国外数学名著系列（续一）
（影印版）65

A. N. Parshin　I. R. Shafarevich (Eds.)

Number Theory IV
Transcendental Numbers

数　论 IV
超越数

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

List of Editors, Authors and Translators

Editor-in-Chief

R.V. Gamkrelidze, Russian Academy of Sciences, Steklov Mathematical Institute, ul. Gubkina 8, 117966 Moscow; Institute for Scientific Information (VINITI), ul. Usievicha 20a, 125219 Moscow, Russia;
e-mail: gam@ipsun.ras.ru

Consulting Editors

A. N. Parshin, I. R. Shafarevich, Steklov Mathematical Institute, ul. Gubkina 8, 117966 Moscow, Russia

Authors

N. I. Fel'dman†
Yu. V. Nesterenko, Department of Mathematics, Moscow State University, Vorobevy Gory, 119899 Moscow; e-mail: nest@nw.math.msu.su

Translator

N. Koblitz, Department of Mathematics, University of Washington, Box 354350, Seattle, WA 98195, USA; e-mail: koblitz@math.washington.edu

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Preface

This book was written over a period of more than six years. Several months after we finished our work, N. I. Fel'dman (the senior author of the book) died. All additions and corrections entered after his death were made by his coauthor.

The assistance of many of our colleagues was invaluable during the writing of the book. They examined parts of the manuscript and suggested many improvements, made useful comments and corrected many errors. I much have pleasure in acknowledging our great indebtedness to them.

Special thanks are due to A. B. Shidlovskii, V. G. Chirskii, A. I. Galochkin and O. N. Vasilenko. I would like to express my gratitude to D. Bertrand and J. Wolfart for their help in the final stages of this book.

Finally, I wish to thank Neal Koblitz for having translated this text into English.

August 1997 *Yu. V. Nesterenko*

Transcendental Numbers

N. I. Fel'dman and Yu. V. Nesterenko

Translated from the Russian
by Neal Koblitz

Contents

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Notation

$\mathbb{N}$ is the set of natural numbers
$\mathbb{N}_0 = \mathbb{N}\bigcup\{0\}$
$\mathbb{Z}$ is the set of integers
$\mathbb{Q}$ is the set of rational numbers
$\mathbb{R}$ is the set of real numbers
$\mathbb{C}$ is the set of complex numbers
$\mathbb{A}$ is the set of algebraic numbers
$\mathbb{Z}_{\mathbb{A}}$ is the set of all algebraic integers
$\mathbb{Z}_{\mathbb{Z}}$ is the set of all algebraic integers of the field $\mathbb{K}$
$\mathbb{K}(z_1, \dots, z_m)$ is the set of all rational functions in the variables $z_1, \dots, z_m$ over the field $\mathbb{K}$
$\mathbb{K}[z_1, \dots, z_m]$ is the set of all polynomials in the variables $z_1, \dots, z_m$ over the field $\mathbb{K}$
$H(P(z)) = H(P)$ is the height of the polynomial $P(z) \in \mathbb{C}[z_1, \dots, z_m]$, i.e., the maximum absolute value of its coefficients
$L(P(z)) = L(P)$ is the length of the polynomial $P(z) \in \mathbb{C}[z_1, \dots, z_m]$, i.e., the sum of the absolute values of its coefficients
$\deg_{z_i} P$ is the degree in z_i of the polynomial P
$\deg P$ is the total degree of the polynomial P
$t(P) = \deg P + \ln H(P)$
$h(I)$ is the rank of the homogeneous ideal $I \subset \mathbb{Z}[x_0, \dots, x_m]$
$\deg I$ is the degree of the homogeneous ideal $I \subset \mathbb{Z}[x_0, \dots, x_m]$
$H(I)$ is the height of the homogeneous ideal $I \subset \mathbb{Z}[x_0, \dots, x_m]$
$t(I) = \deg I + \ln H(I)$
$|I(\omega)|$ is the magnitude of the homogeneous ideal $I \subset \mathbb{Z}[x_0, \dots, x_m]$ at the point $\omega \in \mathbb{C}^{m+1}$
$\deg \alpha$ is the degree of the algebraic number α
$H(\alpha)$ is the height of the algebraic number α
$L(\alpha)$ is the length of the algebraic number α
$\mathrm{Norm}(\alpha)$ is the product of all of the conjugates of the algebraic number α
$\lceil\overline{\alpha}\rceil$ is the maximum absolute value of the conjugates of the algebraic number α
$|\alpha|_{\mathfrak{p}}$ is the $\mathfrak{p}$-adic norm of the algebraic number α
$\|a\|$ is the distance from the number $a \in \mathbb{R}$ to the nearest integer
$\|x\| = \max_{1\le i\le m} |x_i|$ is the sup-norm of the vector $x = (x_1, \dots, x_m) \in \mathbb{C}^m$
$[a]$ is the greatest integer function of the real number a
$\operatorname{tr}\deg \mathbb{K}$ is the transcendence degree of the field $\mathbb{K}$
$\tau(\mathbb{K})$ is the transcendence type of the field $\mathbb{K} \subset \mathbb{C}$
$s(a)$ is the size of the complex number a
$\mu(a)$ is the exponent of irrationality of the real number a

Introduction

0.1. Preliminary Remarks. The purpose of this survey is to describe the basic concepts, results, and methods of transcendental number theory, along with its relationships to other branches of mathematics.

Definition 0.1. A number $\alpha \in \mathbb{C}$ is said to be *algebraic* if there exists a polynomial $P(z) \in \mathbb{Z}[z]$ not identically zero such that $P(\alpha) = 0$. We shall let $\mathbb{A}$ denote the set of all algebraic numbers. A complex number that is not algebraic is said to be *transcendental*. Clearly, a number ζ is transcendental if and only if the numbers $1, \zeta, \ldots, \zeta^n$ are linearly independent over $\mathbb{Q}$ for any n.

In addition to theorems about transcendence of certain numbers, transcendental number theory includes irrationality results and also certain types of quantitative results. Among the examples of quantitative theorems are those that give estimates for approximations of algebraic numbers by rational numbers.

The authors do not want to hide their fondness for transcendental number theory, but they do not expect that all readers will at first share this feeling. Hence, we shall start out by trying to explain what we find particularly fascinating in this field.

In the first place, in transcendental number theory many of the basic problems have very simple statements, but require deep and powerful methods to solve them. In many cases it took hundreds or even thousands of years to find solutions, and in other cases, despite the very simple formulation of the problem, we still do not have a solution. For example, it is still unknown whether $e + \pi$ is irrational.

In the second place, the theory of transcendental numbers has an ancient history. The seminal questions were asked by the mathematicians of antiquity.

The methods of transcendental number theory largely come from analysis and algebra. It should also be noted that, conversely, some of the results in this field can be used to help solve problems in other areas of mathematics.

0.2. Irrationality of $\sqrt{2}$. When the mathematicians of ancient times tried to find a common unit of measure for two intervals, they discovered that such a unit does not always exist. For example, the side and the diagonal of a square do not have a common unit of measure (it seems that this was first established by mathematicians of the Pythagorean school). This fact is clearly equivalent to the irrationality of $\sqrt{2}$. A proof can be found in the tenth book of Euclid's *Elements*, so the theorem on the irrationality of $\sqrt{2}$ may be regarded as the earliest theorem (with a proof given) in transcendental number theory. In recognition of the ancient history of this theorem, we shall give several proofs of it.

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