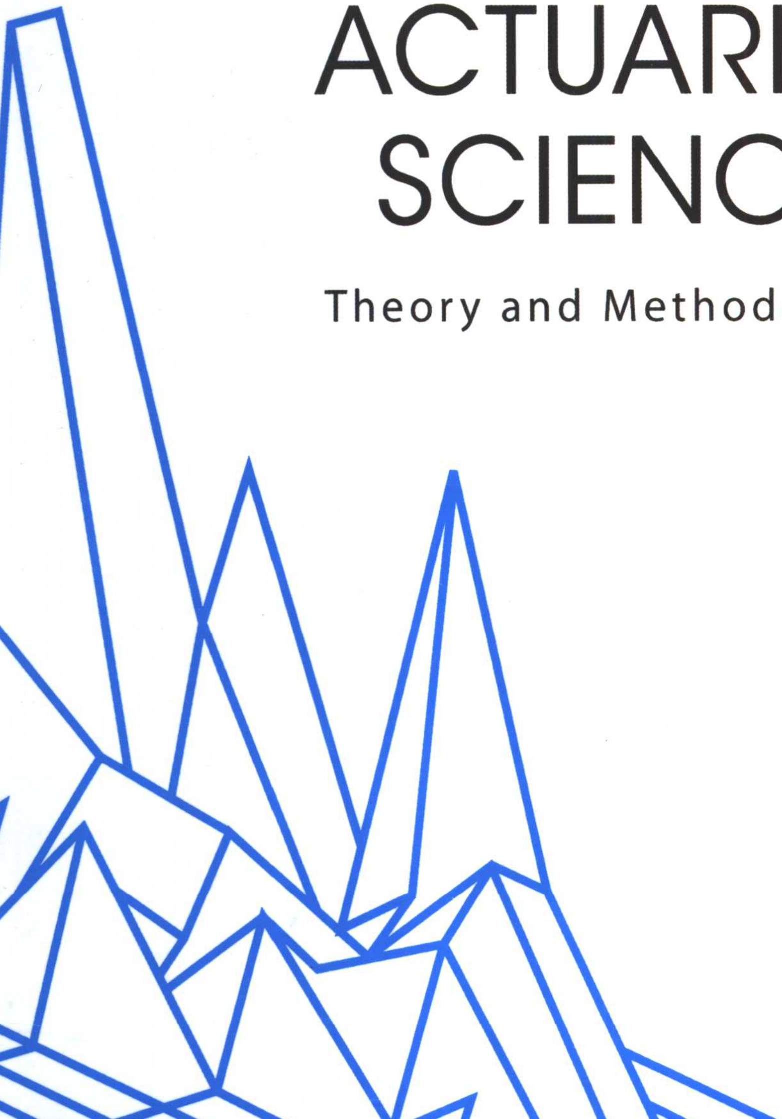
ACTUARIAL SCIENCE

Theory and Methodology



高等教育出版社
HIGHER EDUCATION PRESS

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World Scientific

NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI

图书在版编目 (CIP) 数据

精算学: 理论与方法 = Actuarial Science: Theory and Methodology / 尚汉冀 (Shang, Hanji) 主编.
— 北京 : 高等教育出版社, 2006.4
ISBN 7-04-019232-2

I. 精... II. 尚... III. 精算学—英文 IV. F224.0

中国版本图书馆 CIP 数据核字 (2006) 第 022967 号

Higher Education Press
4 Dewai Dajie, Beijing 100011, P. R. China, and
World Scientific Publishing Co Pte Ltd
5 Toh Tuch Link, Singapore 596224

ISBN 7–04–019232–2

Printed in P. R. China

Preface

Since actuarial education was introduced into China in 1980s, more and more attention have been paid to the theoretical and practical research of actuarial science in China.

In 1998, the National Natural Science Foundation of China approved a 1 million Yuan RMB financial support to a key project ≪Insurance Information Processing and Actuarial Mathematics Theory & Methodology≫ (project 19831020), which is the first key project on actuarial science supported by the government of China. From 1999 to 2003, professors and experts from Fudan University, Peking University, Institute of Software of Academia Sinica, East China Normal University, Shanghai University of Finance and Economics, Shanghai University and Jinan University worked together for this project, and achieved important successes in their research work. In a sense, this book is a summation of what they had achieved.

The book consists of seven chapters. Chapter 1 mainly presents the major results about ruin probabilities, the distribution of surplus before and after ruin for a compound Poisson model with a constant premium rate and a constant interest rate. This chapter also gives asymptotic formulas of the low and upper bounds for the distribution of the surplus immediately after ruin under subexponential claims. Chapter 2 introduces some recent results on compound risk models and copula decomposition. For the compound risk models, it includes the recursive evaluation of compound risk models on mixed type severity distribution in one-dimensional case, the bivariate recursive equation on excess-of-loss reinsurance, and the approximation to total loss of homogeneous individual risk model by a compound Poisson random variable. On the copula decomposition, the uniqueness of bivariate copula convex decomposition is proved, while the coefficient of the terms in the decomposition equation is given.

Chapter 3 is concerned with distortion premium principles and some related topics. Apart from the characterization of a distortion premium principle, this chapter also examines the additivities involved in premium pricing and reveals the relationship among the three types of additivities. Furthermore, reduction of distortion premium to standard deviation principle for certain distribution families is investigated. In addition, ordering problem for real-valued risks (beyond the nonnegative risks) is addressed, which suggests that it is more reasonable to order risks in the dual theory than the original theory.

Chapter 4 illustrates the application of fuzzy mathematics in evaluating and analyzing risks for insurance industry. As an example, fuzzy comprehensive evaluation is used to evaluate the risk of suffering from diseases related to better living conditions. Fuzzy information processing (including information distribution and information diffusion) is introduced in this chapter and plays an important role in dealing with the small sample problem. Chapter 5 presents some basic definitions and principles of Fuzzy Set Theory and the fuzzy tools and techniques applied to actuarial science and insurance practice. The fields of application involve insurance game, insurance decision, etc. Chapter 6 is concerned with some applications of financial economics to actuarial mathematics, especially to life insurance and pension. Combining financial economics, actuarial mathematics with partial differential equation, a general framework has been established to study the mathematical model of the fair valuation of life insurance policy or pension. In particular, analytic solutions and numerical results have been obtained for various life insurance policies and pension plans. Chapter 7 provides a working framework for exploring the risk profile and risk assessment of China insurance. It is for the regulatory objective of building a risk-oriented supervision system based on China insurance market profile and consistent to the international development of solvency supervision.

The authors of various chapters of this book are: Professor Rongming Wang of East China Normal University (Chapter 1), Dr. Jingping Yang of Peking University (Chapter 2), Dr. Xianyi Wu of East China Normal University, Dr. Xian Zhou of Hong Kong University and Professor Jinglong Wang of East China Normal University (Chapter 3), Professor Hanji Shang of Fudan University (Chapter 4), Professor Yuchu Lu of Shanghai University (Chapter 5), Professor Weixi Shen of Fudan University (Chapter 6) and Professor Zhigang Xie of Shanghai University of Finance & Economics (Chapter 7). As the editor, I am most grateful to all authors for their cooperation. I would like to thank Professor Tatsien Li, Professor Zhongqin

Xu and Professor Wenling Zhang. Their support is very important to our research work and to the publication of this book. I also thank Mr. Hao Wang for his effective work in editing the book.

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Contents

Chapter 1

Risk Models and Ruin Theory

Three particular questions of interest in classical ruin theory are (a) *the time of ruin*, (b) *the deficit at ruin*, and (c) *the surplus immediately before ruin*. From a mathematical point of view, a crucial role is played by the amounts of surplus before and after ruin. In this chapter we mainly present the major results about ruin probabilities, the distribution of surplus before and after ruin for a compound Poisson model with a constant premium rate and a constant interest rate. The compound Poisson risk process with a constant interest force is an interesting stochastic model in risk theory and it provides a basic understanding about how investments will affect the ruin probability and related ruin functions. At first, we provide some results on the severity of ruin for a compound Poisson model with a constant interest rate. Then, we investigate the distribution of surplus process immediately before ruin in particular. Equations satisfied by the distributions of surplus immediately before ruin and their Laplace transform are given. Some special cases are also discussed and Lundberg type bounds are presented. Next, by using the techniques of [Kalashnikov and Konstantinides(2000)] and a formula obtained by [Yang and Zhang (2001a)], we give asymptotic formulas of the low and upper bounds for the distribution of the surplus immediately after ruin under subexponential claims. Finally, a class of risk processes in which claims occur as a renewal process is studied. A clear expression for Laplace transform of the finite time ruin probability time is well given when the claim amount distribution is a mixed exponential. As its consequence, a well-known result about ultimate ruin probability in the classical risk model is obtained. All results in Sections 1 and 2 of this chapter are from [Yang and Zhang (2001a)] and [Yang and Zhang (2001b)]. The main results in Sections 3 and 4 are from [Wang *et. al.* (2004)] and [Wang and Liu(2002)].

1.1 On the Distribution of Surplus Immediately after Ruin under Interest Force

In this section, we consider the problem of the severity of ruin for a compound Poisson model with a constant interest rate. By using the techniques of [Sundt and Teugels (1995)], equations satisfied by the distributions of surplus immediately after ruin have been obtained. Some special cases are also discussed. Some results on the severity of ruin given in this section are similar to those in [Sundt and Teugels (1995)] on ruin probability.

1.1.1 *The Risk Model*

We use the same model as in [Sundt and Teugels (1995)]. Let $U_\delta(t)$ denote the value of the reserve at time t. $U_\delta(t)$ is given by

$$dU_\delta(t) = p\,dt + U_\delta(t)\;\delta\,dt - dX(t),$$

where p is a constant which denotes the premium rate that the insurance company receives and δ is the interest force,

$$X(t) = \sum_{j=1}^{N(t)} Y_j\ ,$$

where $N(t)$ denotes the number of claims occurring in an insurance portfolio in the time interval $(0,t]$ and Y_i denotes the amount of the ith claim. We assume that $\{N(t), t \geqslant 0\}$ is a homogeneous Poisson process with intensity λ, and $N(t)$ and Y_j are two independent processes. We further assume that Y_j (j $=$ 1, 2, ...) are positive and mutually independent and $\{Y_j\}$ is an identically distributed random sequence with common distribution F. We also assume that $F(0) = 0$ and $\mu_k = \int_0^\infty x^k dF(x)$. When $k = 1$, we denote $\mu = \mu_1$.

It follows from [Sundt and Teugels (1995)] that

$$U_\delta(t) = u\,e^{\delta t} + p\bar{s}_{\overline{t|}}^{(\delta)} - \int_0^t e^{\delta(t-v)} dX(v)\ ,$$

where $u = U(0) \geqslant 0$ and

$$\bar{s}_{\overline{t|}}^{(\delta)} = \int_0^t e^{\delta v} dv = \begin{cases} t, & \text{if} \quad \delta = 0, \\ \frac{e^{\delta t}-1}{\delta}, & \text{if} \quad \delta > 0. \end{cases}$$

For convenience, we will drop the index δ when the force of interest is zero.

Let $\psi_\delta(u)$ denote the ultimate ruin probability with initial reserve u. That is

$$\psi_\delta(u) = P\left\{ \bigcup_{t\geqslant 0} (U_\delta(t) < 0) \,\middle|\, U_\delta(0) = u \right\}.$$

We use $\bar{\psi}_\delta(u) = 1 - \psi_\delta(u)$ to denote the non-ruin probability (i.e. the probability that ruin never occurs).

It follows immediately from [Sundt and Teugels (1995)] that the integral equation is satisfied

$$\bar{\psi}_\delta(u) = \frac{p}{p+\delta u}\bar{\psi}_\delta(0) + \frac{1}{p+\delta u}\int_0^u \bar{\psi}_\delta(u-y)\{\delta + \lambda(1-F(y))\}dy. \quad (1.1)$$

Next, we are interested in the function $G_\delta(u, y)$, representing the probability of ruin beginning with initial reserve u and that the deficit at the time of ruin is less than $y > 0$.

$$G_\delta(u,y) = \Pr\left(T < \infty, -y < U_\delta(T) < 0 \,\middle|\, U_\delta(0) = u\right),$$

where T is the ruin time defined by

$$T = \inf\{t \geqslant 0 : U_\delta(t) < 0\}.$$

It is easy to see that $\psi_\delta(u) = \lim_{y\to+\infty} G_\delta(u,y)$.

For notational convenience, we define

$$\begin{aligned}
\bar{G}_\delta(u,y) &= \psi_\delta(u) - G_\delta(u,y) \\
&= P\left(T < \infty, U_\delta(T) \leqslant -y \,\middle|\, U_\delta(0) = u\right), \\
\bar{\bar{G}}_\delta(u,y) &= 1 - \bar{G}_\delta(u,y).
\end{aligned}$$

1.1.2 Equations for $\bar{\bar{G}}_\delta(u,y)$

1.1.2.1 *Integral Equations for $\bar{\bar{G}}_\delta(u,y)$, $\bar{G}_\delta(u,y)$ and $G_\delta(u,y)$*

In this subsection, we will try to obtain integral equations for $\bar{\bar{G}}_\delta(u,y)$, $\bar{G}_\delta(u,y)$ and $G_\delta(u,y)$. Using the renewal property of the surplus process, an integral equation, satisfied by the function which we are interested in, can be obtained. This is a commonly used technique in risk theory. Although we cannot, in general, solve the integral equation, some asymptotic results can often be obtained by using the integral equation. The main result of this subsection is given in the following theorem.

Theorem 1.1

$$\bar{\bar{G}}_\delta(u,y) = \frac{p}{p+\delta u}\bar{\bar{G}}_\delta(0,y) + \frac{1}{p+\delta u}\int_0^u \bar{\bar{G}}_\delta(u-z,y)[\delta+\lambda(1-F(z))]dz$$
$$+\frac{\lambda}{p+\delta u}\int_0^y (F(z)-F(u+z))dz, \tag{1.2}$$

$$\bar{G}_\delta(u,y) = \frac{p}{p+\delta u}\bar{G}_\delta(0,y) + \frac{1}{p+\delta u}\int_0^u \bar{G}_\delta(u-z,y)[\delta+\lambda(1-F(z))]dz$$
$$-\frac{\lambda}{p+\delta u}\int_0^u (1-F(z))dz - \frac{\lambda}{p+\delta u}\int_0^y (F(z)-F(u+z))dz, \tag{1.3}$$

$$G_\delta(u,y) = \frac{p}{p+\delta u}G_\delta(0,y) + \frac{1}{p+\delta u}\int_0^u G_\delta(u-z,y)[\delta+\lambda(1-F(z))]dz$$
$$+\frac{\lambda}{p+\delta u}\int_0^y (F(z)-F(u+z))dz. \tag{1.4}$$

Proof. The proof goes along the lines of page 10 of [Sundt and Teugels (1995)]. We only present the main steps of the proof. Notice that, given the first claim time $T_1 = t$ and the first claim amount $Y_1 = z$, the reserve just after the first claim is $ue^{\delta t} + p\cdot\frac{e^{\delta t}-1}{\delta} - z$. Therefore

$$\begin{aligned}\bar{\bar{G}}_\delta(u,y) &= E[\bar{\bar{G}}_\delta(ue^{\delta T_1} + p\frac{e^{\delta T_1}-1}{\delta} - Y_1, y)]\\ &= \int_0^{+\infty} \lambda e^{-\lambda t}\int_{(0,ue^{\delta t}+p\frac{e^{\delta t}-1}{\delta}+y)} \bar{\bar{G}}_\delta\left(ue^{\delta t} + p\frac{e^{\delta t}-1}{\delta} - z, y\right)\\ &\quad dF(z)dt\\ &= \lambda(p+\delta u)^{\frac{\lambda}{\delta}}\int_u^{+\infty}(p+\delta s)^{-\frac{\lambda}{\delta}-1}\int_0^{s+y}\bar{\bar{G}}_\delta(s-z,y)dF(z)ds,\end{aligned}$$

where the last equality is obtained by using the substitution

$$s = ue^{\delta t} + p\frac{e^{\delta t}-1}{\delta}.$$

By taking partial derivative of the above expression with respect to u, and rearrange the terms, we have

$$(p+\delta u)\frac{\partial}{\partial u}\bar{\bar{G}}_\delta(u,y) = \lambda\,\bar{\bar{G}}_\delta(u,y) - \lambda\int_0^{u+y}\bar{\bar{G}}_\delta(u-z,y)dF(z). \tag{1.5}$$

By integrating both sides of (1.5) from 0 to u, we have that the left hand side of (1.5) equals

$$(p+\delta u)\,\overline{\overline{G}}_\delta\,(u,y) - p\,\overline{\overline{G}}_\delta\,(0,y) - \delta\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv$$

and the right hand side of (1.5) equals

$$\lambda\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv + \lambda\int_0^u\int_0^{v+y} \overline{\overline{G}}_\delta\,(v-z,y)d(1-F(z))dv. \qquad (1.6)$$

After some calculation and rearrangement of the terms, the second term of the right hand side of (1.6) equals

$$\begin{aligned}&-\lambda\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv + \lambda\int_0^u (1-F(z))\,\overline{\overline{G}}_\delta\,(u-z,y)dz\\ &+\lambda\int_0^y \overline{\overline{G}}_\delta\,(-z,y)(F(z)-F(u+z))dz.\end{aligned}$$

So we have

$$\begin{aligned}&(p+\delta u)\,\overline{\overline{G}}_\delta\,(u,y) - p\,\overline{\overline{G}}_\delta\,(0,y) - \delta\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv\\ &=\lambda\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv - \lambda\int_0^u \overline{\overline{G}}_\delta\,(v,y)dv\\ &\quad+\lambda\int_0^u (1-F(z))\,\overline{\overline{G}}_\delta\,(u-z,y)dz\\ &\quad+\lambda\int_0^y \overline{\overline{G}}_\delta\,(-z,y)(F(z)-F(u+z))dz.\end{aligned}$$

Therefore,

$$\begin{aligned}\overline{\overline{G}}_\delta\,(u,y) = {}&\frac{p}{p+\delta u}\,\overline{\overline{G}}_\delta\,(0,y)\\ &+\frac{1}{p+\delta u}\int_0^u \overline{\overline{G}}_\delta\,(u-z,y)\big(\delta+\lambda(1-F(z))\big)dz\\ &+\frac{\lambda}{p+\delta u}\int_0^y \overline{\overline{G}}_\delta\,(-z,y)(F(z)-F(u+z))dz.\end{aligned}$$

By the definition of $\overline{\overline{G}}_\delta\,(u,y)$, we know that

$$\overline{\overline{G}}_\delta\,(-z,y)=1 \quad \text{for all} \quad z\in(0,y).$$

Based on this, we obtain Equation (1.2). Equations (1.3) and (1.4) can be obtained from Equations (1.1) and (1.2). □