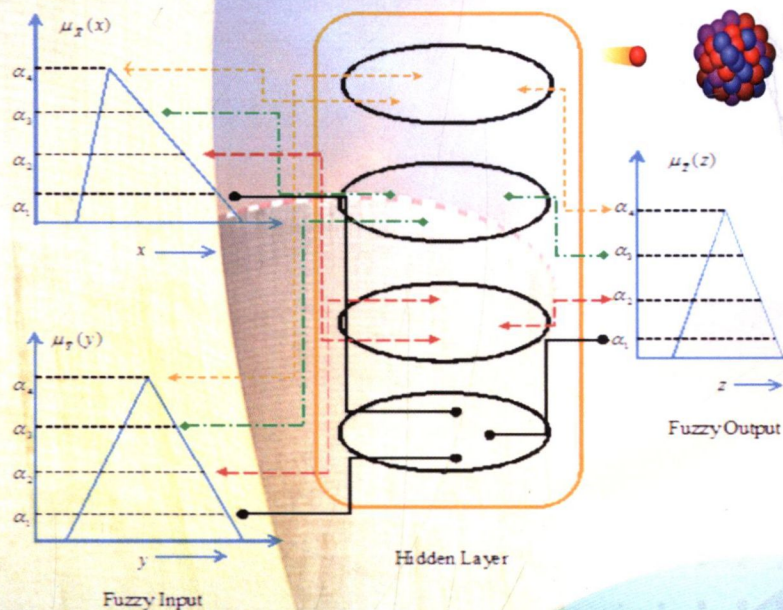


NEUTRON DIFFUSION

CONCEPTS AND UNCERTAINTY
ANALYSIS FOR ENGINEERS
AND SCIENTISTS



S. Chakraverty
Sukanta Nayak



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The successful design and implementation of nuclear reactor cores depends greatly on the abilities of individual engineers to adhere to precise standards of structural reliability and mathematical precision, whether in the prediction of diffusion and absorption coefficients integral to producing the necessary atomic reactions, or in the exact geometric mapping of the reactor facility itself. In designing nuclear reactor facilities, engineers are consistently faced with the challenge of building fully-functioning structures while also contending with the inevitable challenges that result from personal errors in measurement, limited observation time, and inconsistencies that emerge in the designs of individual experiments.

With an eye towards current theoretical models of nuclear energy production, this inclusive manual is designed to foster a systematic understanding of nuclear diffusion theory along with fuzzy/interval/stochastic uncertainty. Drawing on the research and experience of leading figures in the field, readers will find a carefully developed and systematically researched study of the basic concepts of reactor physics, as well as neutron diffusion theory, the modeling of uncertainty methods (fuzzy, interval, stochastic) and their application to diffusion problems. The text serves as an essential reference for students, scholars, practitioners, researchers and academicians in the assorted fields of engineering and science, and in particular nuclear engineering. This guide covers the following concepts:

- Covers the basics of nuclear reactor and neutron diffusion theory.
- Explains the fundamentals of uncertainty analysis, with examples and illustrations.
- Covers recent developments of uncertainty, such as fuzzy set theory and stochastic processes.
- Examines hybrid uncertainty in neutron diffusion problems.
- Provides systematic and understandable coverage of neutron diffusion.
- Shows the use of uncertainty fuzzy set theory and stochastic processes.
- Provides applications to nuclear diffusion problems in science and engineering.
- Demonstrates computational procedures with simulation results.



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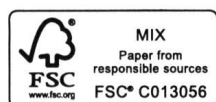
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Neutron Diffusion

Concepts and Uncertainty Analysis for
Engineers and Scientists

Preface

The design of the reactor core is based on the description of the production, transport and absorption of neutrons. As neutrons move within a medium, viz. gas, liquid or solid, they collide with the nuclei of the atoms in the medium. During such collisions, neutrons may be absorbed by the nuclei, which are either elastic or inelastic. Absorption of neutrons may result in a loss or an increase in the number of neutrons by fission. Fission neutrons usually have different energies and move in different directions than incident neutrons. As a result, there may be scattering of neutrons, which changes the position, energy and direction of the motion of the neutrons. Furthermore, the scattering of neutron collision inside a reactor depends upon the geometry of the reactor, diffusion coefficient and absorption coefficient. It may also be noted that the parameters responsible for the diffusion (i.e. the scattering of neutron) may not always be crisp, rather they may be uncertain.

In general, these uncertainties occur due to the vague, imprecise and incomplete information about the variables and parameters, as a result of errors in measurement, observation, experiment or applying different operating conditions or due to maintenance-induced errors, which are uncertain in nature. So, to overcome these uncertainties, one may use the fuzzy/interval or stochastic environmental parameters and variables in place of crisp (fixed) parameters. With these uncertainties, the governing differential equations turn fuzzy/interval or stochastic. Practically, it is sometimes difficult to obtain the solution of fuzzy equations due to the complexity in the fuzzy arithmetic; for example, addition and multiplication are not the inverse operations of subtraction and division, respectively. On the other hand, we may model the problem as stochastic only when sufficient data are available. Moreover, the combination (hybridization) of fuzzy and stochastic is also a new vista. These were found to be important and challenging areas of study in recent years. As such, one may need to understand the nuclear diffusion principles/theories corresponding to reliable and efficient techniques for the solution of such uncertain problems.

Accordingly, the objective of this book is to provide first the basic concepts of reactor physics as well as neutron diffusion theory. The main aim of the book, however, is about handling uncertainty in neutron diffusion problems. Hence, uncertainties (i.e. fuzzy, interval, stochastic) and their applications in nuclear diffusion problems have been included here in a systematic manner, along with the recent developments. This book may be an essential reference for students, scholars, practitioners, researchers and academicians in the assorted fields of engineering and science, particularly nuclear engineering.

Chapter 1 describes the preliminaries of basic reactor principles. Here, a few important and related terminologies for the nuclear reactor are briefly explained. In Chapter 2, neutron diffusion theory and the formulation of the neutron diffusion equation have been presented, which give first-hand scientific insights to study various nuclear design problems. The transportation of the scattered neutrons is modelled and formulated mathematically. In Chapter 3, the fundamentals of uncertainties are discussed. In this chapter, uncertainties are addressed with respect to three categories, viz. interval, fuzzy and probabilistic. The operations of these uncertainties are demonstrated through various examples. Furthermore, Chapter 4 elaborates the uncertain modelling (considering interval/fuzzy parameters as uncertain) of neutron diffusion. The factors involved in the reactor system, which are responsible for uncertainty, are modelled in terms of interval/fuzzy.

One-group models with respect to crisp parameters are explained in Chapter 5. This chapter includes both the analytical and numerical approaches to investigate one-group models, whereas in Chapter 6, uncertain (considering interval/fuzzy parameters as uncertain) one-group models are discussed, and example problems are investigated. In this chapter, the uncertain neutron diffusion equation for a bare square homogeneous reactor is discussed, which has been modelled by the fuzzy finite element method. The multigroup neutron diffusion equation has been generalized in Chapter 7, and again the finite element method has been used to solve the same. In Chapter 8, the uncertain multigroup neutron diffusion model is investigated. Accordingly, a benchmark problem is solved under an uncertain environment, and the sensitivity of the uncertain parameters is also analyzed.

Chapter 9 includes the theory of point kinetic diffusion. Here, different terminologies related to point kinetic diffusion of the one-group bare reactor are discussed, and the point kinetic diffusion equation for crisp parameters is formulated. Next, the stochastic point kinetic diffusion equation is modelled in Chapter 10. The basic concept of the birth–death stochastic process and stochastic point kinetic diffusion are discussed here. Finally, in Chapter 11, hybridized uncertainty (i.e. both probabilistic and fuzzy) is considered, and uncertain point kinetic diffusion is modelled. Furthermore, the hybridized uncertainty for this model is demonstrated through an example problem.

This book aims to provide a systematic understanding of nuclear diffusion theory along with uncertainty, viz. fuzzy/interval/stochastic and hybrid methods. The book will certainly prove to be a benchmark for graduate and postgraduate students, teachers, engineers and researchers in this field. It provides comprehensive results and an up-to-date and self-contained review of the topic along with application-oriented treatment of the use of newly developed methods of different fuzzy, stochastic and hybrid computations in various domains of nuclear engineering and sciences. It is worth mentioning that the presented methods may very well be used in/extended to various other engineering disciplines, viz. electronics, marine, chemical and mining, and sciences such as physics, chemistry and biotechnology, where one wants to model physical problems with respect to non-probabilistic (interval/fuzzy) and hybrid uncertainties for understanding the real scenario.

S. Chakraverty
Sukanta Nayak

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Basic Reactor Principles

The principle of the nuclear reactor is based on the transport of neutrons and their interaction with matter within a reactor. Various terminologies involved in the design of nuclear reactors, based on the fission process, have been included in this chapter.

1.1 Atomic Structure

An atom is a small part of an element which consists of a positively charged nucleus surrounded by negatively charged *electrons*. Atomic nuclei are made up of two fundamental particles, viz. protons and neutrons. The *proton* carries a unit positive charge (+e) and a mass about 1836 times the electronic mass (m_e). In fact, the proton is identical with the nucleus of a hydrogen atom. On the other hand, the *neutron* is electrically neutral and slightly heavier than the proton. The neutrons and protons are together called the *nucleons*.

For each element, protons represent the atomic number of the atomic nucleus and are denoted by the symbol Z . It is also the same as the ordinal number of the element in the familiar periodic table of the elements. The total number of neutrons (N) and protons (Z) inside the nucleus is called the *mass number* (A), that is $A = N + Z$. In general, a nucleus of an atom X of atomic number Z and mass number A is symbolically written as ${}^A_ZX$. For example, the helium atom is denoted by ${}^4_2\text{He}$, with the atomic number 2 and mass number 4. These are also known as α -particles. The protons and neutrons are symbolically denoted as p and n , respectively. The subscript to the left (i.e. atomic number) of the atomic symbol is often omitted and it looks like ${}^AX$.

Nuclei with the same atomic number (Z) but a different mass number (A) are called *isotopes*. A particular element with a given Z may have isotopes of different mass numbers. Their nuclei possess the same number of protons, whereas a different number of neutrons. For example, carbon has the isotopes ${}^{12}\text{C}$, ${}^{13}\text{C}$ and ${}^{14}\text{C}$ with mass numbers 12, 13 and 14, respectively. On the other hand, nuclei with the same mass number, but different atomic numbers, are known as *isobars*, while those with the same number of neutrons are known as *isotones*. Chlorine and potassium are isotones that have 20 neutrons in their nuclei.

The mass of each atom is expressed in terms of the atomic mass unit (u), which is exactly $1/12$ of the mass of the ${}^{12}\text{C}$ atom. Uranium is the most important element for the release of nuclear energy. In nature, it exists in three different forms with mass numbers 134, 235 and 238, respectively. Apart from this, another important element is thorium, with the atomic number 90 and mass number 232 (Hetrick 1971).

1.2 Binding Energy

In nuclear physics, the nuclear masses are determined by using the mass spectrometers (Ghoshal 2010). The actual mass is always less than the sum of the masses of the constituent nucleons. The difference is called the *mass defect*. The mass defect is related to the energy binding the particles in the nucleus. In other words, if we want to break up a nucleus of Z protons and N neutrons completely so that they are all separated from each other, a certain minimum amount of energy is to be supplied to the nucleus. This energy is known as the *binding energy*.

Let us consider the mass of protons, neutrons and electrons as m_p , m_n and m_e , respectively. The mass defect is defined as (Stacey 2007)

$$\text{Mass defect} = [Z(m_p + m_e) + (A - Z)m_n] - M,$$

where

$Z(m_p + m_e) + (A - Z)m_n$ is the masses of the constituents of the atom
 M is the observed mass of the atom

Using the Einstein equation, the binding energy can be calculated as follows:

$$E = \Delta mc^2 \quad (1.1)$$

where

Δm is the mass defect
 c is the velocity of light ($\approx 3 \times 10^8$ m/s)

In other words, binding energy = $\Delta m \times 931.5$ MeV, where $1 \text{ u} = 931.5$ MeV.

1.2.1 Calculation of 1 u

It is well known that 1 mole of ^{12}C has the mass of 12 g or 12×10^{-3} kg. Since 1 mole contains 6.02205×10^{23} atoms (Avogadro number), the mass of each ^{12}C atom is

$$\frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg} = 12 \times 1.66056 \times 10^{-27} \text{ kg}.$$

Hence, the unit of atomic mass is

$$1 \text{ u} = \frac{1}{12} \times \frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg} = 1.66056 \times 10^{-27} \text{ kg}. \quad (1.2)$$

The energy equivalent of this amount of mass is

$$\begin{aligned} 1 \text{ u} &= 1.66056 \times 10^{-27} \times c^2 \\ &= 1.66056 \times 10^{-27} \times 8.98755 \times 10^{16} \\ &= 931.502 \text{ MeV} \end{aligned} \quad (1.3)$$

If Δm is in gram, then E will be in MeV per gram atom.

The binding energy of a nucleus, when divided by the number of nucleons, gives the mean binding energy per nucleon.

The binding energy per nucleon is a measure of the stability of the nucleus. The greater the binding energy per nucleon, the more stable the nucleus.

It is noted that if the value of binding energy is negative then the product of the nucleus or nuclei will be less stable than the reactant nucleus or nuclei. If the binding energy is positive, then the product nucleus is more stable than the reactant nucleus.

1.3 Nuclear Reactions

The reactions in which the nucleus of an atom itself undergoes a spontaneous change or interacts with other nuclei of lighter particles resulting in new nuclei (and one or more lighter particles) are called nuclear reactions. Here, the neutron reactions are mainly categorized into scattering, capture and fission. The exchange of energy between a neutron and a nucleus are based on the scattering reactions. In scattering reactions, scattering is used to describe the movement of the neutron after interaction. Scattering may be either elastic or inelastic. In elastic scattering, the exchange of energy between the neutron and the nucleus is kinetic. But in inelastic scattering, the kinetic energy of the neutron is transferred to the nucleus as potential energy.

Neutrons lose much of their kinetic energy due to the scattering collision with various nuclei in the medium through which the neutrons move and become slow with energies of an electron volt or less. Finally, the kinetic energy may be reduced to such an extent that the average is the same as that of the atoms (or molecules) of the medium. Since the value of the kinetic energy depends on the temperature, it is called thermal energy. Hence, the neutrons whose energies have been reduced to this extent are called thermal neutrons.

In inelastic scattering, the excited compound nucleus can emit its excess energy as gamma (γ) radiation and this process is known as radiative capture or capture. For example, ^{238}U is the most abundant naturally occurring isotope that has the following radiative capture:



Here, the resulting nucleus ^{239}U is radioactive and decays with the emission of a negative beta particle.

Finally, the third type of interaction between neutrons and nuclei is fission, or it is called nuclear fission. This is an essential process in nuclear reactors, which will be discussed in the subsequent sections.

1.4 Nuclear Fusion

The process of combining lighter elements into a stable heavier nucleus is known as *nuclear fusion*. However, such processes can take place at reasonable rates only at very high temperatures of the order of several million degrees, which exist only in the interior of stars.

Such processes are therefore called thermonuclear reactions. Once a fusion reaction is initiated, the energy released in the process is sufficient to maintain the temperature and to keep the process going on.

For example, the hydrogen bomb is based on the fusion of hydrogen nuclei into heavier ones by the thermonuclear reactions with the release of enormous energy.



1.5 Radioactivity

There are another class of nuclei that are unstable. They disintegrate (break up) spontaneously with the emission of electromagnetic radiation of very high energy. As such, the process of spontaneous transformation of a nucleus is known as radioactivity. Some of the unstable high atomic weight elements are radium, thorium and uranium. The emission or decay is associated with the radiation from the atomic nucleus, which is either an alpha particle (helium nucleus) or a beta particle (an electron). In many cases, gamma radiation comes out with the particle emission. The process of disintegration continues and after a number of stages of radiation, a stable nucleus is formed.

1.5.1 Rate of Radioactive Decay

The nuclei of a given radioactive species have a definite probability of decaying in unit time; this decay probability has a constant value characteristic of the particular nuclide. It remains the same, irrespective of the chemical or physical state of the element at all readily accessible temperatures and pressures. In a given specimen, the rate of decay at any instant is directly proportional to the number of radioactive atoms of the nuclide under considerations present at that instant. When N is the number of the particular radioactive atoms present at time t , then the decay rate can be represented as

$$\frac{dN}{dt} = -\lambda N \quad (1.6)$$

where λ is the decay constant of the radioactive nuclide.

If N_0 is the number of the particular radioactive atoms present at time zero and a time t , later, when N of these nuclei remain, Equation 1.6 is written as

$$\ln \frac{N}{N_0} = -\lambda t. \quad (1.7)$$

Equation 1.7 may be changed and encrypted as follows:

$$N = N_0 e^{-\lambda t}. \quad (1.8)$$

The ratio between the number of atoms disintegrating in unit time to the total number of atoms present at that time is called the decay constant of that nuclide.

In 1904, Rutherford introduced a constant known as the half-life period of the radioactive element for evaluating its radioactivity or for comparing its radioactivity with the activities of other radioactive elements. The half-life period of a radioactive element is