英文版

数字信号处理

（英文版）

Digital Signal Processing

○ 蔡坤宝　编著
○ 江泽佳　尹霄丽　主审

電子工業出版社
PUBLISHING HOUSE OF ELECTRONICS INDUSTRY
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内容简介

本书系统地阐述了数字信号处理所涉及的信号与系统分析和系统设计的基本理论、基本分析与设计方法、基本算法和处理技术。全书共 10 章，主要内容包括：离散时间信号与系统的基本概念，离散时间信号与系统的变换域分析，包括 z 变换和离散时间傅里叶变换、连续时间信号的抽样与重建，离散傅里叶变换及其快速算法（FFT），数字滤波器实现的基本结构，IIR 和 FIR 数字滤波器的设计原理与基本设计方法，数字信号处理中的有限字长效应，多抽样率数字信号处理。本书配有多媒体电子课件、英文版教学大纲、习题指导与实验手册。

本书可以作为电子与通信相关专业的本科数字信号处理课程中英文双语教学的教材，或中文授课的英文版教学参考书，也可供从事数字信号处理的工程技术人员学习参考。本书尤其适合初步开展数字信号处理课程中英文双语授课的教师与学生选用。

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前 言

近年来，计算机和高速数字信号处理器的更新换代日新月异。与此同时，信号处理的新理论、新方法和高深的信号处理算法层出不穷。上述两方面的发展互为基础，相互促进，促使数字信号处理正朝着高新技术学科的目标飞速地发展。掌握信号处理的基本理论、分析与设计方法，并应用数字方法灵活地实现信号与系统的分析与设计，业已成为现代信息与通信工程师必备的基本技能。在我国，数字信号处理课程已成为信息与通信工程和电气工程本科生的必修课程，也成为其他学科学生的必修或选修课程。

在20余年信号处理的教学和科研实践中，我为本科开设数字信号处理课程的讲稿经过反复修改不断地得到提升，国内外已出版的优秀教材为本书提供了丰富的资源，遂使讲稿以教材的形式得以出版。本书提供了相对较完备的离散时间信号与系统的基本理论、分析与设计数字信号与系统的基本方法，因此，本书适合作为本科数字信号处理课程双语教学的教材，也适合于实际工作的工程师用做自学材料。

学习本课程的学生应当已经通过了电路分析基础和复变函数课程的考核，在连续时间信号与系统方面已经具备了较好的基础。

本教材的主要内容包含10章和6个附录。

第1章首先介绍信号、系统和信号处理的基本概念，然后讨论信号的分类，最后对数字信号处理的概况进行简要的描述。

第2章阐述离散时间信号与系统。本章首先介绍了序列的运算和若干基本的离散时间信号，然后从离散时间系统的数学定义出发，将系统分为静态与动态系统、线性与非线性系统、非时变与时变系统、因果与非因果系统、稳定与非稳定系统，以及无源与无损系统，接着在将离散时间线性非时变系统表示为线性卷积和的基础上，对线性非时变系统的基本的相互连接、稳定性和因果性进行了讨论；最后，提出了由线性常系数差分方程表征系统的输入与输出关系，并介绍了这些方程的时域解的求解方法。

第3章通过深入讨论将序列按照复变量z或复指数序列集合$\{e^{-j\omega n}\}$展开的表示方法，引出了两种非常有用的离散时间信号的变换域表示方法，即z变换和离散时间傅里叶变换（DTFT）。首先从z变换的定义及其收敛域着手，介绍了若干基本序列的z变换，详细地论述了逆z变换方法，包括围线积分法、部分分式展开法、幂级数展开法和借助于长除法的幂级数展开法，并对z变换的性质进行了详细的论述。接着从DTFT的定义与其收敛准则着手，对DTFT的性质进行了介绍。随后引入了频率响应的概念，对线性非时变系统的特征函数与特征值的概念、因果稳定且实的线性非时变系统的正弦稳态响应进行了阐述。在定义线性非时变系统的传递函数

之后，讨论了依据传递函数的极点与零点图进行频率响应的几何估算。接着介绍了频带有限的连续时间信号的抽样，以及由该信号的抽样值重建原信号。最后论述了 z 变换与拉普拉斯变换之间的关系。

第 4 章详细地论述了离散傅里叶变换（DFT）。DFT 是在时域和频域均为有限长且离散的一种傅里叶变换。本章首先介绍了离散傅里叶级数（DFS）的定义、逆离散傅里叶级数（IDFS）和 DFS 的性质，然后介绍了 DFT 与逆离散傅里叶变换（IDFT），并讨论了 DFT 的性质。接着介绍了用圆周卷积计算线性卷积的方法，讨论了实现线性非时变系统的重叠相加法。此外，对 z 变换的抽样与重建进行了论述，对应用 DFT 的连续时间信号的傅里叶分析进了行详细的阐述，还对包括混迭的缩减、频谱泄漏、频率分辨率和栅栏效应在内的实际应用中的问题进行了详细的论述。本章以 DFT 在连续时间正弦信号分析中应用的论述结尾。

第 5 章集中阐述了基本的快速傅里叶变换（FFT）算法。本章所包含的内容有按时间抽取的 FFT 基 2 算法和按频率抽取的 FFT 基 2 算法。

第 6 章介绍了实现 IIR 和 FIR 数字滤波器的若干基本结构。IIR 数字滤波器的基本结构包括直接 I 型、直接 II 型、级联型和并联型。FIR 滤波器的基本结构包括直接型、级联型、快速卷积型、线性相位型和频率抽样型。

第 7 章详细地论述了 IIR 数字滤波器的设计方法。首先介绍了包括数字滤波器设计的基础知识和技术指标在内的一些初步问题，详细讨论了由相位特性描述的离散时间系统的 4 种类型，即最小相位系统、最大相位系统、最大相位超前系统和最小相位超前系统，并仔细研究了全通系统的定义及其重要的相位特性。然后介绍了模拟滤波器转换为数字滤波器的变换方法，包括冲激响应不变变换法、阶跃响应不变变换法和双线性变换法。接着介绍了模拟原型滤波器的设计，包括模拟巴特沃思低通滤波器和模拟切比雪夫 I 型低通滤波器，也介绍了设计低通 IIR 数字滤波器的两种方法，这涉及冲激响应不变变换法和双线性变换法。最后，考虑带通、带阻和高通 IIR 数字滤波器设计的两种方法，即模拟频带变换法和数字频带变换法。

第 8 章主要涉及 FIR 数字滤波器的设计。从线性相位 FIR 数字滤波器的性质的论述开始，讨论了线性相位 FIR 滤波器的 4 种类型。在论述基本的窗函数后，介绍了设计线性相位 FIR 滤波器的窗函数法，包括低通、高通、带通和带阻线性相位 FIR 滤波器的设计。

第 9 章论述了数字系统的有限字长效应。从二进制数的表示及其量化误差的讨论开始，主要集中讨论了定点二进制表示和由舍入与截断产生的误差。然后介绍了由舍入和 A/D 转换产生的量化误差，提出了量化效应的统计分析及量化噪声在线性非时变系统的传输。接着详细地论述了 IIR 数字滤波器实现中的系数量化效应与舍入效应，讨论了 IIR 滤波器定点实现中动态范围的缩放比例。最后介绍了定点 FFT 实现中的极限环振荡和舍入误差。

第 10 章在导引水准上论述了多抽样率数字信号处理，旨在为学生选修通信系统、控制系统和数字信号处理等更高级别的课程做准备，这些课程或许与数字滤波器组、多维的多抽样率系统和小波变换有关。接着论述了数字信号的降抽样率与升抽样率的基本运算，这些运算是由两个基本的抽样率转换装置（即抽样率降低器和抽样率升高器）来执行的。然后介绍了由抽样率转换装置与数字滤波器的级联构成的抽取器和内插器。最后介绍了有理数因子的抽样率转换和抽样率转换的有效结构。

附录 A~附录 E 提供了 5 个非常有用的表格，这些表格可用做若干变换及其性质和归一化巴特沃思低通滤波器的汇总。附录 F 为各章尾的习题提供了部分答案。

本书向使用本书作为教材的教师提供免费配套习题指导与实验手册、英文版教学大纲和多媒体电子课件，请通过华信教育资源网下载，网址为 http://www.huaxin.edu.cn 或 http://www.hxedu.edu.cn。

在我国的许多大学里，数字信号处理课程平行地伴随着涉及信号处理和系统分析与设计实验的实践课程，在这样的课程中学生将接受基于 MATLAB 的数字信号处理专题的基本训练。无容置疑，牢固掌握本书中的概念对于用好 MATLAB 信号处理工具箱是必不可少的。

在本书正式出版之际，我向重庆大学电气工程学院的江泽佳教授和周守昌教授表示衷心的感谢，感谢他们在我长达 20 多年的教学和学术生涯中热情持久地给予我真诚的指导。另外，我真诚地感谢重庆大学电气工程学院的吴言逊教授，感谢他在 1986 年就鼓励我和我的同事开设数字信号处理课程。重庆大学的江泽佳教授、北京邮电大学电子工程学院的尹霄丽副教授和目前就职于英国伦敦 Bristol 大学的江丕书博士为本书的初稿提供了评审，我深深地感谢他们提出的宝贵意见、建议、修改和校正，他们的工作极大地提高了本书的质量。衷心地感谢重庆大学教务处在我撰写本书的过程中给予的经济资助。感谢重庆大学电气工程学院和通信工程学院提供了和谐的教学环境。此外，感谢以各自的方式曾帮助过我的所有老师、同事、学生和朋友。特别感谢电子工业出版社的策划编辑王羽佳和责任编辑谭海平与段丹辉，有了他们的热情支持与帮助和认真负责的工作才使本书得以顺利出版。最后向我的家属表示由衷的感谢，感谢她们在本书的撰写中给予持久的鼓励和支持。

敬请读者注意，我将十分感激你们指出本书中的任何错误，这些错误以及任何建议和批评请发到我的电子邮箱：ckb@ccee.cqu.edu.cn。

蔡坤宝

2007 年 6 月

Preface

In recent years, computers and high-speed digital signal processors are updated more and more rapidly. At the same time, new theory and methods of signal processing as well as sophisticated signal processing algorithms are emerged endlessly. The development of the above two aspects are based and promoted with each other, with the result that the field of digital signal processing has being explosively progressed as a new high-tech subject. It has become a basic requirement for modern information and communication engineers to master the basic theory and the analysis and design techniques of signal processing, and to implement the analysis and design of signals and systems using digital methods flexibly. In our country, the course of digital signal processing has become a required course for undergraduate students in information and communication engineering and electrical engineering, and an optional or necessary course for students in other disciplines.

In more than twenty-year teaching and research practice on signal processing, my lecture notes on digital signal processing for undergraduate level course were repeatedly promoted by revisions and fueled by many excellent textbooks published at home and abroad, which lead this book to be published in a present form. In this book a relatively complete set of basic principles of discrete-time signals and systems and fundamental techniques for analyzing and designing digital signals and systems are presented. Thus, this book is suitable for undergraduate level course on digital signal processing. It is also suitable for practicing engineers to use this book as self-study material. Particularly, I hope that this book is suitable for a bilingual teaching course on digital signal processing in universities of China.

It is assumed that the student selecting this course has successfully passed the courses of fundamentals of circuit analysis and complex variables, and has had a good foundation of continuous-time signals and systems.

The main contents of this book are contained in 10 chapters and 6 appendixes.

Chapter 1 presents at first an introduction to the basic concept about signals, systems and signal processing. Then the classification of signals is discussed. Finally, an overview of digital signal processing is briefly described.

Chapter 2 is devoted to the discrete-time signals and systems. It starts with the time-domain representation of a discrete-time signal as a sequence of numbers. Then the operations on sequences and some basic discrete-time signals are introduced. The discussion of discrete-time systems begins with the mathematical definition of discrete-time systems, and then the systems are classified into static and dynamic systems, linear and nonlinear systems, time-invariant and time-varying systems, causal and noncausal systems, stable and unstable systems, as well as passive and lossless systems. Based on the representation of discrete-time LTI systems as the linear convolution sum, some fundamental interconnections and the stability and causality of LTI systems are discussed. Finally, an important class of systems whose input-output relation is characterized by linear constant-coefficient difference equations is presented and the time-domain solution of these equations is introduced.

Chapter 3 develops an alternate representation of a sequence in terms of a complex variable z or a set of complex exponentials in the form of $\{e^{-j\omega n}\}$, which leads to two particularly useful transform-domain representations of discrete-time signals, namely, the z-transform and the discrete-time Fourier transform (DTFT). Starting with the definition of the z-transform and its region of convergence, the z-transform of some basic sequences are presented. The inverse z-transform techniques are provided in detail, including the contour integral method, the partial-fraction expansion method, the power series expansion and the power series expansion by long division. Then the properties of the z-transform are discussed in detail. The discrete-time Fourier transform is introduced through its definition and convergence criteria, and its properties are next presented. The concept of the frequency response of discrete-time LTI systems is then introduced. The fundamental concept of the eigenfunction and eigenvalue of LTI systems along with the sinusoidal steady-state response of a causal, stable and real LTI system is treated. Next, the transfer function of LTI systems is defined, followed by a discussion of the geometric evaluation of the frequency response in terms of pole-zero plots of the transfer function. Finally, the sampling of a bandlimited continuous-time signal and the reconstruction of such a signal from its samples are introduced. This chapter is concluded with a discussion of the relations of the z-transform to the Laplace transform.

Chapter 4 is devoted to the discrete Fourier transform (DFT), which is such a Fourier transform with both finite-length and discrete in the time domain and the frequency domain. In this chapter, the definition of discrete Fourier series (DFS), the inverse discrete Fourier series (IDFS) and the properties of DFS are introduced. Then the DFT and the inverse discrete Fourier transform (IDFT) are presented, followed by a discussion of properties of the DFT. Next, a linear convolution evaluated by the circular convolution is discussed in detail, followed by a discussion of the overlap-add method implementing linear time-invariant systems. The sampling and reconstruction in the z-transform is also discussed. The Fourier analysis of continuous-time signals using the DFT is treated in detail. Practical considerations including the aliasing reduction, the spectral leakage, the frequency resolution and the palisade effect are explained. This chapter is concluded with a treatment of the application of the DFT in analyzing continuous-time sinusoidal signals.

Chapter 5 focuses on the fast Fourier transform (FFT) algorithms. Included in this chapter are descriptions of the decimation-in-time FFT algorithm with radix-2 and the decimation-in-frequency FFT algorithm with radix-2.

Chapter 6 introduces some basic structures for the realization of IIR and FIR digital filters. The basic structures for IIR digital filters include the direct form I, direct form II, cascade form and parallel form, while the basic structures for FIR filters include the direct form, cascade form, fast convolution form, linear phase form and frequency sampling form.

Chapter 7 is entirely devoted to the design techniques of IIR digital filters. First, preliminary considerations including some fundamental knowledge and specifications for designing digital filters are presented, and four types of discrete-time systems characterized by phase properties are discussed in detail, including the minimum- and maximum-phase systems, and the maximum phase-lead and minimum phase-lead systems. The definition and important phase property of allpass systems are carefully considered. Second, the transformations of analog-to-digital filter are introduced, including the impulse invariance transformation, step invariance transformation and the bilinear transformation. Then the design of analog prototype filters is introduced, which includes the analog Butterworth lowpass filters and the analog Chebyshev-I lowpass filters. Two techniques of the design of lowpass IIR digital filters are presented, concerning with the impulse invariance and the bilinear transformation. Finally, the design of the bandpass, bandstop and highpass IIR digital filters is considered with two different methods, namely, the analog frequency-band transformations and the digital frequency-band transformations.

Chapter 8 deals with the design of FIR digital filters. It begins with a discussion of the properties of linear phase FIR filters, followed by a discussion of four types of linear-phase FIR filters. After a discussion of basic window functions, the design techniques using windows for linear-phase FIR filters are introduced, including the design of lowpass, highpass, bandpass and bandstop linear-phase FIR filters.

Chapter 9 treats the finite-wordlength effects in digital systems. It starts a discussion with binary number representations and their quantization errors that are mainly concentrated on the fixed-point binary representation and the errors resulting from rounding and truncation. Then, the quantization errors arisen from rounding and A/D conversion are introduced, and the statistical analysis of quantization effects and the transmission of the quantization noise through LTI systems are presented. Next, the coefficient quantization effects and round-off effects in the realization of IIR digital filters are discussed in detail, also the dynamic range scaling in fixed-point implementations of IIR filters are discussed. Finally, the limit-cycle oscillations and the round-off errors in fixed-point FFT implementation are introduced.

Chapter 10 gives a discussion of the multirate digital signal processing at an introductory level, which is designed to prepare students for upper-level courses in communication systems, control systems and the digital signal processing that may relate to digital filter banks, multidimensional multirate systems and wavelet transforms. With such an aim, the discussion begins with basic operations of downsampling and upsampling for digital signals, which are performed by using two basic sampling rate conversion devices, namely, downsampler and upsampler. Then, the decimator and interpolator formed by a cascade of the sampling rate conversion devices and digital filters are presented. Finally, the sampling rate conversion by a rational factor and some efficient structures for sampling rate conversion are introduced.

This book contains seven appendixes. Appendixes from A to E provide six very useful tables that can be used as a summary for several transforms and their properties and for the normalized Butterworth lowpass filters. Appendix F provides partial answers for many end-of-chapter problems.

The book is accompanied by a detailed solution manual for all end-of-chapter problems in the book. A copy of the manual is only available to instructors adopting this book for use in classrooms and may be obtained by writing to the publisher.

This book is designed for junior or senior students taking the bilingual teaching course on digital signal processing in universities of China. Before taking this course, they have generally received basic or much deeper training on the use of MATLAB software from prerequisite courses. In many universities of China, the course of the digital signal processing is parallelly accompanied by a practical course concerning experiments on signal processing and system analysis and design, in which students will accept fundamental training on the topics of digital signal processing based-on MATLAB. It is firmly believed that a good understanding of the concepts considered in this book is essential for intelligent use of the signal processing toolbox in MATLAB.

Please note that I would appreciate readers bringing to my attention any errors that may appear in the printed version. These errors or any suggestions and comments can be communicated to me by E-mail addressed to ckb@ccee.cqu.edu.cn.

As this book is published formally, I would like to express my sincere thanks to Professor Zejia Jiang and Professor Shouchang Zhou in the College of Electrical Engineering of Chongqing University for enthusiastically and persistently instructing me in my teaching and academic career in a long period of more than twenty years. In addition, I would like to express my sincere thanks to Professor Yanxun Wu in the College of Electrical Engineering of Chongqing University for his great encouragement to my colleagues and myself to offer the digital signal processing course in 1986. Professor Zejia Jiang, associate Professor Xiaoli Yin in the Electronic Engineering College of Beijing Telecommunications University and Dr. Pisu Jiang who is now at University of Bristol in London provided reviews of the manuscript. I deeply thank all of them for their valuable comments, suggestions, correction and proofreading, which have improved the book tremendously. I wish to express my gratitude to the Office of Educational Administration of Chongqing University for financial support in writing this book, and to Colleges of Electrical Engineering and Communication Engineering of Chongqing University for providing harmonious teaching environment. In addition, I would like to express my thanks to all of my teachers, colleagues, students and friends for helping me in their own individual ways. Particularly, I am indebted to planning editor Yujia Wang and responsible editors Haiping Tan and Danhui Duan of Publishing House of Electronics Industry in China. Their enthusiastic support and help, and earnest word lead this book to be published successfully.

Finally, I would like to thank my families for their endless encouragement, patience and support.

Kunbao Cai

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1 Introduction

1.1 What Is a Signal?

Signals, in one form or another, constitute a basic ingredient of our daily lives. For example, a common form of human communication takes place through the use of speech signals, which may be in a face-to-face conversation or over a telephone channel. Another common form of human communication is visual in nature, with the signals taking the form of images of people or objects around us. Indeed, there are so many signals encountered in our living environment that the list of signals is almost endless.

Generally speaking, signals are a carrying body to convey information, while the information is contents embodied in signals. However, signals, in a narrow sense, are mathematically defined as a function of one or more independent variables that conveys information on nature of a physical phenomenon. When the function depends on a single independent variable, the signal is said to be one-dimensional. For Example, speech and music signals represent air pressure as a function of time at a point in space. When the function depends on two or more independent variables, the signal is said to be multidimensional signal. For example, a black-and-white picture is a representation of light intensity as a function of two spatial coordinates; a video signal in television consists of a sequence of images, called frames, and is a function of three independent variables that are two spatial coordinates and time.

Generally, a signal is a function of independent variables such as time, distance, position, temperature, pressure and etc. It is a common convention that the independent variable of the mathematical representation of a single variable signal will be represented to as time in this textbook, although it may in fact not represent time.

1.2 What Is a System?

A system, in its most general form, is defined as a combination and interconnection of several components to perform a designed task. For examples, the human physiology system, ecological system, communication system, electric power system and global positioning system are all the real-world systems, in a wide sense. However, a system, in a narrow sense, is mathematically defined as a transformation or operator that maps an input signal into an output signal. Specifically, a discrete-time system can be denoted as

$$y(n) = T[x(n)] \tag{1.1}$$

where $x(n)$ is the input signal, $y(n)$ is the output signal and 'T' is an operator which represents a rule or computation applied to the input signal to yield the output signal. Such a system is often depicted using a block diagram shown in Figure 1.1.

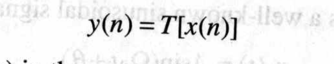

Figure 1.1 Block diagram representation for the system

A basic structure of commonly used communication systems is depicted in Figure 1.2. There are three basic elements in this system, namely, transmitter, channel and receiver. Functionally, the transmitter changes the message signal into a form suitable for transmission over the channel, the channel is the physical medium that connects the transmitter and receiver, and the receiver processes the channel output to produce an estimate of the message signal for a user.

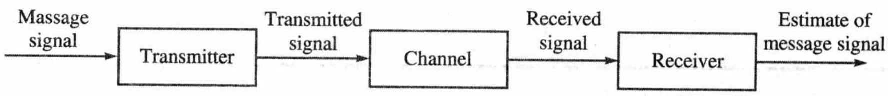

Figure 1.2 Basic structure of a communication system

1.3 What Is Signal Processing?

Signal processing is concerned with the representation, transformation, and manipulation of signals and the information they contain. For example, we may wish to separate two or more signals that have been mixed together, or we may want to enhance some signal components or estimate some parameters of a signal model. In communication systems, it is commonly necessary to do pre-processing such as modulation, signal conditioning, and compression prior to transmission over a channel and then to post process at the receiver. The technology for signal processing was almost exclusively continuous-time analog technology until 1960s. The rapid development of digital computers and micro processors together with some important theoretical progress such as the fast Fourier transform (FFT) algorithm caused a major shift to digital technologies, giving rise to the field of digital signal processing.

1.4 Classification of Signals

In this textbook, we will restrict our attention to one-dimensional signals which are defined as single-valued functions of independent variable time. "Single-valued" means that for every specified instant of time there is a unique value of the function except for the discontinuities of the function. The value of a signal at a specified time is called its amplitude. The variation of the amplitude as a function of the independent time variable is called its waveform. The classification of signals is a basic problem in the field of signal processing, because different types of signals concern with different representations and processing methods.

1.4.1 Deterministic and Random Signals

According to the certainty of some features of general signals, the signals can be classified into two sets, that is, deterministic signals and random signals.

1. Deterministic Signals

A deterministic signal is such a signal about which there is no uncertainty with respect to its value at any specified time. Thus, a deterministic signal can be completely described by a known function of time.

A typical deterministic signal is a well-known sinusoidal signal, that is,

$$x_c(t) = A\sin(\Omega_0 t + \theta) \tag{1.2}$$

where A is its amplitude, Ω_0 is its angular frequency with units radians per second (rad/s), and θ is its initial phase with units radians (rad).

2. Random Signals

A random signal is such a signal about which there is uncertainty before it occurs. In other words, a random signal is generated in a random fashion and cannot be predicted ahead of time. Thus, a random signal cannot be described by a deterministic function. According to the theory of random processes, such a signal may be viewed as one realization of an ensemble of signals, with each signal in the ensemble having a different waveform. Moreover, each signal within the ensemble has a certain probability of occurrence. The ensemble of signals is called a random process.

A typical random signal is the random initial-phase sinusoidal signal, that is,

$$x_c(t) = A\sin(\Omega_0 t + \varphi) \tag{1.3}$$

where the amplitude A and the angular frequency Ω_0 are constants, and the initial phase φ is a random variable with a probability density function $p(\varphi) = 1/2\pi$. Although the amplitude and angular frequency of the signal are constants, the initial phase cannot be predicted before it is generated.

Another random signal is thermal noise generated in the amplifier of a radio or television receiver. Its amplitude fluctuates between positive and negative values in a complete random fashion.

1.4.2 Continuous-Time and Discrete-Time Signals

According to the continuity of the independent time variable for signals, the signals can be classified into two classes as follows.

1. Continuous-Time Signals

A signal $x(t)$ is said to be a continuous-time signal, if it is defined in the continuous-time domain. However, it is not necessary for the amplitude of the signal to be continuous at any time instants. In other words, a continuous-time signal may be undefined at a finite number of discrete time instants.

Furthermore, the continuous-time signals can be classified into two subclasses. One is the analog signals whose time variable and amplitude are all continuous. Such a signal is shown in Figure 1.3(a). Another is the quantized boxcar or stairstep signals whose time variable is continuous while its amplitude takes discrete values with finite precision. Such a signal is shown in Figure 1.3(b).

2. Discrete-Time Signals

A discrete-time signal is defined only at discrete time instants. Thus, the independent variable of the signal takes discrete values only, which are usually uniformly spaced on the time axis. However, the amplitude of a discrete-time signal may take infinite-precision values or finite-precision values. A discrete-time signal with discrete-valued amplitude represented by a finite number of digits is the so-called digital signal.

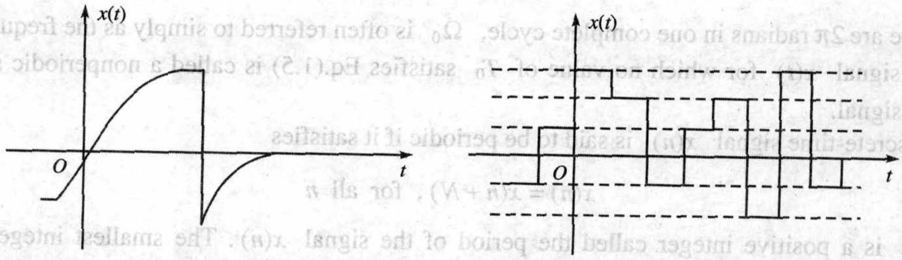

Figure 1.3 Continuous-time signals: (a) an analog signal; (b) a quantized boxcar signal

A discrete-time signal is often derived from a continuous-time signal by sampling it at a uniform rate, that is,

$$x(n) = x_c(nT) = x_c(t)|_{t=nT}, \quad n = 0, \pm 1, \pm 2, \cdots \tag{1.4}$$

where T denotes the sampling period with units seconds (s), n denotes an integer that may assume positive and negative values, however, it corresponds to time. Such a signal is shown in Figure 1.4(a),