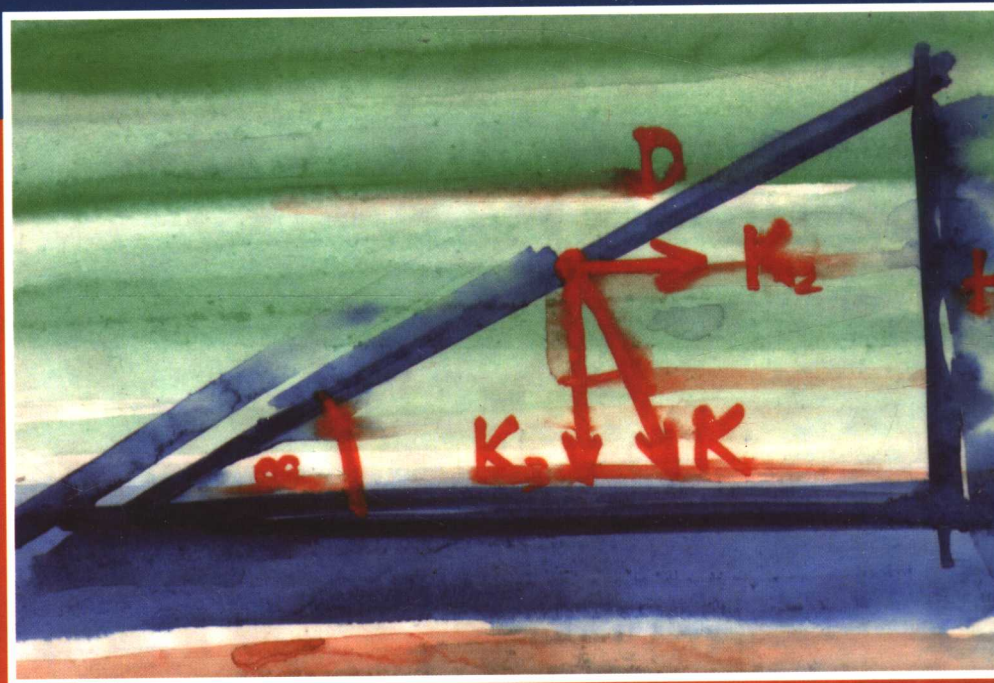


CLASSICAL THEORETICAL PHYSICS



Greiner

CLASSICAL MECHANICS

Systems of Particles and
Hamiltonian Dynamics

经典力学

粒子系和哈密顿动力学

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Walter Greiner

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Systems of Particles and
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Foreword by D. Allan Bromley

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Foreword

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics—with mathematics, the most fundamental of sciences—using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicians remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck, and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through the use of consistent notation, logical ordering of topics, and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

These volumes on classical physics, finally available in English, complement Greiner's texts on quantum physics, most of which have been available to English-speaking audiences for some time. The complete set of books will thus provide a coherent view of physics that includes, in classical physics, thermodynamics and statistical mechanics, classical dynamics, electromagnetism, and general relativity; and in quantum physics, quantum mechanics, symmetries, relativistic quantum mechanics, quantum electro- and chromodynamics, and the gauge theory of weak interactions.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that . . .," which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point, and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools

under study are applied to actual problems of interest to working physicists. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1789–1857) in his *Positive Philosophy* who noted, “To understand a science it is necessary to know its history.” This is all too often forgotten in modern physics teaching, and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner’s lectures, which underlie these volumes, are internationally noted for their clarity, for their completeness, and for the effort that he has devoted to making physics an integral whole. His enthusiasm for his sciences is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again, the reader will find that, after dipping into a particular volume to review a specific topic, he or she will end up browsing, caught up by often fascinating new insights and developments with which he or she had not previously been familiar.

Having used a number of Greiner’s volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

D. Allan Bromley
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New Haven, Connecticut, USA

Preface

Theoretical physics has become a many faceted science. For the young student, it is difficult enough to cope with the overwhelming amount of new material that has to be learned, let alone obtain an overview of the entire field, which ranges from mechanics through electrodynamics, quantum mechanics, field theory, nuclear and heavy-ion science, statistical mechanics, thermodynamics, and solid-state theory to elementary-particle physics; and this knowledge should be acquired in just eight to ten semesters, during which, in addition, a diploma or master's thesis has to be worked on or examinations prepared for. All this can be achieved only if the university teachers help to introduce the student to the new disciplines as early on as possible, in order to create interest and excitement that in turn set free essential new energy.

At the Johann Wolfgang Goethe University in Frankfurt am Main, we therefore confront the student with theoretical physics immediately, in the first semester. Theoretical Mechanics I and II, Electrodynamics, and Quantum Mechanics I—An Introduction are the courses during the first two years. These lectures are supplemented with many mathematical explanations and much support material. After the fourth semester of studies, graduate work begins, and Quantum Mechanics II—Symmetries, Statistical Mechanics and Thermodynamics, Relativistic Quantum Mechanics, Quantum Electrodynamics, Gauge Theory of Weak Interactions, and Quantum Chromodynamics are obligatory. Apart from these, a number of supplementary courses on special topics are offered, such as Hydrodynamics, Classical Field Theory, Special and General Relativity, Many-Body Theories, Nuclear Models, Models of Elementary Particles, and Solid-State Theory.

This volume of lectures, *Classical Mechanics: Systems of Particles and Hamiltonian Dynamics*, deals with the second and more advanced part of the important field of classical mechanics. We have tried to present the subject in a manner that is both interesting to the student and easily accessible. The main text is therefore accompanied by many exercises and examples that have been worked out in great detail. This should make the book useful also for students wishing to study the subject on their own.

Beginning the education in theoretical physics at the first university semester, and not as dictated by tradition after the first one and a half years in the third or fourth semester, has brought along quite a few changes as compared to the traditional courses in that discipline. Especially necessary is a greater amalgamation between the actual physical problems and the necessary mathematics. Therefore, we treat in the first semester vector algebra and analysis, the solution of ordinary, linear differential equations, Newton's mechanics of a mass point, and the mathematically simple mechanics of special relativity.

Many explicitly worked-out examples and exercises illustrate the new concepts and methods and deepen the interrelationship between physics and mathematics. As a matter of fact, the first-semester course in theoretical mechanics is a precursor to theoretical physics. This changes significantly the content of the lectures of the second semester addressed here. Theoretical mechanics is extended to systems of mass points, vibrating strings and membranes, rigid bodies, the spinning top, and the discussion of formal (analytical) aspects of mechanics, that is, Lagrange's, Hamilton's formalism, and Hamilton–Jacobi formulation of mechanics. Considered from the mathematical point of view, the new features are partial differential equations, Fourier expansion, and eigenvalue problems. These new tools are explained and exercised in many physical examples. In the lecturing praxis, the deepening of the exhibited material is carried out in a three-hour-per-week *theoretica*, that is, group exercises where eight to ten students solve the given exercises under the guidance of a tutor.

We have added some chapters on modern developments of nonlinear mechanics (dynamical systems, stability of time-dependent orbits, bifurcations, Lyapunov exponents and chaos, systems with chaotic dynamics), being well aware that all this material cannot be taught in a one-semester course. It is meant to stimulate interest in that field and to encourage the students' further (private) studies.

The last chapter is devoted to the history of mechanics. It also contains remarks on the lives and work of outstanding philosophers and scientists who contributed importantly to the development of science in general and mechanics in particular.

Biographical and historical footnotes anchor the scientific development within the general context of scientific progress and evolution. In this context, I thank the publishers Harri Deutsch and F.A. Brockhaus (*Brockhaus Enzyklopädie*, F.A. Brockhaus, Wiesbaden—marked by [BR]) for giving permission to extract the biographical data of physicists and mathematicians from their publications.

We should also mention that in preparing some early sections and exercises of our lectures we relied on the book *Theory and Problems of Theoretical Mechanics*, by Murray R. Spiegel, McGraw-Hill, New York, 1967.

Over the years, we enjoyed the help of several former students and collaborators, in particular, H. Angermüller, P. Bergmann, H. Betz, W. Betz, G. Binnig, J. Briechele, M. Bundschuh, W. Caspar, C. v. Charewski, J. v. Czarnecki, R. Fickler, R. Fiedler, B. Fricke, C. Greiner, M. Greiner, W. Grosch, R. Heuer, E. Hoffmann, L. Kohaupt, N. Krug, P. Kurowski, H. Leber, H.J. Lustig, A. Mahn, B. Moreth, R. Mörschel, B. Müller, H. Müller, H. Peitz, G. Plunien, J. Rafelski, J. Reinhardt, M. Rufa, H. Schaller, D. Schebesta, H.J. Scheefer, H. Schwerin, M. Seiwert, G. Soff, M. Soffel, E. Stein, K.E. Stiebing, E. Stämmler, H. Stock, J. Wagner, and R. Zimmermann. They all made their way in science and society, and meanwhile work as professors at universities, as leaders in industry, and in other places. We particu-

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