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傅里叶分析与小波分析导论

(英文版)

Introduction to
Fourier Analysis
and Wavelets

Mark A. Pinsky

THE BROOKS/COLE SERIES IN
ADVANCED MATHEMATICS
Paul J. Sally, Jr., EDITOR

(美) Mark A. Pinsky 著
西 北 大 学



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PREFACE

To my parents,
Harry A. Pinsky and Helen M. Pinsky,
who led me to the path of learning

This book provides a self-contained treatment of classical Fourier analysis at the upper undergraduate or beginning graduate level. I assume that the reader is familiar with the rudiments of Lebesgue measure and integral on the real line. My viewpoint is mostly classical and concrete, preferring explicit calculations to existential arguments. In some cases, several different proofs are offered for a given proposition to compare different methods.

The book contains more than 175 exercises that are an integral part of the text. It can be expected that a careful reader will be able to complete all of these exercises. Starred sections contain material that may be considered supplementary to the main themes of Fourier analysis. In this connection, it is fitting to comment on the role of Fourier analysis, which plays the dual role of queen and servant of mathematics. Fourier-analytic ideas have an inner harmony and beauty quite apart from any applications to number theory, approximation theory, partial differential equations, or probability theory. In writing this book it has been difficult to resist the temptation to develop some of these applications as a testimonial of the power and flexibility of the subject. The following list of "extra topics" are included in the starred sections: Stirling's formula, Laplace asymptotic method, the isoperimetric inequality, equidistribution modulo one, Jackson/Bernstein theorems, Wiener's density theorem, one-sided heat equation with Robin boundary condition, the uncertainty principle, Landau's asymptotic lattice point formula, Gaussian sums and the Schrödinger equation, the central limit theorem, the Berry-Esséen theorem and the law of the iterated logarithm. While none of these topics is "mainstream Fourier analysis," each of them has a definite relation to some part of the subject.

A word about the organization of the first two chapters, which are essentially independent of one another. Readers with some sophistication but little previous knowledge of Fourier series can begin with Chapter 2 and anticipate a self-contained treatment of the n -dimensional Fourier transform and many of its applications. By contrast, readers who wish an introductory treatment of Fourier series should begin with Chapter 1, which provides a reasonably complete introduction to Fourier analysis on the circle. In both cases I emphasize the Riesz-Fischer and Plancherel theorems, which demonstrate the

natural harmony of Fourier analysis with the Hilbert spaces $L^2(\mathbb{T})$ and $L^2(\mathbb{R}^n)$. However much of modern harmonic analysis is carried out in the L^p spaces for $p \neq 2$, which is the subject of Chapter 3. Here we find the interpolation theorems of Riesz-Thorin and Marcinkiewicz, which are applied to discuss the boundedness of the Hilbert transform and its application to the L^p convergence of Fourier series and integrals. In Chapter 4 I merge the subjects of Fourier series and Fourier transforms by means of the Poisson summation formula in one and several dimensions. This also has applications to number theory and multiple Fourier series, as noted above.

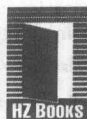
Chapter 5 explores the application of Fourier methods to probability theory. Limit theorems for sums of independent random variables are equivalent to the study of iterated convolutions of a probability measure on the line, leading to the central limit theorem for convergence and the Berry-Esséen theorems for error estimates. These are then applied to prove the law of the iterated logarithm.

The final Chapter 6 deals with wavelets, which form a class of orthogonal expansions that can be studied by means of Fourier analysis—specifically the Plancherel theorem from Chapter 2. In contrast to Fourier series and integral expansions, which require one parameter (the frequency), wavelet expansions involve two indices—the scale and the location parameter. This allows additional freedom and leads to improved convergence properties of wavelet expansions in contrast with Fourier expansions. I include a brief application to Brownian motion, where the wavelet approach furnishes an easy access to the precise modulus of continuity of the standard Brownian motion.

Many of the topics in this book have been “class-tested” to a group of graduate students and faculty members at Northwestern University during the academic years 1998–2000. I am grateful to this audience for the opportunity to develop and improve my original efforts.

I owe a debt of gratitude to Paul Sally, Jr., who encouraged this project from the beginning. Gary Ostedt gave me full editorial support at the initial stages followed by Bob Pirtle and his efficient staff. Further thanks are due to Robert Fefferman, whose lectures provided much of the inspiration for the basic parts of the book. Further assistance and feedback was provided by Marshall Ash, William Beckner, Miron Bekker, Leonardo Colzani, Galia Dafni, George Gasper, Umberto Neri, Cora Sadosky, Aurel Stan, and Michael Taylor. Needless to say, the writing of Chapter 1 was strongly influenced by the classical treatise of Zygmund and the elegant text of Katznelson. The latter chapters were influenced in many ways by the books of Stein and Stein/Weiss. The final chapter on wavelets owes much to the texts of Hernandez/Weiss and Wojtaszczyk.

Mark A. Pinsky



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CONTENTS

39			
43			
45			
45			
49			
51			
54			
57			
57			
59			
61			
61			
62			
65			
66			
70	1	FOURIER SERIES ON THE CIRCLE	1
73	1.1	Motivation and Heuristics	1
74	1.1.1	Motivation from Physics	1
75	1.1.1.1	The Vibrating String	1
78	1.1.1.2	Heat Flow in Solids	2
80	1.1.2	Absolutely Convergent Trigonometric Series	3
80	1.1.3	*Examples of Factorial and Bessel Functions	6
80	1.1.4	Poisson Kernel Example	7
80	1.1.5	*Proof of Laplace's Method	9
80	1.1.6	*Nonabsolutely Convergent Trigonometric Series	11
81	1.2	Formulation of Fourier Series	13
82	1.2.1	Fourier Coefficients and Their Basic Properties	13
84	1.2.2	Fourier Series of Finite Measures	19
85	1.2.3	*Rates of Decay of Fourier Coefficients	20
86	1.2.3.1	Piecewise Smooth Functions	21
86	1.2.3.2	Fourier Characterization of Analytic Functions	22
86	1.2.4	Sine Integral	24
86	1.2.4.1	Other Proofs That $\text{Si}(\infty) = 1$	24
89	1.2.5	Pointwise Convergence Criteria	25
89	1.2.6	*Integration of Fourier Series	29
91	1.2.6.1	Convergence of Fourier Series of Measures	30
94	1.2.7	Riemann Localization Principle	31
97	1.2.8	Gibbs-Wilbraham Phenomenon	31
97	1.2.8.1	The General Case	34
100	1.3	Fourier Series in L^2	35
102	1.3.1	Mean Square Approximation—Parseval's Theorem	35
106	1.3.2	*Application to the Isoperimetric Inequality	38

IV CONTENTS

1.3.3	*Rates of Convergence in L^2	39
1.3.3.1	Application to Absolutely-Convergent Fourier Series	43
1.4	Norm Convergence and Summability	45
1.4.1	Approximate Identities	45
1.4.1.1	Almost-Everywhere Convergence of the Abel Means	49
1.4.2	Summability Matrices	51
1.4.3	Fejér Means of a Fourier Series	54
1.4.3.1	Wiener's Closure Theorem on the Circle	57
1.4.4	*Equidistribution Modulo One	57
1.4.5	*Hardy's Tauberian Theorem	59
1.5	Improved Trigonometric Approximation	61
1.5.1	Rates of Convergence in $C(\mathbb{T})$	61
1.5.2	Approximation with Fejér Means	62
1.5.3	*Jackson's Theorem	65
1.5.4	*Higher-Order Approximation	66
1.5.5	*Converse Theorems of Bernstein	70
1.6	Divergence of Fourier Series	73
1.6.1	The Example of du Bois-Reymond	74
1.6.2	Analysis via Lebesgue Constants	75
1.6.3	Divergence in the Space L^1	78
1.7	*Appendix: Complements on Laplace's Method	80
1.7.0.1	First Variation on the Theme-Gaussian Approximation	80
1.7.0.2	Second Variation on the Theme-Improved Error Estimate	80
1.7.1	*Application to Bessel Functions	81
1.7.2	*The Local Limit Theorem of DeMoivre-Laplace	82
1.8	Appendix: Proof of the Uniform Boundedness Theorem	84
1.9	*Appendix: Higher-Order Bessel functions	85
1.10	Appendix: Cantor's Uniqueness Theorem	86
2	FOURIER TRANSFORMS ON THE LINE AND SPACE	89
2.1	Motivation and Heuristics	89
2.2	Basic Properties of the Fourier Transform	91
2.2.1	Riemann-Lebesgue Lemma	94
2.2.2	Approximate Identities and Gaussian Summability	97
2.2.2.1	Improved Approximate Identities for Pointwise Convergence	100
2.2.2.2	Application to the Fourier Transform	102
2.2.2.3	The n -Dimensional Poisson Kernel	106

	2.2.3	Fourier Transforms of Tempered Distributions	108
	2.2.4	*Characterization of the Gaussian Density	109
	2.2.5	*Wiener's Density Theorem	110
2.3		Fourier Inversion in One Dimension	112
	2.3.1	Dirichlet Kernel and Symmetric Partial Sums	112
	2.3.2	Example of the Indicator Function	114
	2.3.3	Gibbs–Wilbraham Phenomenon	115
	2.3.4	Dini Convergence Theorem	115
	2.3.4.1	Extension to Fourier's Single Integral	117
	2.3.5	Smoothing Operations in \mathbb{R}^1 —Averaging and Summability	117
	2.3.6	Averaging and Weak Convergence	118
	2.3.7	Cesàro Summability	119
	2.3.7.1	Approximation Properties of the Fejér Kernel	121
	2.3.8	Bernstein's Inequality	122
	2.3.9	*One-Sided Fourier Integral Representation	124
	2.3.9.1	Fourier Cosine Transform	124
	2.3.9.2	Fourier Sine Transform	125
	2.3.9.3	Generalized h -Transform	125
2.4		L^2 Theory in \mathbb{R}^n	128
	2.4.1	Plancherel's Theorem	128
	2.4.2	*Bernstein's Theorem for Fourier Transforms	129
	2.4.3	The Uncertainty Principle	131
	2.4.3.1	Uncertainty Principle on the Circle	133
	2.4.4	Spectral Analysis of the Fourier Transform	134
	2.4.4.1	Hermite Polynomials	134
	2.4.4.2	Eigenfunction of the Fourier Transform	136
	2.4.4.3	Orthogonality Properties	137
	2.4.4.4	Completeness	138
2.5		Spherical Fourier Inversion in \mathbb{R}^n	139
	2.5.1	Bochner's Approach	139
	2.5.2	Piecewise Smooth Viewpoint	145
	2.5.3	Relations with the Wave Equation	146
	2.5.3.1	The Method of Brandolini and Colzani	149
	2.5.4	Bochner-Riesz Summability	152
	2.5.4.1	A General Theorem on Almost-Everywhere Summability	153
2.6		Bessel Functions	154
	2.6.1	Fourier Transforms of Radial Functions	157
	2.6.2	L^2 -Restriction Theorems for the Fourier Transform	158
	2.6.2.1	An Improved Result	159
	2.6.2.2	Limitations on the Range of p	161
2.7		The Method of Stationary Phase	162
	2.7.1	Statement of the Result	163
	2.7.2	Application to Bessel Functions	164
	2.7.3	Proof of the Method of Stationary Phase	165
	2.7.4	Abel's Lemma	167

3	FOURIER ANALYSIS IN L^p SPACES	169
3.1	Motivation and Heuristics	169
3.2	The M. Riesz-Thorin Interpolation Theorem	169
	3.2.0.1 Generalized Young's Inequality	174
	3.2.0.2 The Hausdorff-Young Inequality	174
	3.2.1 Stein's Complex Interpolation Theorem	175
3.3	The Conjugate Function or Discrete Hilbert Transform	176
	3.3.1 L^p Theory of the Conjugate Function	177
	3.3.2 L^1 Theory of the Conjugate Function	179
	3.3.2.1 Identification as a Singular Integral	183
3.4	The Hilbert Transform on \mathbb{R}	184
	3.4.1 L^2 Theory of the Hilbert Transform	185
	3.4.2 L^p Theory of the Hilbert Transform, $1 < p < \infty$	186
	3.4.2.1 Applications to Convergence of Fourier Integrals	187
	3.4.3 L^1 Theory of the Hilbert Transform and Extensions	188
	3.4.3.1 Kolmogorov's Inequality for the Hilbert Transform	192
	3.4.4 Application to Singular Integrals with Odd Kernels	194
3.5	Hardy-Littlewood Maximal Function	197
	3.5.1 Application to the Lebesgue Differentiation Theorem	200
	3.5.2 Application to Radial Convolution Operators	202
	3.5.3 Maximal Inequalities for Spherical Averages	203
3.6	The Marcinkiewicz Interpolation Theorem	206
3.7	Calderón-Zygmund Decomposition	209
3.8	A Class of Singular Integrals	210
3.9	Properties of Harmonic Functions	212
	3.9.1 General Properties	212
	3.9.2 Representation Theorems in the Disk	214
	3.9.3 Representation Theorems in the Upper Half-Plane	216
	3.9.4 Herglotz/Bochner Theorems and Positive Definite Functions	219
4	POISSON SUMMATION FORMULA AND MULTIPLE FOURIER SERIES	222
4.1	Motivation and Heuristics	222
4.2	The Poisson Summation Formula in \mathbb{R}^1	223
	4.2.1 Periodization of a Function	223
	4.2.2 Statement and Proof	225
	4.2.3 Shannon Sampling	228
4.3	Multiple Fourier Series	230
	4.3.1 Basic L^1 Theory	231
	4.3.1.1 Pointwise Convergence for Smooth Functions	233
	4.3.1.2 Representation of Spherical Partial Sums	233

4.3.2	Basic L^2 Theory	235
4.3.3	Restriction Theorems for Fourier Coefficients	236
4.4	Poisson Summation Formula in \mathbb{R}^d	238
4.4.1	*Simultaneous Nonlocalization	239
4.5	Application to Lattice Points	241
4.5.1	Kendall's Mean Square Error	241
4.5.2	Landau's Asymptotic Formula	243
4.5.3	Application to Multiple Fourier Series	244
4.5.3.1	Three-Dimensional Case	245
4.5.3.2	Higher-Dimensional Case	247
4.6	Schrödinger Equation and Gauss Sums	247
4.6.1	Distributions on the Circle	248
4.6.2	The Schrödinger Equation on the Circle	250
4.7	Recurrence of Random Walk	252
5	APPLICATIONS TO PROBABILITY THEORY	256
5.1	Motivation and Heuristics	256
5.2	Basic Definitions	256
5.2.1	The Central Limit Theorem	260
5.2.1.1	Restatement in Terms of Independent Random Variables	261
5.3	Extension to Gap Series	262
5.3.1	Extension to Abel Sums	266
5.4	Weak Convergence of Measures	268
5.4.1	An Improved Continuity Theorem	269
5.4.1.1	Another Proof of Bochner's Theorem	270
5.5	Convolution Semigroups	272
5.6	The Berry-Esséen Theorem	276
5.6.1	Extension to Different Distributions	279
5.7	The Law of the Iterated Logarithm	280
6	INTRODUCTION TO WAVELETS	284
6.1	Motivation and Heuristics	284
6.1.1	Heuristic Treatment of the Wavelet Transform	285
6.2	Wavelet Transform	286
6.2.0.1	Wavelet Characterization of Smoothness	290
6.3	Haar Wavelet Expansion	291
6.3.1	Haar Functions and Haar Series	291
6.3.2	Haar Sums and Dyadic Projections	292
6.3.3	Completeness of the Haar Functions	295
6.3.3.1	Haar Series in C_0 and L_p Spaces	296
6.3.3.2	Pointwise Convergence of Haar Series	298

6.3.4	*Construction of Standard Brownian Motion	299
6.3.5	*Haar Function Representation of Brownian Motion	301
6.3.6	*Proof of Continuity	301
6.3.7	*Lévy's Modulus of Continuity	302
6.4	Multiresolution Analysis	303
6.4.1	Orthonormal Systems and Riesz Systems	304
6.4.2	Scaling Equations and Structure Constants	310
6.4.3	From Scaling Function to MRA	313
6.4.3.1	Additional Remarks	315
6.4.4	Meyer Wavelets	318
6.4.5	From Scaling Function to Orthonormal Wavelet	319
6.4.5.1	Direct Proof that $V_1 \ominus V_0$ Is Spanned by $\{\Psi(t - k)\}_{k \in \mathbb{Z}}$	324
6.4.5.2	Null Integrability of Wavelets Without Scaling Functions	325
6.5	Wavelets with Compact Support	326
6.5.1	From Scaling Filter to Scaling Function	327
6.5.2	Explicit Construction of Compact Wavelets	330
6.5.2.1	Daubechies Recipe	331
6.5.2.2	Hernandez-Weiss Recipe	333
6.5.3	Smoothness of Wavelets	334
6.5.3.1	A Negative Result	336
6.5.4	Cohen's Extension of Theorem 6.5.1	338
6.6	Convergence Properties of Wavelet Expansions	341
6.6.1	Wavelet Series in L^p Spaces	341
6.6.1.1	Large Scale Analysis	345
6.6.1.2	Almost-Everywhere Convergence	346
6.6.1.3	Convergence at a Preassigned Point	347
6.6.2	Jackson and Bernstein Approximation Theorems	347
6.7	Wavelets in Several Variables	352
6.7.1	Two Important Examples	352
6.7.1.1	Tensor Product of Wavelets	354
6.7.2	General Formulation of MRA and Wavelets in \mathbb{R}^d	354
6.7.2.1	Notations for Subgroups and Cosets	355
6.7.2.2	Riesz Systems and Orthonormal Systems in \mathbb{R}^d	356
6.7.2.3	Scaling Equation and Structure Constants	357
6.7.2.4	Existence of the Wavelet Set	358
6.7.2.5	Proof That the Wavelet Set Spans $V_1 \ominus V_0$	361
6.7.2.6	Cohen's Theorem in \mathbb{R}^d	362
6.7.3	Examples of Wavelets in \mathbb{R}^d	362
References		365
Notations		369
Index		373