On Economic Inequality

Amartya Sen

Enlarged edition with a substantial annexe
'*On Economic Inequality* after a Quarter Century'

James Foster and Amartya Sen

CLARENDON PRESS • OXFORD
1997

Oxford University Press, Great Clarendon Street, Oxford OX2 6DP

Oxford New York
Athens Auckland Bangkok Bogota Bombay
Buenos Aires Calcutta Cape Town Dar es Salaam
Delhi Florence Hong Kong Istanbul Karachi
Kuala Lumpur Madras Madrid Melbourne
Mexico City Nairobi Paris Singapore
Taipei Tokyo Toronto

and associated companies in
Berlin Ibadan

Oxford is a trade mark of Oxford University Press

Published in the United States
by Oxford University Press Inc., New York

First edition published in cloth and paperback 1973
First edition reprinted in paperback 1978

British Library Cataloguing in Publication Data
Data available

Library of Congress Cataloging in Publication Data
Data available

ISBN 0–19–829297–X
ISBN 0–19–828193–5 (Pbk)

1 3 5 7 9 10 8 6 4 2

Typeset by BookMan Services
Printed in Great Britain
on acid-free paper by
Biddles Ltd.,
Guildford and King's Lynn

Dedication to the first edition

To Antara and Nandana

with the hope that when they grow up
they will find less of it no matter
how they decide to measure it

Preface to the First Edition

The idea of inequality is both very simple and very complex. At one level it is the simplest of all ideas and has moved people with an immediate appeal hardly matched by any other concept. At another level, however, it is an exceedingly complex notion which makes statements on inequality highly problematic,[1] and it has been, therefore, the subject of much research by philosophers, statisticians, political theorists, sociologists and economists. While this book is concerned with economic inequality only, the presentation reflects this duality. I have had to employ a fair number of technical concepts and use some mathematical operations, but the concepts have also been explained in non-technical terms and the mathematical results have been given intuitive explanation. It is hoped that the non-technical reader will not be put off by the formalities. The importance of the formal results lies ultimately in their relevance to normal communication and to things that people argue about and fight for.

While the technical and non-technical sections have not been put into separate compartments, it should be possible for someone not interested in technicalities to skip (or skim through) the formal sections and to go directly from the intuitive presentation of the axioms to the intuitive explanation of the results. The section headings used throughout the book should help the reader in this sorting out.

In many ways this book is a development of some ideas I studied in my *Collective Choice and Social Welfare*.[2] The framework of thought presented there I have tried to apply here to the specific field of economic inequality. The approaches to social evaluation that I rejected then, I reject more strongly now, and what I defended in that work, I have tried to develop

[1] See Bernard Williams, 'The Idea of Equality', in P. Laslett and W. G. Runciman, *Philosophy, Politics and Society*, Second Series, Blackwell, Oxford.

[2] Holden-Day, San Francisco, 1970, and Oliver & Boyd, Edinburgh, 1971, Mathematical Economics Texts, No. 5.

more fully in this one in the particular context of inequality. No apologies for that, but I ought to put my cards on the table.

I owe debts to many. While preparing the Radcliffe Lectures, I was working with Partha Dasgupta and David Starrett on a joint paper on the measurement of economic inequality.[3] I am grateful to them not only because I have incorporated into the lectures some results from our joint paper (in particular, Theorems 3.1 and 3.2), but also because I have learnt a great deal from them and I have used that knowledge quite freely.

The Radcliffe Lectures, which were delivered last May, were informally presented, and in the discussions that followed I have gained much. I should particularly mention the searching questions raised by David Epstein, John Muellbauer, Graham Pyatt and John Williamson. In revising the lectures for this book, I have expanded some sections, incorporating not merely those things that I could not put into the lectures because of shortage of time or because of stylistic limitations (footnotes sound nasty in a lecture), but also some additional bits which are essentially responses to the queries raised. I have also benefited from discussions following my lectures on related topics at Essex University (Economics Department Seminar, January 1972), Columbia University (Joint Seminar of Economics and Philosophy Departments, March 1972), Harvard University (Political Economy Lecture, March 1972), the Delhi School of Economics (Special Lectures, August 1972), and the Indian Statistical Institute (Research Seminars, August 1972). I am grateful to Tony Atkinson, Pranab Bardhan, Nikhiles Bhattacharya, Sanjit Bose, Terence Gorman, Peter Hammond, and Richard Layard, for helpful comments and criticisms. This is a long list, and there must have been others.

For astonishingly skilful typing against the heavy odds of my impossible writing, I am very grateful to Celia Turner and Luba Mumford.

[3] 'Notes on the Measurement of Inequality', *Journal of Economic Theory*, Vol. 3 (1973).

Finally I am most grateful to the University of Warwick, and in particular to Professor Graham Pyatt, for the honour of an invitation to deliver the Radcliffe Lectures for this year.

London School of Economics A.K.S.
November, 1972

Preface to the Enlarged Edition

The first edition of this book was based on my Radcliffe Lectures at the University of Warwick, given nearly a quarter century ago, in 1972. It was meant as a contribution to the newly developing technical literature on economic inequality, while also attempting to integrate that literature with substantive issues that make inequality a matter of great practical interest. Even though a large part of the book was devoted to analytical and mathematical reasoning, the axioms used as well as the results presented were interpreted in intuitive terms. The work was based on the belief that 'the importance of the formal results lies ultimately in their relevance to normal communication and to things that people argue about and fight for' (p. vii).

In this enlarged edition, with a substantial annexe (as large as the original book), the motivational commitments remain much the same. Over the last quarter of a century, issues of inequality have become even more central (and also more contentious) in public debates and arguments. At the same time, an enormous—and often formidable—technical literature has grown and flourished in the pure theory of the measurement and evaluation of economic inequality. Some of the analytical issues partially examined in the original edition of this book have become much more fully explored, and some results presented there have been consolidated or substantially extended. And many new issues have been identified and successfully investigated.

The Annexe is largely an attempt to examine and assess the present state of the analytical literature on the measurement of inequality and poverty. I have worked on it jointly with James Foster, who has co-authored it. Foster has been an ideal collaborator, not only because of his superb skills and congenial temperament, but also because of his mastery of the relevant literature. Indeed, Foster has himself made several of the major contributions in the recent theoretical develop-

ments in the measurement and evaluation of inequality and poverty.

In writing this annexe, James Foster and I have had to weigh the intellectual interest in, and the practical importance of, the diverse investigations and results that have been presented in the monumental literature that has developed over the last quarter of a century on this subject. Our focus has been on the 'substance' of the analytical results, rather than on technical details. For those interested in pursuing a more fiercely technical course, we have tried to provide reasonably comprehensive references to the formal literature, with identification of the issues addressed and the general nature of the results obtained. We have also tried to clarify, in accessible terms, the main technical issues underlying the formal literature.

The 1972 Radcliffe Lectures were much influenced by lines of formal reasoning developed in social choice theory, pioneered by Kenneth Arrow.[1] I was then—as I still am—greatly involved in this field. The analyses in the first edition of this book used a distinctly 'social choice perspective'.[2] The 1973 edition of *On Economic Inequality* had, among other things, included proposals for making social choice theory more directly relevant to policy judgements as well as to public debates and social criticism. As it happens, the literature on social choice theory has also dramatically expanded since the early 1970s (to a great extent in the direction hoped for in the 1973 edition).[3] The Annexe takes note, *inter alia*, of these

[1] K. J. Arrow, *Social Choice and Individual Values* (Wiley, New York, 1951). From a different direction, A. B. Atkinson's works on inequality measurement much influenced the 1972 Radcliffe Lectures, as did the exploration of social justice by John Harsanyi, Serge Kolm, John Rawls, and Patrick Suppes.

[2] *On Economic Inequality* was, in many ways, a follow-up of my earlier book, *Collective Choice and Social Welfare* (Holden-Day, San Francisco, 1970; republished, North-Holland, Amsterdam, 1979).

[3] For an account and critical assessment of the technical literature in social choice theory until about the middle 1980s, see my 'Social Choice Theory' in K. J. Arrow and M. Intriligator (eds.), *Handbook of Mathematical Economics* (North-Holland, Amsterdam, 1986); see also K. Suzumura, *Rational Choice, Collective Decisions and Social Welfare* (Cambridge University Press, Cambridge, 1983).

explorations and results, and examines their bearing on the evaluation and measurement of inequality and poverty.

Earlier versions of the Annexe were read by Sudhir Anand, Tony Atkinson, and Tony Shorrocks, and their comments and suggestions have been particularly helpful in revising it. Over the years, we have also benefited from interactions with Kenneth Arrow, Fabrizio Barca, Kaushik Basu, Charles Blackorby, Andrea Brandolini, Satya Chakravarty, Frank Cowell, G. A. Cohen, Partha Dasgupta, Angus Deaton, David Donaldson, Jean Drèze, Bhaskar Dutta, Ronald Dworkin, Gary Fields, Peter Hammond, Wulf Gaertner, Nanak Kakwani, Ravi Kanbur, Peter Lambert, John Muellbauer, Robert Nozick, Martha Nussbaum, Siddiq Osmani, Prasanta Pattanaik, Derek Parfit, Douglas Rae, Martin Ravallion, John Rawls, V. K. Ramachandran, John Roemer, Thomas Scanlon, David Starrett, Nicholas Stern, Kotaro Suzumura, Larry Temkin, Philippe Van Parijs, John Weymark, Peyton Young, and Stefano Zamagni, among others, and both James Foster and I would like to take this opportunity of thanking them all for their help. James Foster would also like to express his deep appreciation to Irene Raj Foster for her help and support, and I join James warmly in this. We have received research assistance of the highest quality from Arun Abraham, and we are grateful to him.

Acknowledgement is also due to the MacArthur Foundation for supporting the research on which the Annexe has drawn. I am, furthermore, indebted to STICERD, at the London School of Economics, and to the Bank of Italy, for giving me research facilities when I respectively visited them.

The material included in the first edition of this book has been left quite unchanged in this enlarged edition. Even the old page numbers have been retained as far as possible (to facilitate reference). The new Annexe ('*On Economic Inequality* after a Quarter Century'), by James Foster and myself, follows those pages, and takes the story from there on.

Cambridge, Massachusetts A.K.S.
September, 1996

Contents

1. Welfare Economics, Utilitarianism, and Equity 1
2. Measures of Inequality 24
3. Inequality as a Quasi-Ordering 47
4. Work, Needs, and Inequality 77

Annexe: '*On Economic Inequality* after a Quarter Century' by James Foster and Amartya Sen 107

Bibliography 221
Index of Names 253
Subject Index 257

1

Welfare Economics, Utilitarianism, and Equity

'Of all human sciences the most useful and most imperfect appears to me to be that of mankind: and I will venture to say the single inscription on the Temple of Delphi[1] contained a precept more important and more difficult than is to be found in all the huge volumes that moralists have ever written.' Thus wrote Jean-Jacques Rousseau in the Preface to his *A Dissertation on the Origin and Foundation of the Inequality of Mankind*, dedicated to the Republic of Geneva on the 12th of June 1754. While the essay, alas, failed to qualify for the prize of the Dijon Academy for which it was considered (and which his less rebellious earlier piece on 'arts and sciences' had received in 1750), the ideas contained in it did help to crystalize the demands that gripped the revolution of 1789.

The relation between inequality and rebellion is indeed a close one, and it runs both ways. That a perceived sense of inequity is a common ingredient of rebellion in societies is clear enough, but it is also important to recognize that the perception of inequity, and indeed the content of that elusive concept, depend substantially on possibilities of actual rebellion. The Athenian intellectuals discussing equality did not find it particularly obnoxious to leave out the slaves from the orbit of discourse, and one reason why they could do it was because they could get away with it. The concepts of equity and justice have changed remarkably over history, and as the intolerance of stratification and differentiation has grown, the

1 The Delphic injunction, it may be recalled, was the somewhat severe advice: 'Know thyself!'

very concept of inequality has gone through radical transformation.

In these lectures I am concerned with *economic* inequality only, and that again in a specific context,[2] but I should argue that the historical nature of the notion of inequality is worth bearing in mind before going into an analysis of economic inequality as it is viewed by economists today. Ultimately the relevance of our ideas on this subject must be judged by their ability to relate to the economic and political preoccupations of our times.

Objective and normative features

The main focus of these lectures will be on the problem of the measurement of inequality of income distribution in aggregative terms, though I shall try to go into some of the policy issues, especially in the context of the socialist economy. On the question of the measurement of inequality, we might begin with a methodological point. The measures of inequality that have been proposed in the economic literature fall broadly into two categories. On the one hand there are measures that try to catch the extent of inequality in some *objective* sense, usually employing some statistical measure of relative variation of income,[3] and on the other there are indices that try to measure inequality in terms of some *normative* notion of social welfare so that a higher degree of inequality corresponds to a lower level of social welfare for a given total of income.[4] It is possible to argue that there are some advantages in taking the former approach, so that one can distinguish between (a) 'seeing' more or less inequality, and (b) 'valuing' it more or less in ethical terms. In the second approach inequality ceases to be an objective notion and the problem of measurement is enmeshed with that of ethical evaluation.

[2] In particular I shall be concerned primarily with the distribution of *income* and not directly with *wealth*.

[3] The usual measures include the variance, the coefficient of variation, the Gini coefficient of the Lorenz curve, and other formulae, which will be discussed in Chapter 2.

[4] For examples of the normative approach to the measurement of income distribution, see Dalton (1920), Champernowne (1952), Aigner and Heins (1967), Atkinson (1970a), Tinbergen (1970), and Bentzel (1970).

This methodological point essentially reflects the dual nature of our conception of inequality. There is, obviously, an objective element in this notion; a fifty-fifty division of a cake between two persons is clearly more equal in some straightforward sense than giving all to one and none to the other. On the other hand, in some complex problems of comparing alternative income distributions among a large number of people, it becomes very difficult to speak of inequality in a purely objective way, and the measurement of the inequality level could be intractable without bringing in some ethical concepts.

Which of the two approaches it would be correct to pursue is not an easy question to answer, and the two approaches in terms of their practical use would not be all that different from each other. Even if we take inequality as an objective notion, our interest in its measurement must relate to our normative concern with it, and in judging the relative merits of different objective measures of inequality, it would indeed be relevant to introduce normative considerations. At the same time, even if we take a normative view of the measures of income inequality, this is not necessarily meant to catch the totality of our ethical evaluation. It would presumably aim to express one particular aspect of the normative comparison, and which particular aspect will depend on the objective features of the inequality problem. To say that 'x involves less inequality than y', even if meant to be a normative statement, will not imply an unqualified recommendation to choose x rather than y, but would presumably be combined with other considerations (e.g., those involving total income and such features) to arrive at an overall judgement.[5] In one way or another, usable measures of inequality must combine factual features with normative ones.

Types of measurement

A second methodological issue concerns the type of measurement that is being sought. Various degrees of measurement

[5] In terms of the classification of value judgements used in Sen (1967b), inequality judgements are *non-compulsive evaluative* judgements.

are conceivable. The strictest type of measure is a ratio-scale like weight or height, in which it makes sense to say that one object weighs twice as much as another (and it does not matter whether we measure it in kilogrammes or pounds). A somewhat looser measure is that of an interval-scale, in which ratios make no sense but the ratios of differences do. The gap between 100° Centigrade and 90° Centigrade is recorded as twice that between 90°C and 85°C no matter whether we express these temperatures in Centigrade or in Fahrenheit (in which they correspond respectively to 212°F, 194°F and 185°F), but the ratio of the temperatures themselves will vary according to the scale chosen.

This interval-scale measure is usually referred to in utility theory as 'cardinal', and if a set of numbers x represents the utilities of different objects, a positive linear transformation of these numbers such as $y = a + bx$, with $b > 0$, can also be used.[6] A looser measure than this corresponds to what is called an 'ordinal' scale in utility theory, where any positive monotonic transformation will do as well, e.g., a set of numbers 1, 2, 3, 4 can be replaced by 100, 101, 179, 999, respectively, since the ranking of the numbers is all that matters.

A closely related measure to the 'ordinal' scale does not involve any numerical representation whatsoever, and just an ordering of all the alternatives is presented, e.g., a set of four alternatives, x_1, x_2, x_3 and x_4, may be ranked as x_3 highest, x_2 and x_1 next together and x_4 last. This kind of an ordering involves a ranking with two specific properties, viz., completeness and transitivity. Completeness requires that if we take any pair of alternatives then in terms of the ranking relation R, either xRy holds or yRx holds, or both. Interpreting R as the relation 'at least as good as', if xRy holds but not yRx then we can say that x is strictly better than y and indicate this as xPy; the case of yPx is exactly the opposite of this. If both xRy and yRx hold, we can declare x and y as 'indifferent' and refer to this as xIy. The property of transitivity demands that if we take any three alternatives x, y, z, and xRy and yRz both hold,

[6] For example, if F is the temperature in the Fahrenheit scale and C that in the Centigrade scale, we have: $F = 32 + 1.8C$.

then so does xRz. It might appear that an ordering can be easily converted into an 'ordinal' numerical measure, and this is indeed so for a finite set of alternatives, but is not invariably possible for an infinite set.[7] In fact, an ordering is a weaker requirement than the existence of an ordinal numerical representation.

Quasi-orderings and inequality judgements

A still weaker measure would be a case where the ranking relation R is not necessarily complete, i.e., not all pairs are rankable vis-à-vis each other. If a relation like this is transitive but not necessarily complete, it is called a quasi-ordering. Another case also weaker than an ordering is one where the ranking relation is complete but not necessarily transitive, of which a special case is one where the strict preference is transitive but indifference is not so.[8]

Most statistical measures of the inequality level assume a high degree of measurement, usually a ratio-scale or at least an interval-scale. This is true not only of the so-called objective measures, but also of normative evaluation (see Chapter 2). It is, however, possible to argue that the implicit notion of inequality that we carry in our mind is, in fact, much less precise and may correspond to an incomplete quasi-ordering. We may not indeed be able to decide whether one distribution x is more or less unequal than another, but we may be able to compare some other pairs perfectly well. The notion of inequality has many aspects, and a coincidence of them may permit a clear ranking, but when these different aspects conflict an incomplete ranking may emerge. There are reasons to believe that our idea of inequality as a ranking relation may indeed be inherently incomplete. If so, to find a measure of inequality that involves a complete ordering may produce artificial problems, because a measure can hardly be more precise than the

[7] The problem arises from not necessarily having a sufficient stock of real numbers to give each alternative an appropriate number in special cases such as lexicographic orderings over a many-dimensional real space. On this see Debreu (1959), Chapter 4.

[8] See Fishburn (1970), Sen (1970a), and Pattanaik (1971).

concept it represents. It will be argued in Chapter 3 that this might well account for some of the difficulties with the standard measures of inequality.

In this context it is perhaps worth saying that the historical connection between the notion of inequality and discontent—and more so rebellion—suggests that the need is for a measure that comes into its own with sharp contrasts, even though it may not provide a scale sensitive enough to order finely distinguished distributions. The unfortunate fact is that in putting up a scale of measurement or ranking, the economist's and the statistician's inclination is to look for an ordering complete in all respects, so that the translation of the notion of inequality from the sphere of political debate, which gives the notion its importance, to the sphere of well-defined economic representation may tend to confuse the mathematical properties of the underlying concept. Indeed inequality measurement is by no means the only field of economic analysis in which the predisposition towards a complete ordering has proved to be a major liability.

Non-conflict economics and Pareto optimality

How much guidance—it is reasonable to ask—can we expect to get from modern welfare economics in analysing problems of inequality? The answer, alas, is: not a great deal. Much of modern welfare economies is concerned with precisely that set of questions which avoid judgements on income distribution altogether. The concentration seems to be on issues that involve no conflict between different individuals (or groups, or classes), and for someone interested in inequality this can hardly make the air electric with expectations.

The so-called 'basic' theorem of welfare economics is concerned with the relation between competitive equilibria and Pareto optimality.[9] The concept of Pareto optimality was evolved precisely to cut out the need for distributional judgements. A change implies a Pareto-improvement if it makes no one worse off and someone better off. A situation is Pareto optimal if there exists no other attainable situation such that a move to it would be a Pareto-improvement. That is, Pareto optimality only guarantees that no change is possible such that someone would become better off without making anyone worse off. If the lot of the poor cannot be made any better without cutting into the affluence of the rich, the situation would be Pareto optimal despite the disparity between the rich and the poor.

[9] For the relevant theorems with proofs see Debreu (1959) and Arrow and Hahn (1972), and for an illuminating informal discussion see Koopmans (1957).

Suppose we are considering the division of a cake. Assuming that each person prefers to have more of the cake rather than less of it, every possible distribution will be Pareto optimal, because any change that makes someone better off is going to make someone else worse off. Since the only issue in this problem is that of distribution, Pareto optimality has no cutting power at all. The almost single-minded concern of modern welfare economics with Pareto optimality does not make that engaging branch of study particularly suitable for investigating problems of inequality.

Social welfare functions

At a more general level, however, there has been quite a bit of discussion in recent years on distributional judgements going beyond Pareto optimality, and indeed the famous Bergson–Samuelson social welfare function was partly motivated by the recognition that policy decisions in economics would require the economist to go beyond Pareto optimality. In its most general form the Bergson–Samuelson social welfare function is any ordering of the set of alternative social states. If X is the set of social states, then a Bergson–Samuelson social welfare function is an ordering R defined over the entire X. In numerical terms it was conceived of as a functional relation W that specifies a welfare value $W(x)$ for each social state x belonging to the set X. The measure of W has been usually taken to be 'ordinal'.

While this is the most general conception of the social welfare function, something more has to be said about the nature of the function $W(x)$ to get some results of practical importance out of this concept. A favourite assumption has been that the social welfare function is 'individualistic' in the sense of making social welfare W a function of individual utilities, i.e., $W(x) = F(U_1(x), \ldots, U_n(x))$, where U_i stands for the utility function of individual i, for $i = 1, \ldots, n$.[10] Further, assuming that W increases with any U_i given the set of utilities of all other individuals, Pareto optimality can be built into the exercise of maximizing W. But the main object of the social welfare function is to take us *beyond* this limited concept by ranking all the Pareto optimal states vis-à-vis each other. The distributional judgements would then depend on the precise social welfare function chosen.

While the conception of a function such as F permits the use of cardinal utilities of individuals as well as of interpersonal comparisons, orthodox welfare economics has been somewhat neurotic about avoiding both these activities. Much of the concentration has, therefore, been on arriving at social welfare, or at any rate at an ordering R of the set of social states X, based exclusively on the set of individual orderings of X. Representing the ordering of individual i as R_i, this line of thinking leads to the search for a functional relation $R = f(R_1, \ldots, R_n)$.

A natural question to ask in this context is whether certain general conditions can be imposed on the relation between the set of individual preferences and the social ordering. In a justly celebrated theorem, Arrow (1951) has shown that a set of extremely mild-looking restrictions eliminate the possibility of having any such functional relation f whatsoever. I do not intend here to go into Arrow's 'impossibility theorem', which

10 See, for example, Bergson (1938), Lange (1942) and Samuelson (1947). Lange, however, seems to have thought that even if social welfare were based '*directly* [on] the distribution of commodities or incomes between the individuals, without reference to the individuals' utilities', social welfare could still 'be expressed in the form of a *scaler function of the vector u*, i.e., W(u)' (p. 30). While it is true that for any distribution of commodities or incomes there would be one and only one vector u and one and only one W, we could still have two distributions leading to the same vector u but to two different values of W, so that in this case W could not really be viewed as a function of u.

has produced much awe, some belligerence, and an astounding amount of specialized energy devoted to finding an escape route from the dilemma. Instead I wish to present a theorem which does not rule out all functional relations f but only those that express any distributional judgements whatsoever, thereby ruling out any meaningful discussion of inequality within the logical framework of this model. The object of presenting and discussing this result is to clarify a basic weakness of the approach in handling problems of distribution and inequality.

A result concerning distributional judgements

Given Arrow's 'impossibility' result, it is clear that the system needs some give. This we provide by relaxing the requirement that social preference R be an ordering, in particular the requirement that R be 'transitive' (i.e., that xRy and yRz should imply xRz). Instead we demand only that the strict preference relation P be transitive (without indifference being necessarily transitive). We continue to require that R should be 'complete', i.e., either x is regarded as at least as good as y, or y regarded as at least as good as x (or both, in which case indifference holds), and of course that R should be 'reflexive', which is the entirely reasonable demand that x be regarded as at least as good as itself. Altogether we impose five conditions on the relation f between individual preference orderings and social preference relation R.

Condition Q (Quasi-transitive Social Preference): The social preference R must be reflexive, complete and quasi-transitive, i.e., the range of f must be confined to preference relations R that are reflexive and complete and which involve a transitive strict preference relation P.

Condition U (Unrestricted Domain): Any logically possible combination of individual preference orderings can be admitted.

Condition I (Independence of Irrelevant Alternatives): Social preference R over any pair x, y depends only on individual preferences over x, y.

Condition P (Pareto Rule): For any pair x, y if all individuals find x to be at least as good as y and some individual finds x

to be strictly better than y, then x is socially strictly preferred to y; and if all individuals are indifferent between x and y, then so is society.

Condition A (Anonymity): A permutation of individual orderings over the individuals keeps the social preference unchanged.

The first condition permits systematic social choice. The second permits individuals to have any preference pattern. The third establishes a relation between individual and social preferences that can be viewed pair by pair. The fourth is simply the familiar Pareto rule. The last condition—originally introduced by May (1952) in the context of the simple majority rule—requires that no special importance should be attached to who in particular holds which preference, all that matters being the combination of preferences that are held (no matter who holds what). These conditions may look reasonable enough, but together they rule out distributional judgements *in toto*.[11]

Theorem 1.1
The only functional relation f satisfying Conditions Q, U, I, P, and A must make all Pareto-incomparable states socially indifferent.

There are various alternative ways of proving this theorem, and I give here the sketch of a proof which I have spelt out elsewhere.[12] Define a person k as 'semidecisive'[13] if his preferring any x to any y implies that socially x is regarded as at least as good as y. He is 'almost semidecisive' if xRy holds whenever he prefers x to y and furthermore everybody else prefers y to x. By using Conditions Q, U, P, and I, it can be shown that if a person is almost semidecisive over some ordered pair (x, y) then he must be semidecisive over every ordered pair. I shall not spell out the entire argument here, but only demonstrate how the argument works. Assume that everyone other than k prefers y to x and also y to z, and let person k prefer x to y and y to z. By the Pareto rule, yPz. If we now assume that zPx, by quasi-transitivity (Condition Q) we would end up getting yPx; but since k is almost semidecisive over (x, y), clearly xRy. So zPx is false, and since R must be complete, xRz holds. By Condition I this must depend on individual preferences only over (x, z). Since only k's preference over (x, z) has been specified, k must be semidecisive over (x, z). Proceeding this way it can be shown that k would be semidecisive over every ordered pair in the set of social alternatives S.[14]

[11] This theorem was presented in a slightly different version in Sen (1970a) as Theorem 5*3.

[12] Sen (1970a), pp. 75–7.

[13] This is a weakening of Arrow's (1963) definition of a set of individuals being 'decisive'.

Next, a set V of individuals is 'almost decisive' over a pair (x, y) if as a result of everyone in V preferring x to y, and everyone not in V preferring y to x, the social ranking is xPy. The group of all individuals is, of course, an almost decisive set by virtue of the Pareto principle. Let the smallest almost decisive set for any pair in S be V^*, and let V^* be almost decisive over (x, y). Partition V^* into V_1^*, consisting of one person, and V_2^* the rest. The rest of the people not in V^* form set N. Let everyone in V_1^* prefer x to y and y to z, everyone in V_2^* prefer z to x and x to y, and everyone in N prefer y to z and z to x. Since V^* is almost decisive, clearly xPy. If we take zPy, this would make V_2^* an almost decisive set, which is impossible since V^* was the *smallest* almost decisive set. Hence yRz. If we now take zPx, then by quasi-transitivity we would get zPy and end up in a contradiction. Hence xRz. But then the solitary man in V_1^* is almost semidecisive over (x, z), and therefore must be semidecisive over every ordered pair of alternatives.

So far Condition A (anonymity) has not been used at all. Using that we see that *everyone* must be semidecisive over *every* ordered pair. But then for x to be socially preferred to y, it is necessary that no one regards y to be better than x. That is, we need then that everyone regards y as being at least as

[14] See the proof of Lemma 5*f in Sen (1970a).

good as x, which means that either (i) everyone is indifferent between x and y, or (ii) someone prefers x to y and everyone regards x to be at least as good as y. By Condition P, (i) implies that x and y are socially indifferent and (ii) implies that x is socially preferred to y. So x is socially preferred to y if and only if x is Pareto-superior to y. This means that if x is not Pareto-superior to y, then y is socially at least as good as x. And if x and y are Pareto-incomparable, then each is socially as good as the other and they must be socially indifferent.

Interpretation of Theorem 1.1

Theorem 1.1 makes Pareto comparisons the only basis of social choice. Since Pareto optimal points by definition are either Pareto-indifferent or Pareto-incomparable, they must all be declared socially indifferent. Even if one person prefers one state to another—however mildly—and all others have the opposite preference, the two states must still be declared to be *equally* good from the social point of view given the axioms of Theorem 1.1. We are back to a situation where judgements on inequality are not permitted and Pareto optimality is both necessary *and* sufficient for overall social optimality. Anyone wishing to make distributional judgements must reject something or other in the framework of Theorem 1.1.

Which of the five conditions is guilty? I would argue that the real trouble lies in the very conception of a social welfare function, which makes social preference dependent on individual orderings only, using neither valuations of intensities of preference, nor interpersonal comparisons of welfare. Avoiding interpersonal comparisons has been the dominant tradition in economics since the depression of the nineteen-thirties, for reasons that must have been—I suspect—unconnected with the depression itself, since the celebrated lambasting of interpersonal comparisons by Robbins (1932), (1988), and others, which started it all, could have hardly been inspired by the sight of obvious human misery. Be that as it may, the attempt to handle social choice without using interpersonal

comparability or cardinality had the natural consequence of the social welfare function being defined on the set of individual orderings. And this is precisely what makes this framework so remarkably unsuited to the analysis of distributional questions. The conditions used in Theorem 1.1 simply precipitate this fundamental weakness.

The point can be illustrated in terms of the exercise of dividing a cake of volume 100 between two persons 1 and 2, with $y_1 + y_2 = 100$, assuming that each prefers more to less. Armed only with individual orderings we know that person 1 prefers a 50–50 division to a 0–100 division, while person 2 prefers the latter. Now comparing the 50–50 division with a 49–51 division, we still have exactly the same ranking on the part of both individuals. We cannot say that the preferences were much sharper in the first case than in the second, since cardinality of individual utilities is not admitted; and this, combined with the ruling out of interpersonal comparisons, kills twice over any prospect of being able to make a statement of the kind that the gain of person 1 in going from 0 to 50 may be larger than the loss of person 2 in coming down from 100 to 50, or even from 51 to 50. The ruling out of interpersonal comparisons even eliminates the possibility of our being able to say that person 2 is better off than person 1 under a 0–100 division. In fact all the characteristics of individual welfare levels in the distribution problem are precisely left out of account in this framework, and it is no wonder that a set of fine-looking conditions can complete the kill and eliminate distributional judgements altogether. Thus Theorem 1.1.

Interpersonal comparisons

The crucial question really concerns interpersonal comparability, since cardinality alone—it is easy to check—will not help us much. With cardinality we can compare each person's gains and losses with alternative values of his own gains and losses, but distributional judgements would seem to demand some ideas of the relative gains and losses of different persons and also of their relative levels of welfare. Indeed Arrow's 'impossibility theorem', which I referred to earlier, remains virtually intact even when cardinality is introduced in the absence of interpersonal comparability, as has been shown.[15] Theorem 1.1 has the same characteristic.

It seems reasonable, therefore, to argue that if the approach of social welfare functions is to give us any substantial help in measuring inequality, or in evaluating alternative measures of inequality, then the framework must be broadened to include interpersonal comparisons of welfare. The question will be asked at this stage whether such comparisons are at all legitimate, and if so in what sense. Despite the widespread allergy to interpersonal comparisons among professional economists, it is I think fair to say that such comparisons can be given a precisely defined meaning. In fact, various alternative frameworks are possible.[16] One in particular will be pursued here.[17]

If I say 'I would prefer to be person A rather than person B in this situation', I am indulging in an interpersonal comparison. While we do not really have the opportunity (or perhaps the misfortune, as the case may be) of in fact becoming A or B, we can think quite systematically about such a choice, and indeed we seem to make such comparisons frequently.

Representing (x, i) as being individual i (with his tastes and mental qualities as well) in social state x, a preference relation $\tilde{R}$ defined over all such pairs provides an 'ordinal' structure of interpersonal comparisons.[18] To obtain interpersonally comparable cardinal welfare levels, one would have to go beyond such a ranking $\tilde{R}$ and introduce additional features for the sake of cardinalization.[19] The numerical representation of $\tilde{R}$ will be unique only up to an increasing monotonic transformation if the measure is ordinal, and unique up to a positive linear transformation if it is cardinal. For any individual welfare function chosen $U(x, i)$ will stand for the welfare level of being person i in state x. While bearing in mind this general framework for interpersonal comparisons, we shall, however, represent this as $U_i(x)$ and think of this as our view of the welfare function of individual i. If the framework of complete interpersonal comparability is used, we must also specify that if any particular transformation of U_i is done for any individual i, then a corresponding transformation would have to be done to everyone else's welfare function as well. For example, given an accepted configuration of welfare functions of the different individuals, if one person's welfare function is doubled, then so should be the welfare function of everyone else as well. While the precise set of welfare functions chosen remains arbitrary, which is unavoidable given the fact that welfare has no natural 'unit' or 'origin', arbitrary *relative* variations are not permitted in the framework of 'full comparability'.[20]

[15] Theorem 8*2 in Sen (1970a).

[16] See Vickrey (1945), Fleming (1952), and Harsanyi (1956). On the philosophical side, see in particular Kant (1788), Sidgwick (1874), Hare (1952), Rawls (1958), (1971), and Suppes (1966).

[17] Cf. Sen (1970a) and Pattanaik (1971).

[18] Formally, $\tilde{R}$ is a ranking of the Cartesian product of X (the set of social states) and J (the set of individuals).

[19] For a discussion of the axiomatic approach to cardinalization, see Fishburn (1970).

Utilitarianism

Once the information content of individual preferences has been broadened to include interpersonally comparable cardinal welfare functions, many methods of social judgement become available. The most widely used approach is that of utilitarianism in which the sum of the individual utilities is taken as the measure of social welfare, and alternative social states are ordered in terms of the value of the sum of individual utilities. Pioneered by Bentham (1789), this approach has been widely used in economics for social judgements, notably by Marshall (1890), Pigou (1920), and Robertson (1952). In the context of the measurement of inequality of income distribution, and in that of judging alternative distributions of income, it has been used by Dalton (1920), Lange (1938),

[20] The logical and intuitive bases of alternative frameworks of interpersonal comparisons of individual welfare are discussed in some detail in Sen (1970a), Chapter 7, 7*, 9 and 9*.

Lerner (1944), Aigner and Heins (1967), and Tinbergen (1970), among others.[21]

The trouble with this approach is that maximizing the sum of individual utilities is supremely unconcerned with the interpersonal distribution of that sum. This should make it a particularly unsuitable approach to use for measuring or judging inequality. Interestingly enough, however, not only has utilitarianism been fairly widely used for distributional judgements, it has—somewhat amazingly—even developed the reputation of being an egalitarian criterion. This seems to have come about through a peculiar dialectical process whereby such adherents of utilitarianism as Marshall and Pigou were attacked by Robbins and others for their supposedly egalitarian use of the utilitarian framework. This gave utilitarianism a ready-made reputation for being equality-conscious.

The whole thing arises from a very special coincidence under some extremely simple assumptions. The maximization of the sum of individual utilities through the distribution of a given total of income between different persons requires equating the marginal utilities from income of different persons, and if the special assumption is made that everyone has the same utility function, then equating marginal utilities amounts to equating total utilities as well. Marshall and others noted this particular aspect of utilitarianism, though they were in no particular hurry to draw any radical distributive policy prescription out of this. But when the attack on utilitarianism came, this particular aspect of it was singled out for an especially stern rebuke.

While this dialectical process gave utilitarianism its ill-deserved egalitarian reputation, the true character of that approach can be seen quite easily by considering a case where one person A derives exactly twice as much utility as person

21 There is the question as to whether it is correct to identify individual 'welfare levels' with individual 'utilities' as the classical utilitarians saw these concepts. On this see Little (1950), Robertson (1952), and Sen (1970a). In this work, I shall treat the two as identical following the traditional practice in economics.

B from any given level of income, say, because B has some handicap, e.g., being a cripple. In the framework of interpersonal comparisons outlined earlier, this simply means that the person making the judgement regards A's position as being twice as good as B's position for any given level of income. In this case the rule of maximizing the sum-total of utility of the two would require that person A be given a higher income than B. It may be noted that, even if income were equally divided, under the assumptions made A would have received more utility than B; and instead of reducing this inequality, the utilitarian rule of distribution compounds it by giving more income to A, who is already better off.

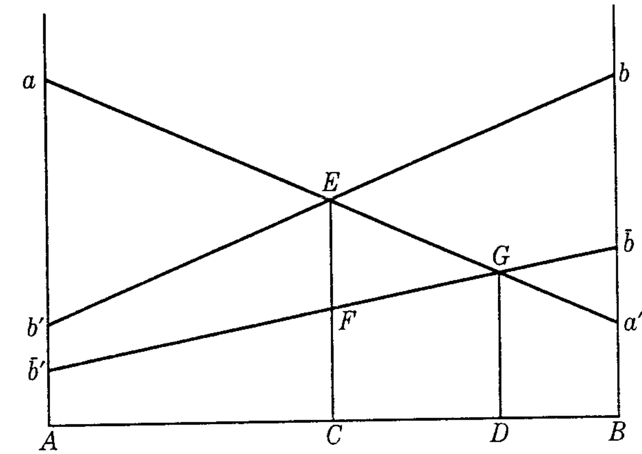

Diagram 1.1

Diagram 1.1 illustrates the problem. The total amount of income to be divided between the two is AB. The share of A is measured in the direction AB and that of B in the direction BA and any point such as C or D reflects a particular division of total income between the two. The marginal utility of A is measured by aa' and that of B by bb', and as drawn they are exact mirror-images of each other. The maximum total of utility is secured by dividing income equally as given by point C with $AC = BC$. So far so good. Assume now that B's marginal utility schedule is exactly half that of A, so that his marginal utility is no longer given by bb' but by $\bar{b}\bar{b}'$. If the income distribution is left unchanged, A's total utility will be $AaEC$ and B's only $BbFC$, and B will be much worse off. To compensate this an egalitarian criterion will now shift income from A to B. Would utilitarianism recommend this? It would recommend precisely the opposite, viz., a transfer of income from poor B to rich A! The new optimal point will be D with A enjoying a total utility of $AaGD$ and B merely $BbGD$.

It seems fairly clear that fundamentally utilitarianism is very far from an egalitarian approach. It is, therefore, odd that virtually all attempts at measuring inequality from a welfare point of view, or exercises in deriving optimal distributional rules, have concentrated on the utilitarian approach.

It might be thought that this criticism would not apply at all if utilitarianism were combined with the assumption that everyone has the same utility function. But this is not quite the case. The distribution of welfare between persons is a relevant aspect of any problem of income distribution, and our evaluation of inequality will obviously depend on whether we are concerned only with the loss of the *sum* of individual utilities through a bad distribution of income, or also with the inequality of welfare levels of different individuals. Its lack of concern with the latter tends to make utilitarianism a blunt approach to measuring and judging different extents of inequality even if the assumption is made that everyone has the same utility function. As a framework of judging inequality, utilitarianism is indeed a non-starter, despite the spell that this approach seems to have cast on this branch of normative economics.

The Weak Equity Axiom

To bring in egalitarian considerations into the form of the social welfare judgements, we might propose various alternative axioms, of which the following is an interesting case.

The Weak Equity Axiom: Let person i have a lower level of welfare than person j for each level of individual income. Then in distributing a given total of income among n individuals including i and j, the optimal solution must give i a higher level of income than j.

This Axiom, which we shall call WEA, puts a restriction on the class of group welfare functions that can be considered. Note that the requirement does not specify how much more is to be given to the deprived person but merely that he should receive more income as a compensation, possibly partial, and even a minute extra amount would satisfy WEA. In this sense the requirement is rather mild.

Three qualifications should be specified here. First, the normative appeal of WEA would very likely depend on the precise interpretation of interpersonal comparisons. The framework in terms of which WEA seems to me to make a great deal of sense is the one that is being used in this work, viz., considering the possibility of being in different persons' positions and then choosing among them. Thus interpreted WEA amounts to saying that if I feel that for any given level of income I would prefer to be in the position of person A (with his tastes and his other non-income characteristics) than in that of person B, then I should recommend that B should get a higher income level than A.

Second, the more equity-conscious one is and the less concerned with the 'aggregate', the more should WEA appeal. It might be argued that if a unit of income gives much more marginal utility to A than to B despite A being in general better off than B, perhaps one should give the additional unit of income to A rather than to B. This type of 'marginalist' comparison is in the spirit of utilitarianism, whereas the philosophy behind WEA lies in a completely different direction. Part of the difference is purely normative, but there are technical problems of measurability and interpersonal comparability that have a bearing on this and which will be discussed in Chapter 2.

Third, it is possible to give person B so much more income that, despite having a lower welfare function, he may end up being much better off than person A. Such possibilities are not ruled out by WEA. It just indicates a *direction* of adjustment, but if the adjustment is quantitatively excessive the inequality may well finish up in the opposite direction. Other conditions have to be introduced to rule out such an occurrence. WEA is a pretty mild force towards equity, and it is at best a necessary but not a sufficient condition for achieving that objective.

It is clear from our earlier discussion that: utilitarianism will violate WEA in many cases. Indeed the example portrayed in Diagram 1.1 shows this quite convincingly. To keep track of the more significant analytical results we elevate this piece of rustic wisdom into a theorem.

Theorem 1.2

There exist social choice situations such that the utilitarian rule of choice would violate the Weak Equity Axiom.

The proof is straightforward; look again at Diagram 1.1.[22]

WEA and concavity

The utilitarian rule is to maximize simply the sum of individual utilities:

$$W = \Sigma_{i=1}^{n} U_i(x) \tag{1.1}$$

To bring in a built-in bias towards equality, the functional relation between social welfare W and individual utilities may be assumed to be strictly concave. That is if U^1 and U^2 are two n-tuples of individual utilities, then for any t with $0 < t < 1$, we may require that:

$$[tW(U^1) + (1-t)W(U^2)] < W(tU^1 + (1-t)U^2) \quad (1.\ 2)$$

This would imply that any 'averaging' of utilities, thereby reducing disparity, would tend to raise social welfare, which does of course push us in the egalitarian direction.

It is interesting to enquire into the relation between the Weak Equity Axiom and strict concavity, since both have egalitarian aspects. It should be clear, however, that the two conditions are in fact independent of each other. WEA is a condition of *optimal* choice in a restricted class of choice situations, and there is no real hope of being able to get the fulfilment of strict concavity (or even weaker conditions like

22 Utilitarianism can satisfy WEA only if the ranking of total utilities is the opposite of that of marginal utilities at equal levels of income.

concavity or quasi-concavity) everywhere in the W-function on the basis of these restricted choice results.[23]

What about the converse? This does not follow either. WEA would require that the inequality-increasing result of the utilitarian case in the situation portrayed in Diagram 1.1 would have to be completely knocked out and instead the inequality-decreasing result brought in. And a W-function that is strictly concave in a mild way and is very close to the linear W of the utilitarian case would not be able to do this.

If not convinced by this reasoning, consider the following example. Take two utility functions identical except for a proportional displacement:

$$U_2(y) = mU_1(y), \text{ for all income level } y, \text{ with } m < 1 \tag{1.3}$$

Assuming that $U_1(y)$ is strictly positive for all positive values of y, person 2 is worse off than person 1 for all y. The group welfare function W is of the following form:

$$W = \frac{1}{\alpha}[(U_1)^\alpha + (U_2)^\alpha] \tag{1.4}$$

For strict concavity we need $\alpha < 1$. If the problem is to maximize W subject to:

$$y_1 + y_2 \leq Y, \tag{1.5}$$

the optimal distribution would have the property:

$$[U(y_1)^{\alpha-1}\, U'(y_1) - [U(y_2)]^{\alpha-1}\, U'(y_2) m^\alpha = 0, \tag{1.6}$$

putting $U(y) = U_1(y)$. This implies:

$$U'(y_1)/U'(y_2) = m^\alpha [U(y_1)/U(y_2)]^{1-\alpha} \tag{1.7}$$

Note that if $\alpha > 0$, then $m^\alpha < 1$, and if $\alpha < 0$, then $m^\alpha > 1$. Since U increases and U' decreases with income y, it is clear that this condition will fulfil $y_1 < y_2$ if and only if $\alpha < 0$.[24] Since

23 Note also that in some cases (see footnote 22) it is possible for WEA to be satisfied *by* utilitarianism, which must always violate strict concavity.

24 In this case social welfare W is bounded from above. There is an analogy here with Ramsey's (1928) social welfare picture with a level of 'bliss'.