time one but not at time zero. We shall refer to any quantity not known at time zero as *random* because it depends on the random experiment of tossing a coin.

We introduce the two positive numbers

$$u = \frac{S_1(H)}{S_0}, \quad d = \frac{S_1(T)}{S_0}. \quad (1.1.1)$$

We assume that $d < u$; if we instead had $d > u$, we may achieve $d < u$ by relabeling the sides of our coin. If $d = u$, the stock price at time one is not really random and the model is uninteresting. We refer to u as the *up factor* and d as the *down factor*. It is intuitively helpful to think of u as greater than one and to think of d as less than one, and hence the names *up factor* and *down factor*, but the mathematics we develop here does not require that these inequalities hold.

We introduce also an *interest rate* r . One dollar invested in the money market at time zero will yield $1 + r$ dollars at time one. Conversely, one dollar borrowed from the money market at time zero will result in a debt of $1 + r$ at time one. In particular, the interest rate for borrowing is the same as the interest rate for investing. It is almost always true that $r \geq 0$, and this is the case to keep in mind. However, the mathematics we develop requires only that $r > -1$.

An essential feature of an efficient market is that if a trading strategy can turn nothing into something, then it must also run the risk of loss. Otherwise, there would be an *arbitrage*. More specifically, we define *arbitrage* as a trading strategy that begins with no money, has zero probability of losing money, and has a positive probability of making money. A mathematical model that admits arbitrage cannot be used for analysis. Wealth can be generated from nothing in such a model, and the questions one would want the model to illuminate are provided with paradoxical answers by the model. Real markets sometimes exhibit arbitrage, but this is necessarily fleeting; as soon as someone discovers it, trading takes places that removes it.

In the one-period binomial model, to rule out arbitrage we must assume

$$0 < d < 1 + r < u. \quad (1.1.2)$$

The inequality $d > 0$ follows from the positivity of the stock prices and was already assumed. The two other inequalities in (1.1.2) follow from the absence of arbitrage, as we now explain. If $d \geq 1 + r$, one could begin with zero wealth and at time zero borrow from the money market in order to buy stock. Even in the worst case of a tail on the coin toss, the stock at time one will be worth enough to pay off the money market debt and has a positive probability of being worth strictly more since $u > d \geq 1 + r$. This provides an arbitrage. On the other hand, if $u \leq 1 + r$, one could sell the stock short and invest the proceeds in the money market. Even in the best case for the stock, the cost of

replacing it at time one will be less than or equal to the value of the money market investment, and since $d < u \leq 1 + r$, there is a positive probability that the cost of replacing the stock will be strictly less than the value of the money market investment. This again provides an arbitrage.

We have argued in the preceding paragraph that if there is to be no arbitrage in the market with the stock and the money market account, then we must have (1.1.2). The converse of this is also true. If (1.1.2) holds, then there is no arbitrage. See Exercise 1.1.

It is common to have $d = \frac{1}{u}$, and this will be the case in many of our examples. However, for the binomial asset-pricing model to make sense, we only need to assume (1.1.2).

Of course, stock price movements are much more complicated than indicated by the binomial asset-pricing model. We consider this simple model for three reasons. First of all, within this model, the concept of arbitrage pricing and its relation to risk-neutral pricing is clearly illuminated. Secondly, the model is used in practice because, with a sufficient number of periods, it provides a reasonably good, computationally tractable approximation to continuous-time models. Finally, within the binomial asset-pricing model, we can develop the theory of conditional expectations and martingales, which lies at the heart of continuous-time models.

Let us now consider a *European call option*, which confers on its owner the right but not the obligation to buy one share of the stock at time one for the *strike price* K . The interesting case, which we shall assume here, is that $S_1(T) < K < S_1(H)$. If we get a tail on the toss, the option expires worthless. If we get a head on the coin toss, the option can be *exercised* and yields a profit of $S_1(H) - K$. We summarize this situation by saying that the option at time one is worth $(S_1 - K)^+$, where the notation $(\dots)^+$ indicates that we take the maximum of the expression in parentheses and zero. Here we follow the usual custom in probability of omitting the argument of the random variable S_1 . The fundamental question of option pricing is how much the option is worth at time zero before we know whether the coin toss results in head or tail.

The *arbitrage pricing theory* approach to the option-pricing problem is to replicate the option by trading in the stock and money markets. We illustrate this with an example, and then we return to the general one-period binomial model.

Example 1.1.1. For the particular one-period model of Figure 1.1.2, let $S(0) = 4$, $u = 2$, $d = \frac{1}{2}$, and $r = \frac{1}{4}$. Then $S_1(H) = 8$ and $S_1(T) = 2$. Suppose the strike price of the European call option is $K = 5$. Suppose further that we begin with an initial wealth $X_0 = 1.20$ and buy $\Delta_0 = \frac{1}{2}$ shares of stock at time zero. Since stock costs 4 per share at time zero, we must use our initial wealth $X_0 = 1.20$ and borrow an additional 0.80 to do this. This leaves us with a cash position $X_0 - \Delta_0 S_0 = -0.80$ (i.e., a debt of 0.80 to the money market). At time one, our cash position will be $(1 + r)(X_0 - \Delta_0 S_0) = -1$

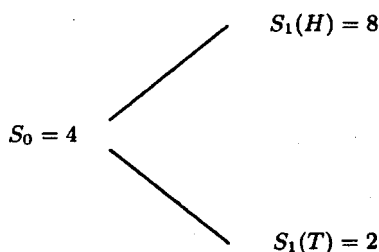


Fig. 1.1.2. Particular one-period binomial model.

(i.e., we will have a debt of 1 to the money market). On the other hand, at time one we will have stock valued at either $\frac{1}{2}S_1(H) = 4$ or $\frac{1}{2}S_1(T) = 1$. In particular, if the coin toss results in a head, the value of our portfolio of stock and money market account at time one will be

$$X_1(H) = \frac{1}{2}S_1(H) + (1+r)(X_0 - \Delta_0 S_0) = 3;$$

if the coin toss results in a tail, the value of our portfolio of stock and money market account at time one will be

$$X_1(T) = \frac{1}{2}S_1(T) + (1+r)(X_0 - \Delta_0 S_0) = 0.$$

In either case, the value of the portfolio agrees with the value of the option at time one, which is either $(S_1(H) - 5)^+ = 3$ or $(S_1(T) - 5)^+ = 0$. We have *replicated* the option by trading in the stock and money markets.

The initial wealth 1.20 needed to set up the replicating portfolio described above is the *no-arbitrage price of the option at time zero*. If one could sell the option for more than this, say, for 1.21, then the seller could invest the excess 0.01 in the money market and use the remaining 1.20 to replicate the option. At time one, the seller would be able to pay off the option, regardless of how the coin tossing turned out, and still have the 0.0125 resulting from the money market investment of the excess 0.01. This is an arbitrage because the seller of the option needs no money initially, and without risk of loss has 0.0125 at time one. On the other hand, if one could buy the option above for less than 1.20, say, for 1.19, then one should buy the option and set up the reverse of the replicating trading strategy described above. In particular, sell short one-half share of stock, which generates income 2. Use 1.19 to buy the option, put 0.80 in the money market, and in a separate money market account put the remaining 0.01. At time one, if there is a head, one needs 4 to replace the half-share of stock. The option bought at time zero is worth 3, and the 0.80 invested in the money market at time zero has grown to 1. At time one, if there is a tail, one needs 1 to replace the half-share of stock.