Electricity and Magnetism

Third Edition

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Electricity and magnetism

Preface to the third edition

De manera que acordé, aunque contra mi voluntad, meter segunda vez la pluma en tan estraña lavor é tan agena de mi facultad, hurtando algunos ratos á mi principal estudio, con otras horas destinadas para recreacioń, puesto que no han de faltar nuevos detractores á la nueva edicion.
1499 *Fernando de Rojas*

So I agreed, albeit unwillingly (since there cannot fail to be fresh critics of a new edition), again to exercise my pen in so strange a labour, and one so foreign to my ability, stealing some moments from my principal study, together with other hours destined for recreation.

FOR the third edition of this textbook the material has been completely revised and in many parts substantially rewritten. S.I. units are used throughout; references to c.g.s. units have been almost wholly eliminated, but a short conversion table is given in Appendix D. The dominance of solid-state devices in the practical world of electronics is reflected in a major change in the subject order.

Chapters 1–9 set out the macroscopic theory of electricity and magnetism, with only minor references to the atomic background, which is discussed in Chapters 10–17. A simple treatment of lattice vibrations is introduced in Chapter 10 in considering the dielectric properties of ionic solids. The discussion of conduction electrons and metals has been expanded into two chapters, and superconductivity, a topic previously excluded, is the subject of Chapter 13. Minor changes have been made in the three chapters (14–16) on magnetism. The discussion of semiconductor theory precedes new chapters on solid-state devices, but we have endeavoured to present such devices in a manner which does not presuppose a knowledge in depth of the theory. The remaining chapters, on amplifiers and oscillators, vacuum tubes, a.c. measurements, noise, and magnetic resonance, bring together the discussion of electronics and its applications.

The authors are grateful to many colleagues in Oxford and readers elsewhere for helpful comments on previous editions which have been incorporated in the present volume. In particular we are indebted to Dr. G. A. Brooker for numerous and detailed comments and suggestions; to Drs F. V. Price and J. W. Hodby, whose reading of new material on electronics in draft form resulted in substantial improvement of the

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Contents

1. Electrostatics I

1.1. The electrical nature of matter

THE fundamental laws of electricity and magnetism were discovered by experimenters who had little or no knowledge of the modern theory of the atomic nature of matter. It should therefore be possible to present these laws in a textbook by dealing at first purely in macroscopic phenomena and then introducing gradually the details of atomic theory as required, developing the subject almost in the historical order of discovery. Instead, in this book a basic knowledge of atomic theory is assumed from the beginning, and the macroscopic phenomena are related to atomic properties throughout.

Conductors and insulators

For the purpose of electrostatic theory all substances can be divided into two fairly distinct classes: *conductors*, in which electrical charge can flow easily from one place to another; and *insulators*, in which it cannot. In the case of solids, all metals and a number of substances such as carbon are conductors, and their electrical properties can be explained by assuming that a number of electrons (roughly one per atom) are free to wander about the whole volume of the solid instead of being rigidly attached to one atom. Atoms which have lost one or more electrons in this way have a positive charge, and are called ions. They remain fixed in position in the solid lattice. In solid substances of the second class, insulators, each electron is firmly bound to the lattice of positive ions, and cannot move from point to point. Typical solid insulators are sulphur, polystyrene, and alumina.

When a substance has no net electrical charge, the total numbers of positive and negative charges within it must just be equal. Charge may be given to or removed from a substance, and a positively charged substance has an excess of positive ions, while a negatively charged substance has an excess of electrons. Since the electrons can move so much more easily in a conductor than the positive ions, a net positive charge is usually produced by the removal of electrons. In a charged conductor the electrons will move to positions of equilibrium under the influence of the forces of mutual repulsion between them, while in an insulator they are fixed in position and any initial distribution of charge will remain almost indefinitely. In a good conductor the movement of charge is almost instantaneous, while in a good insulator it is extremely slow. While there is no such thing as a perfect conductor or perfect insulator, such concepts are useful in developing electrostatic theory; metals form a good approximation to the former, and substances such as sulphur to the latter.

1.2. Coulomb's law and fundamental definitions

The force of attraction between charges of opposite sign, and of repulsion between charges of like sign, is found to be inversely proportional to the square of the distance between the charges (assuming them to be located at points), and proportional to the product of the magnitudes of the two charges. This law was discovered experimentally by Coulomb in 1785. In his apparatus the charges were carried on pith balls, and the force between them was measured with a torsion balance. The experiment was not very accurate, and a modern method of verifying the inverse square law with high precision will be given later (§ 1.3). From here on we shall assume it to be exact.

If the charges are q_1 and q_2, and r is the distance between them, then the force F on q_2 is along r. If the charges are of the same sign, the force is one of repulsion, whose magnitude is

$$F = C\frac{q_1 q_2}{r^2}.$$

The vector equation for the force is

$$\mathbf{F} = C\frac{q_1 q_2}{r^3}\mathbf{r}. \tag{1.1}$$

Here $\mathbf{F}$, $\mathbf{r}$ are counted as positive when directed from q_1 to q_2. Eqn (1.1) is the mathematical expression of Coulomb's law.

The units of $\mathbf{F}$ and $\mathbf{r}$ are those already familiar from mechanics; it remains to determine the units of C and q. Here there are two alternatives: either C is arbitrarily given some fixed numerical value, when eqn (1.1) may be used to determine the unit of charge, or the unit of charge may be taken as some arbitrary value, when the constant C is to be determined by experiment. The *Système International* (S.I.), which will be used throughout this book, makes use of the second method. The force F is in newtons, the distance r in metres, and an arbitrary unit, the coulomb, is used to measure the charges q_1 and q_2. The coulomb is directly related to the unit of current, the ampere, which is one coulomb per second; the ampere is defined by the forces acting between current-carrying conductors (see § 4.1). Eqn (1.1) for Coulomb's law is then analogous to that for gravitational attraction, except that it deals with electrical charges instead of masses; the constant of proportionality C must be determined by experiment. In the S.I., the constant C is written as $1/4\pi\epsilon_0$, the factor 4π being introduced here so that it occurs in formulae involving spherical rather than plane geometry. Eqn (1.1) therefore becomes

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \mathbf{r}, \tag{1.2}$$

where **F** is in newtons (N), **r** in metres (m), and q in coulombs (C). The quantity ϵ_0 is known as the 'permittivity of a vacuum' (see § 1.5); its experimental value (see § 5.8) is $8{\cdot}85 \times 10^{-12}$ coulomb^2 newton^{-1} metre^{-2} ($C^2\,N^{-2}\,m^{-2}$)—a more convenient name for this unit (see § 1.6) is farad metre^{-1} ($F\,m^{-1}$).

Electric field and electric potential

The force which a charge q_2 experiences when in the neighbourhood of another charge q_1 may be ascribed to the presence of an 'electric field' of strength **E** produced by the charge q_1. Since the force on a charge q_2 is proportional to the magnitude of q_2, we define the field strength **E** by the equation

$$\mathbf{F} = \mathbf{E} q_2. \tag{1.3}$$

From this definition and Coulomb's law it follows that **E** does not depend on q_2, and is a vector quantity, like **F**. From eqn (1.2) we find that

$$\mathbf{E} = \frac{q_1}{4\pi\epsilon_0 r^3} \mathbf{r} \tag{1.4}$$

is the electric field due to the charge q_1.

If a unit positive charge is moved an infinitesimal distance **ds** in a field of strength **E**, then the work done by the field is $\mathbf{E}.\mathbf{ds}$, and the work done against the field is $-\mathbf{E}.\mathbf{ds}$. This follows from the fact that the force on unit charge is equal to the electric field strength **E**. The work done against the field in moving a unit positive charge from a point A to a point B will therefore be

$$V = -\int_A^B \mathbf{E}.\mathbf{ds}.$$

This is a scalar quantity known as the electric potential. If the field strength **E** is due to a single charge q at O, as in Fig. 1.1, then the force on unit charge at an arbitrary point P is along OP, and **ds** is the vector element P_1P_2. Now $\mathbf{E}.\mathbf{ds} = E \cos\theta \,\mathrm{d}s = E\,\mathrm{d}r$, and hence

$$V_B - V_A = -\int_A^B E\,\mathrm{d}r = -\frac{q}{4\pi\epsilon_0}\int_{r_1}^{r_2} \frac{\mathrm{d}r}{r^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

Thus the difference of potential between A and B depends only on the positions of A and B, and is independent of the path taken between them.

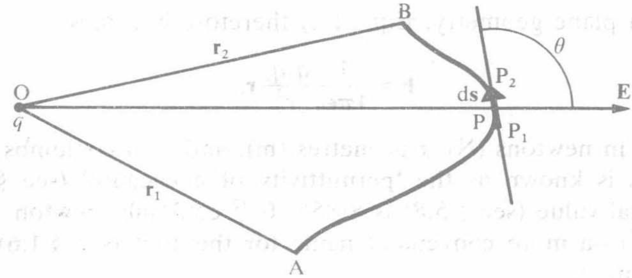

FIG. 1.1. Calculation of the potential difference between points A and B due to the field of a point charge q at O.

The potential at a point distance r from a charge q is the work done in bringing up unit charge to the point in question from a point at zero potential. By convention, the potential is taken as zero at an infinite distance from all charges, that is, $V = 0$ for $r = \infty$. Therefore the potential at a point distance r from a charge q is

$$V = q/4\pi\epsilon_0 r. \tag{1.5}$$

The difference in potential $\mathrm{d}V$ between P_1 and P_2 (Fig. 1.1) distance $\mathbf{ds}$ apart is

$$\mathrm{d}V = -\mathbf{E}.\mathbf{ds} = -(E_x\,\mathrm{d}x + E_y\,\mathrm{d}y + E_z\,\mathrm{d}z).$$

Hence

$$\mathbf{E} = -\text{grad } V = -\nabla V, \tag{1.6}$$

where in Cartesian coordinates grad $V = \mathbf{i}\,\partial V/\partial x + \mathbf{j}\,\partial V/\partial y + \mathbf{k}\,\partial V/\partial z$ and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors parallel to the x-, y-, and z-axes. The components of $\mathbf{E}$ along the three axes are

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}.$$

The negative sign shows that of itself a positive charge will move from a higher to a lower potential, and work must be done to move it in the opposite direction. (For vector relations, see Appendix A.)

The work done in taking a charge q round a closed path in an electrostatic field is zero. This can be seen from Fig. 1.2. The work done in taking the charge q round the path ABCA is

$$W = -q\oint \mathbf{E}.\mathbf{ds} = q(V_B - V_A) + q(V_C - V_B) + q(V_A - V_C) = 0,$$

and is independent of the path taken provided it begins and ends at the same point. Therefore the electric potential is a single-valued function of the space coordinates for any stationary distribution of electric charges; it has only one value at any point in the field.

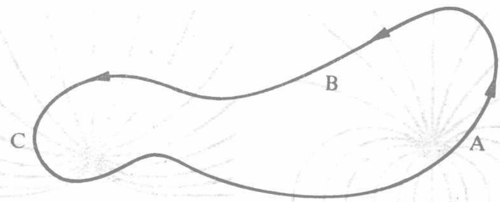

FIG. 1.2. The work done in taking an electric charge round a closed path in an electrostatic field is zero.

From the vector identity curl grad $V = 0$ or $\nabla \wedge (\nabla V) = 0$ (see Appendix § A.9, eqn (A.19)) it follows that curl $\mathbf{E} = 0$. Here curl $\mathbf{E}$ is a vector whose components are

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right).$$

These components can be shown to be zero by the use of elementary circuits (cf. Appendix § A.6), and the fact that no work is done in taking a charge round a closed path. The relation curl $\mathbf{E} = 0$ holds because $\mathbf{E}$ can be expressed as the gradient of a scalar potential: $\mathbf{E} = $ grad V. This is true in electrostatics, but does not hold when a changing magnetic flux threads the circuit (see § 5.1).

Since potential is a scalar quantity the potential at any point is simply the algebraic sum of the potentials due to each separate charge. On the other hand, $\mathbf{E}$ is a vector quantity, and the resultant field is the vector sum of the individual fields. Hence it is nearly always simpler to work in terms of potential rather than field; once the potential distribution is found, the field at any point is found by using eqn (1.6).

Units

From eqn (1.3) we obtain the unit of electric field strength. An electric field of 1 unit exerts a force of 1 newton on a charge of 1 coulomb. Electric field strengths can therefore be expressed in newton coulomb^{-1} (N C^{-1}).

The unit of potential is defined as follows: When 1 joule (J) of work is done in transferring a charge of 1 C from A to B, the potential difference between A and B is 1 volt (V). From eqn (1.6) E can be expressed in volt metre^{-1} (V m^{-1}), and this is the unit which is customarily used. It is easily verified that the two alternative units for E are equivalent.

Lines of force

A line drawn in such a way that it is parallel to the direction of the field at any point is called a line of force. Fig. 1.3 shows the lines of force for

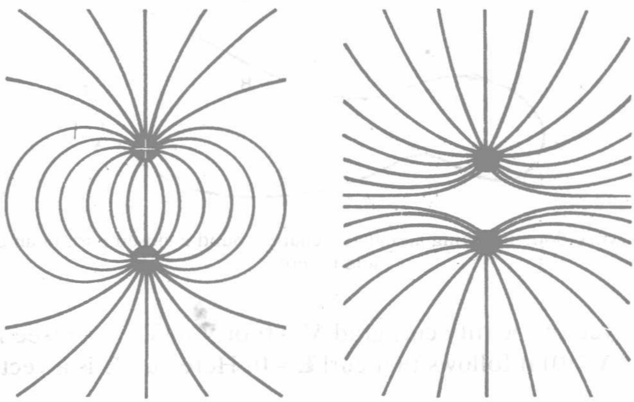

FIG. 1.3. (a) Lines of force between equal charges of opposite sign. (b) Lines of force between equal charges of the same sign.

two equal charges. Lines of force do not intersect one another since the direction of the field cannot have two values at one point; they are continuous in a region containing no free charges, and they begin and end on free charges. The number of lines of force drawn through unit area normal to the direction of $\mathbf{E}$ is equal to the value of $\mathbf{E}$ at that point.

If a series of curves is drawn, each curve passing through points at a given potential, these equipotential curves cut the lines of force orthogonally. Equipotential curves are generally drawn for equal increments of potential; then $\mathbf{E}$ is greatest where the equipotentials are closest together.

1.3. Gauss's theorem

Let S be a closed surface surrounding a charge q, and let q be distant r from a small area $\mathrm{d}\mathbf{S}$ on the surface S at A (Fig. 1.4(a)). The electric field strength $\mathbf{E}$ at A has the value

$$E = q/4\pi\epsilon_0 r^2.$$

The number of lines of force passing through an element of area $\mathrm{d}S$ is

$$\mathbf{E}.\mathrm{d}\mathbf{S} = E\cos\theta\,\mathrm{d}S = \frac{q\cos\theta\,\mathrm{d}S}{4\pi\epsilon_0 r^2},$$

where the outward normal to the surface element makes an angle θ with $\mathbf{E}$. Now the solid angle subtended by $\mathrm{d}S$ at O is $\mathrm{d}\Omega = \cos\theta\,\mathrm{d}S/r^2$, and the value of $E\cos\theta\,\mathrm{d}S$ is therefore $q\,\mathrm{d}\Omega/4\pi\epsilon_0$. Hence the total number of lines of force passing through the whole surface is

$$\int E\cos\theta\,\mathrm{d}S = \frac{q}{4\pi\epsilon_0}\int \mathrm{d}\Omega = \frac{q}{\epsilon_0}, \tag{1.7a}$$