



教育科学“十五”国家规划课题研究成果

# Linear Algebra

刘金宪 韩骁兵  
苏连青 李丽霞



高等教育出版社  
HIGHER EDUCATION PRESS

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# 线 性 代 数

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Liu Jinxian Han Xiaobing

Su Lianqing Li Lixia

Higher Education Press

## 内容提要

本书是教育科学“十五”国家规划课题研究成果,对线性代数的内容做了较准确的、深入浅出的英文表述。内容包括行列式、矩阵、向量、方程组解的结构、矩阵的特征值与特征向量等。数学专业技术符号系统与国内现行教学规范一致。分节配备了习题并附有答案。本书适合作为高等院校同名课程双语教学的配套教材,也可以作为英语专业数学课程的教科书,以及数学与应用数学、信息与计算科学专业学科英语的阅读读物。

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## 总 序

为了更好地适应当前我国高等教育跨越式发展需要,满足我国高校从精英教育向大众化教育的重大转移阶段中社会对高校应用型人才培养的各类要求,探索和建立我国高等学校应用型人才培养体系,全国高等学校教学研究中心(以下简称“教研中心”)在承担全国教育科学“十五”国家规划课题——“21世纪中国高等教育人才培养体系的创新与实践”研究工作的基础上,组织全国100余所培养应用型人才为主的高等院校,进行其子项目课题——“21世纪中国高等学校应用型人才培养体系的创新与实践”的研究与探索,在高等院校应用型人才培养的教学内容、课程体系研究等方面取得了标志性成果,并在高等教育出版社的支持和配合下,推出了一批适应应用型人才培养需要的立体化教材,冠以“教育科学‘十五’国家规划课题研究成果”。

2002年11月,教研中心在南京工程学院组织召开了“21世纪中国高等学校应用型人才培养体系的创新与实践”课题立项研讨会。会议确定由教研中心组织国家级课题立项,为参加立项研究的高等院校搭建高起点的研究平台,整体设计立项研究计划,明确目标。课题立项采用整体规划、分步实施、滚动立项的方式,分期分批启动立项研究计划。为了确保课题立项目标的实现,组建了“21世纪中国高等学校应用型人才培养体系的创新与实践”课题领导小组(亦为高校应用型人才立体化教材建设领导小组)。会后,教研中心组织了首批课题立项申报,有63所高校申报了近450项课题。2003年1月,在黑龙江工程学院进行了项目评审,经过课题领导小组严格的把关,确定了首批9项子课题的牵头学校、主持学校和参加学校。2003年3月至4月,各子课题相继召开了工作会议,交流了各校教学改革的情况和面临的具体问题,确定了项目分工,并全面开始研究工作。计划先集中力量,用两年时间形成一批有关人才培养模式、培养目标、教学内容和课程体系等理论研究成果报告和研究报告基础上同步组织建设的反映应用型人才特色的立体化系列教材。

与过去立项研究不同的是,“21世纪中国高等学校应用型人才培养体系的创新与实践”课题研究在审视、选择、消化与吸收多年来已有应用人才培养探索与实践成果基础上,紧密结合经济全球化时代高校应用型人才工作的实际需要,努力实践,大胆创新,采取边研究、边探索、边实践的方式,推进高校应用型人才工作,突出重点目标,并不断取得标志性的阶段成果。

教材建设作为保证和提高教学质量的重要支柱和基础,作为体现教学内

容和教学方法的知识载体,在当前培养应用型人才中的作用是显而易见的。探索、建设适应新世纪我国高校应用型人才培养体系需要的教材体系已成为当前我国高校教学改革和教材建设工作面临的十分重要的任务。因此,在课题研究过程中,各课题组充分吸收已有的优秀教学改革成果,并和教学实际结合起来,认真讨论和研究教学内容和课程体系的改革,组织一批学术水平较高、教学经验较丰富、实践能力较强的教师,编写出一批以公共基础课和专业、技术基础课为主的有特色、适用性强的教材及相应的教学辅导书、电子教案,以满足高等学校应用型人才的需要。

我们相信,随着我国高等教育的发展和高校教学改革的不断深入,特别是随着教育部“高等学校教学质量和教学改革工程”的启动和实施,具有示范性和适应应用型人才培养的精品课程教材必将进一步促进我国高校教学质量的提高。

全国高等学校教学研究中心

2003年4月

# 前 言

随着对外开放的日益深入以及全球一体化的快速推进,外语作为信息交流的重要工具正在全方位地向日常工作渗透。双语教学将技术教学与外语有机结合,有利于学生综合素质的全面提高,顺应时代发展的方向。适合双语教学,同时又与国内专业技术教学内容相适应的外文版教材,是目前教学第一线所急需的。本书就是考虑到这种需要而做的一种尝试。本书在数学知识的深广度上,与国内高校同名课程现行教学内容相当,专业技术符号系统与国内现行教学规范一致。内容包括行列式、矩阵、向量、线性方程组、矩阵的特征值与特征向量等。有些内容打了星号,这是考虑到本书有多种不同的用途。比如作为科技英语专业的数学教科书,受学时数量的限制就可以不讲带星号的内容。

双语教学使专业技术知识的学习与语言方面的困难交织在一起,增加了课程的难度。由于快速向高等教育大众化进军,近几年学生状况也有了一些新的变化。考虑到这两方面的情况,编写过程中特别注意把知识点的难度台阶分解拆细,使得表述深入浅出、便于自学。本书作者们来自数学与英语两个专业,编写过程中力求数学概念严谨清晰、语言表达规范顺畅。本书适合作为高等院校同名课程双语教学的配套教材,也可以作为英语专业的数学课程的教科书,以及数学专业、信息与计算专业的学科英语的阅读读物。

伴随着河北科技大学数学课程双语教学试验的进程,这一组配套教材的编写与反复修改已经进行了五个年头。本书是集体努力的结果,先后参加编写工作的人员有(以姓名笔划为序):仇计清、李丽霞、刘金宪、苏连青、郑克旺、韩骁兵、蔡习宁等。其中刘金宪、韩骁兵、苏连青、李丽霞任主编,郑克旺、仇计清、蔡习宁任审稿。

自1998年以来,本书在教学实践中已使用过5届,其间做过两次全面的修改。由于作者水平所限,疏漏甚至错误在所难免,恳请业界同仁不吝赐教。校领导、教务处、理学院、外语学院对数学课双语教学以及本书的编写一直给予大力的支持,数学系领导与老师们也给予多方面的帮助。在此一并表示由衷的谢意。最后,我们要特别感谢的是高等教育出版社李艳馥等编辑人员和天津大学熊洪允教授,他们宝贵的修改意见、严谨的学风使编者们受益匪浅。

编者

2003年8月1日

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# CHAPTER I

## DETERMINANTS

### § 1 Determinants of Order 2 and 3

#### 1.1 Determinants of order 2

Let  $a_{11}, a_{12}, a_{21}, a_{22}$  be numbers, the word numbers mean real numbers throughout the book.

We denote the algebraic sum  $a_{11}a_{22} - a_{12}a_{21}$  by the notation

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

which is called a *determinant of order 2*, or a  $2 \times 2$  *determinant*. That is to say

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}. \quad (1)$$

The sign of each term in the algebraic sum may be kept in mind

$$\begin{array}{c} \begin{array}{|c|c|} \hline a_{11} & a_{12} \\ \hline a_{21} & a_{22} \\ \hline \end{array} \\ \begin{array}{cc} (-) & (+) \end{array} \end{array}$$

by a diagonal line pattern. The product of two elements on the solid line is with positive sign, and the product of two elements on the dotted line with negative sign.

The determinant has 2 *rows*, 2 *columns* and 4 *elements*  $a_{11}, a_{12}, a_{21}, a_{22}$ .

For instance, the first column is  $\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$  and the second row is  $(a_{21}, a_{22})$ .

**Example 1.** Compute the second order determinant  $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$ .

**Solution.**

$$\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 2 \cdot 4 - (-1) \cdot 3 = 11.$$

**Example 2.** Let  $D_2 = \begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix}$ .

(a) What values would  $\lambda$  take on such that  $D_2 = 0$ ?

(b) What values would  $\lambda$  take on such that  $D_2 \neq 0$ ?

**Solution.**

$$D_2 = \begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3).$$

(a)  $D_2 = 0$  when  $\lambda = 0$  or  $\lambda = 3$ .

(b)  $D_2 \neq 0$  when  $\lambda \neq 0$  and  $\lambda \neq 3$ . □

The following properties are easily verified by direct computation, which you should carry out completely.

*If each element of one column, say the second column, is the sum of two numbers, then*

$$D = \begin{vmatrix} a_{11} & a_{12} + b_{12} \\ a_{21} & a_{22} + b_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix}.$$

The analogous property also holds with respect to the first column. We give the proof for the additivity with respect to the second column to show how easy it is. Namely, we have

$$\begin{aligned} \begin{vmatrix} a_{11} & a_{12} + b_{12} \\ a_{21} & a_{22} + b_{22} \end{vmatrix} &= a_{11}(a_{22} + b_{22}) - a_{21}(a_{12} + b_{12}) \\ &= (a_{11}a_{22} - a_{12}a_{21}) + (a_{11}b_{22} - a_{21}b_{12}) \\ &= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix} \end{aligned}$$

which is precisely the desired additivity.

*If  $k$  is a number, then*

$$\begin{vmatrix} a_{11} & ka_{12} \\ a_{21} & ka_{22} \end{vmatrix} = k \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} ka_{11} & a_{12} \\ ka_{21} & a_{22} \end{vmatrix}.$$

If the two columns are completely the same, then the determinant is equal to 0.

If one adds a same multiple of every element of one column to the corresponding element of the other column, then the value of the determinant does not change.

In other words, let  $k$  be a number, then

$$\begin{vmatrix} a_{11} + ka_{12} & a_{12} \\ a_{21} + ka_{22} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},$$

and similarly when we add a multiple of the first column to the second.

If the two columns are interchanged, then the determinant changes by a sign. In other words, we have

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = - \begin{vmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{vmatrix}.$$

The determinant obtained by interchanging rows and columns of  $D$  is called the *transpose* of  $D$ , and is denoted by  $D'$ . That is to say, let

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

then its transpose

$$D' = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}.$$

A determinant  $D$  is equal to its transpose, i. e.,

$$D = D'.$$

It is explicit that

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}.$$

## 1.2 Determinants of order 3

The notation

$$D_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

is called a *determinant of order 3*. It has 3 rows and 3 columns. We define  $D_3$  according to the formula known as the *expansion by a row*, say the first row. That is, we define

$$\begin{aligned} D_3 &= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \\ &= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}. \end{aligned} \quad (2)$$

We may describe this sum in another notation as follows. Let  $M_{ij}$  be the determinant obtained from  $D_3$  by deleting the  $i$ -th row and the  $j$ -th column. Then the sum expressing  $D_3$  can be rewritten as

$$D_3 = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}.$$

In other words, each term consists of the product of an element of the first row and the  $2 \times 2$  determinant obtained by deleting the first row and the  $j$ -th column, and putting the appropriate sign to this term as shown.

These can be seen clearly in the following example.

**Example 3.** Compute the third order determinant

$$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 4 \\ -3 & 2 & 5 \end{vmatrix}.$$

**Solution.**

$$M_{11} = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}, M_{12} = \begin{vmatrix} 1 & 4 \\ -3 & 5 \end{vmatrix}, M_{13} = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix},$$

and our formula (2) for the determinant of order 3 yields



$$\begin{aligned}
 D_3 &= 2 \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 4 \\ -3 & 5 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} \\
 &= 2(5 - 8) - (5 + 12) = -23. \quad \square
 \end{aligned}$$

The right hand side in the representation (2) is called the *formula of expansion by the first row*. We can also use the second row, and write a similar sum, namely

$$-a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \quad (3)$$

$$= -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}.$$

Again, each term is the product of  $a_{2j}$  and the  $2 \times 2$  determinant obtained by deleting the second row and the  $j$ -th column, and putting the appropriate sign in front of each term. This sign is determined according to the pattern

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}.$$

One can see directly that the determinant can be expanded according to any row by multiplying out all terms, and expanding the  $2 \times 2$  determinants, thus obtaining the determinant as an alternating sum of six terms:

$$\begin{aligned}
 D_3 &= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} \\
 &\quad + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}. \quad (4)
 \end{aligned}$$

Furthermore, we can also expand according to a column following the same principle. For instance, expanding out according to the first column:

$$a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \quad (5)$$

$$= a_{11}M_{11} - a_{21}M_{21} + a_{31}M_{31}$$

yields precisely the same six terms as in the expression (4).

**Example 4.** Compute the determinant

$$\begin{vmatrix} 3 & 0 & 1 \\ 1 & 2 & 5 \\ -1 & 4 & 2 \end{vmatrix}$$