SECOND EDITION

Derivative Securities

JARROW & TURNBULL

DERIVATIVE SECURITIES

SECOND EDITION

ROBERT JARROW
Finance Faculty
Johnson Graduate School of Management
Cornell University

STUART TURNBULL
Canadian Imperial Bank of Commerce

South-Western College Publishing
Thomson Learning™

Australia • Canada • Denmark • Japan • Mexico • New Zealand • Philippines
Puerto Rico • Singapore • South Africa • Spain • United Kingdom • United States

Derivative Securities, 2*e* by Robert Jarrow and Stuart Turnbull

Vice President/Publisher: Jack W. Calhoun
Acquisitions Editor: Michael B. Mercier
Developmental Editor: Thomas S. Sigel
Marketing Manager: Lisa Lysne
Production Editor: Kara ZumBahlen
Manufacturing Coordinator: Charlene Taylor
Cover Design: Michael H. Stratton
Cover Illustration: ArtVille/David Wasserman
Production House: Beckwith Bookworks
Compositor: A & B Typesetting
Printer: R.R. Donnelley

Printed in the United States of America
2 3 4 5 02 01 00

For more information contact South-Western College Publishing, 5101 Madison Road, Cincinnati, Ohio, 45227 or find us on the Internet at http://www.swcollege.com

Library of Congress Cataloging-in-Publication Data

Jarrow, Robert A.
Derivative Securities/Robert A. Jarrow, Stuart M. Turnbull.—
2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-538-87740-5 (hard: alk. paper)
1. Derivative securities. I. Turnbull, Stuart M. II. Title.
HG6024.A3J37 2000 99-046348
332.63′2—dc21

This book is printed on acid-free paper.

To our wives.

About the Authors

Robert A. Jarrow is the Ronald P. and Susan E. Lynch Professor of Investment Management at the Johnson Graduate School of Management, Cornell University. He is also a managing director and the director of research at Kamakura Corporation. He was the 1997 IAFE/SunGard Financial Engineer of the year. He is a graduate of Duke University, Dartmouth College, and the Massachusetts Institute of Technology. He is an IAFE Senior Fellow. Professor Jarrow is renowned for his pioneering work on the Heath-Jarrow-Morton model for pricing interest rate derivatives. His current research interests include the pricing of exotic interest rate options and other credit derivatives as well as investment management theory. His publications include four books, *Options Pricing*, *Finance Theory*, *Modelling Fixed Income Securities and Interest Rate Options*, and *Derivative Securities*, as well as over 70 publications in leading finance and economic journals. Professor Jarrow is a founding co-editor of *Mathematical Finance* and a co-editor of *The Journal of Derivatives*. He is an associate editor of the *Journal of Financial and Quantitative Analysis*, *Review of Derivatives Research*, *Journal of Fixed Income*, *The Financial Review*, *The Journal of Risk*, and *The Review of Futures Markets*. He is an advisory editor for *Asia-Pacific Financial Markets*.

Stuart M. Turnbull is Vice President of Global Analytics, the Canadian Imperial Bank of Commerce. He is in charge of developing the next generation of credit and market risk management models. Prior to joining the Bank, he was the Bank of Montreal Professor of Banking and Finance, Queen's University (Canada), and a Research Fellow, Institute for Policy Analysis (Toronto). He is a graduate of the Imperial College of Science and Technology (London) and the University of British Columbia. He is the author of *Option Valuation*, and (with Robert A. Jarrow) *Derivative Securities*. He has published over 30 articles in major finance and economic journals, and in law and economics journals, as well as many articles in practitioner journals. He is co-author with Robert Jarrow of the J-T model of pricing credit derivatives, which is widely used by financial institutions. He has been a consultant to many financial institutions. He is an associate editor of *Mathematical Finance*, the *Journal of Financial Engineering*, the *International Journal of Theoretical and Applied Finance*, and has served as an associate editor for the *Journal of Finance*.

PREFACE

Our purpose in writing this text is to make the theory and practice of pricing and hedging derivative securities accessible to undergraduates and MBA students, as well as practitioners. We have five explicit objectives.

First, the book reflects both *option* pricing theory and practice and *futures* pricing theory and practice. In practice, options on an underlying asset are often hedged with futures on the same underlying asset. This is because futures are viewed as more liquid than the underlying asset. For example, in the over-the-counter swap market Eurodollar futures are considered the preferred instrument for hedging, and in the stock index options market futures on the same stock index are the preferred hedging vehicle.

Current pricing theory, which is based on martingale methods, adopts a unified approach to all forms of derivatives, such as options and futures. Thus theory and practice dictate a new approach to teaching derivative securities. In this text we adopt this new direction, presenting a unified approach to pricing and hedging derivative securities. As such, our text differs from existing books, which either concentrate on options with little or no futures material or on futures with little or no options material.

Second, the book covers diverse areas such as equity and index derivatives, foreign currency derivatives, and commodity derivatives, as well as interest rate derivatives. Given the recent expansion in the trading of exotic options, any relevant text must include this material.

Third, the book addresses the issue of how to incorporate credit risk into the pricing and risk management of derivatives. Our text provides a clear, self-contained explanation of how to incorporate credit risk and gives many numerical examples.

Fourth, the book is a useful guide for pricing and hedging any derivative security, included in the text or not. To achieve this goal, the book includes all proofs and/or necessary references. As such, the level of detail included is higher than that contained in other books in this area, but without any additional mathematical complexity.

Fifth, the book integrates user-friendly software that allows the reader to price equity, foreign currency, and interest rate derivatives, as well as many types of exotic options. This software can be downloaded from our web page, located at *http://jarrow.swcollege.com*.

We believe that in satisfying our five objectives, our textbook offers a unique and comprehensive treatment of derivatives accessible to undergraduates, MBA students, executives, and regulators.

The accessibility of our textbook to students with various backgrounds is facilitated by the organizational design. We explore two key modeling paradigms: (1) the discrete time binomial model, and (2) the continuous time models of Black-Scholes and Heath-Jarrow-Morton. We use the discrete time binomial model to introduce all the relevant concepts. This is done because of its mathematical simplicity. We provide detailed proofs of all the key results. We include numerous examples, each highlighted and isolated from the text for easy reference and identification.

The continuous time models of Black-Scholes and Heath-Jarrow-Morton are provided for two reasons. First, they are used extensively in the industry, so consequently any serious student of derivatives must know this material. Second, they provide closed form solutions for many types of options and futures. These solutions can be programmed on a computer and quickly calculated. This facilitates an understanding of the different properties of financial derivatives. We include numerous examples using the software provided with the text.

Changes and New Components

The present edition has been revised with an eye toward improving presentation, streamlining the internal design for easier reading, and updating the applications and data, as well as including the latest developments in the field. New features include:

- Simplified notation.
- Simpler and more intuitive proofs.
- A new section explaining Ito's lemma (Chapter 8).
- A proof of the restriction that no-arbitrage imposes when modeling the evolution of forward interest rates. This restriction is used in the Heath-Jarrow-Morton model (Chapter 16).
- A new section (Chapter 18) on some of the recent disasters associated with derivative securities, including Barings Bank and Kidder Peabody & Co. This section includes a description of the Tinkerbell phenomenon first identified by Lee Wakeman (1997).

In addition, a web site now accompanies and supports this second edition. Visit *http://jarrow.swcollege.com* to access the following:

- **Spreadsheet software** This downloadable software gives students the opportunity to implement techniques presented in the text.
- **Test bank** Written by Arkadev Chatterjea from Indiana University, this new component provides instructors with multiple choice, true or false, and essay questions. Many questions are based upon the Chartered Financial Analyst (CFA) examinations administered by the Association for Investment Management and Research (AIMR). The test bank is password protected.
- **PowerPoint slides** Prepared by Robert Jarrow and Arkadev Chatterjea, these comprehensive slides may be used by instructors and students to reinforce concepts presented in the book.

Richard DeFusco, University of Nebraska
Donald Fehrs, University of Notre Dame
Richard Flavell, Lombard Risk Systems Limited
Louis Gagnon, Royal Bank of Canada
Bruce Grundy, Wharton School of Business/Australian Graduate School of Management
Chandrasekhar Gukhal, Cornell University
Edmond Levy, HSBC Midland Bank
Dean Leistikow, Fordham University
Robert Mark, Canadian Imperial Bank of Commerce
Joseph McCarthy, Bryant College
Tom McCurdy, University of Toronto
Thomas Miller, Kennesaw State University
Ieuan G. Morgan, Queen's University
Edwin H. Neave, Queen's University
M. Nimalendran, University of Florida
Joseph Ogden, SUNY at Buffalo
Hun Park, University of Illinois
Eduardo Schwartz, UCLA
Adel Turki, Purdue University
Robert Trevor, Macquarie University
John Wisbey, Lombard Risk Systems Limited
Guofu Zhou, Washington University

The following people deserve special thanks:

Phelim Boyle, University of Waterloo
Peter Carr, Nations Bank, Montgomery Securities
Arkadev Chatterjea, Indiana University
Dilip Madan, University of Maryland
Angelo Melino, University of Toronto
Frank Milne, Queen's University
Lee Wakeman, Risk Analysis and Control

Credit for the design and implementation of the input and output screens for the software goes to Warren Tom (UBS, Connecticut). We are very grateful to Warren for his many suggestions and dedication. We thank Barb Drake, Linda Freeman, and Donna Phoenix for a wonderful job in typing the manuscript. And special thanks go to our editors Mike Mercier, Thomas Sigel, and Kara ZumBahlen without whom this revised textbook would still only be in manuscript form.

Robert A. Jarrow
Cornell University

Stuart M. Turnbull
Canadian Imperial Bank of Commerce

CONTENTS

PREFACE xvii

PART I THE BASICS

1 INTRODUCTION TO DERIVATIVE SECURITIES 2

1.0 Introduction 2

1.1 Forward Contracts 3
Formalization 4

1.2 Futures Contracts 6
Standardization 6
Clearing House 8
Settlement Price 9
Daily Settlement and Margins 10
Regulation 11
Why Standardization? Why Daily Settlement? 11
Basis 12
Newspaper Quotes 13

1.3 Options 15
Call Options 15
Put Options 17
American versus European Options 19

1.4 Organized Option Markets 20

1.5 Option Newspaper Quotes 22

1.6 Interest Rates and Bond Prices 23
Zero-Coupon Bond Prices 23
Discount Rates 24
Simple Interest Rates 25
Discretely Compounded Interest Rates 26
Continuously Compounded Interest Rates 29

1.7 Summary 30

2 SIMPLE ARBITRAGE RELATIONSHIPS FOR FORWARD AND FUTURES CONTRACTS 34

2.0 Introduction 34

2.1 Definition of Arbitrage 34

2.2 Assumptions 35

2.3 Forward and Spot Prices 37
No Cash Flows on the Underlying Asset Over the Life of the Forward Contract 37
Formal Derivation (Cash-and-Carry) 39
Value of a Forward Contract 41

2.4 Known Cash Flows to the Underlying Asset 42
Formal Derivation 44
Value of a Forward Contract 46

2.5 Forward Contracts on Constant Dividend Yield and Interest-Paying Assets 47
Forward Contracts on a Stock Index 47
Foreign Exchange Forward Contracts 48

2.6 Forward Contracts on Commodities 51
Storage Costs 52
Convenience Yield 54
The Implied Repo Rate 55
Forward Contracts on Electricity 55

2.7 Forward and Futures Prices Compared 56
Equality of Forward and Futures Prices 59
Empirical Evidence 61

2.8 Summary 62

Appendix: Present Value of Dividends Over Life of Forward Contract 67

3 SIMPLE ARBITRAGE RELATIONSHIPS FOR OPTIONS 68

3.0 Introduction 68

3.1 Call and Put Options 68

3.2 Put Options 74

3.3 Relationship Between European Call and Put Options 79

3.4 Relationship Between American Call and Put Options 82

3.5 Summary 84

PART II THE BINOMIAL MODEL

4 ASSET PRICE DYNAMICS 90

4.0 Introduction 90

4.1 The Lognormal Distribution 91

4.2 The Basic Idea (Binomial Pricing) 96

4.3 Formal Description (Binomial Pricing) 98

4.4 The Binomial Approximation to the Lognormal Distribution 99

4.5 Extensions 105

4.6 Stochastic Differential Equation Representation 105

4.7 Complications 107
Lognormal Distribution 107
Continuous Trading 107
Continuously Changing Prices 108

4.8 Summary 108

Appendix: The Expected Value of the Future Stock Price 112

5 THE BINOMIAL PRICING MODEL 114

5.0 Introduction 114

5.1 Single-Period Example 115

5.2 Multiperiod Example 119

5.3 The Binomial Pricing Model 123
The Binomial Model 123
Constructing the Synthetic Option 124
Risk-Neutral Valuation 126
Put Options 130

5.4 Hedge Ratio (Delta) 133

5.5 Lattice Parameters 133

5.6 The Black-Scholes Option Pricing Model 137

5.7 Forward and Futures Prices 138
Formalization 143

5.8 Replicating an Option on Spot with Futures 146
Formalization 148
Hedge Ratios 149

5.9 Summary 150

6 MARTINGALE PRICING 155

6.0 Introduction 155

6.1 Relative Prices and Martingales 155
The Money Market Account 156
Risk-Neutral Valuation 156

6.2 Martingales and No Arbitrage 157

6.3 Futures Prices 161
Formal Description 165

6.4 Summary 166

Appendix: Proof of the Proposition 171

7 AMERICAN OPTIONS 175

7.0 Introduction 175

7.1 Cum-Dividend/Ex-Dividend Prices 176

7.2 American Call Options 178
No Dividends 178
Dividends 181

7.3 American Put Options 183
Time Value 183
Dividends 185

7.4 Valuation 187
American Call Options 187
Computational Complexity 191
American Put Options 192

7.5 Options on Forward Contracts 195
Call Options 196
Put Options 198
Valuation 199

7.6 Summary 203

PART III THE BLACK-SCHOLES MODEL AND EXTENSIONS

8 THE BLACK-SCHOLES MODEL 210

8.0 Introduction 210

8.1 Continuous Time Representation of Stock Price Changes 211

8.2 Interest Rates 213

8.3 Ito's Lemma 213

8.4 The Equivalent Martingale Probability Distribution 215
8.5 European Options 217
8.6 Hedging 218
8.7 Properties of the Black-Scholes Model 224
8.8 Use of the Black-Scholes Model 227
Historic Volatility 228
Implied Volatility 231
8.9 Option Strategies 233
8.10 Partial Differential Equation 235
Derivation 235
Delta, Gamma, and Theta 236
8.11 Summary 237
Appendix A 245
Appendix B 247
Appendix C: Unequally Spaced Observations 248
Appendix D 249

9 EXTENSIONS TO THE BLACK-SCHOLES MODEL 251
9.0 Introduction 251
9.1 Known Dividend Model 251
9.2 Pseudo-American Model 255
9.3 The Roll Model 257
9.4 Constant Dividend Yield Model 258
9.5 Options on Futures and Forward Contracts 261
Futures Contracts 261
Forward Contracts 264
9.6 Summary 265
Appendix: Continuous Dividend Yield 271

10 REPLICATION AND RISK EXPOSURE WITH MODEL MISSPECIFICATION 272
10.0 Introduction 272
10.1 Problems with Delta Hedging 272
10.2 A General Approach 278
10.3 Delta Hedging 282
The Delta Neutral Position 283
Formalization 285

10.4 Delta-Gamma Hedging 286
Formalization 289
Theta Neutral 289
10.5 Delta-Gamma-Vega Hedging 290
Formalization 293
10.6 Model Misspecification 294
10.7 Summary 294

11 FOREIGN CURRENCY 302
11.0 Introduction 302
11.1 Foreign Currency Derivatives 302
Foreign Currency Options 303
Options on Foreign Currency Futures 305
Uses of Currency Derivatives 308
11.2 Single-Period Example 310
11.3 Multiperiod Extension 315
11.4 Formalization 318
Martingale Pricing 320
Risk-Neutral Valuation 320
Replicating Portfolio 320
11.5 Lattice Parameters 321
11.6 Closed Form Solutions (Modified Black-Scholes) 325
11.7 American Options 330
11.8 Options on Foreign Currency Futures 334
Closed Form Solutions 336
11.9 Replicating Options on Spot with Futures 338
11.10 Summary 342
Appendix: The Quadratic Approximation 349

12 STOCK INDEXES AND COMMODITIES 350
12.0 Introduction 350
12.1 Derivatives on Stock Market Indexes 350
12.2 Stock Market Index Futures 352
Futures Prices 352
12.3 Spread Trading 355
Same Index 355
Different Indexes 356
12.4 Index Options 357

12.5 Pricing Index Options 361
Binomial Pricing Model 362
Closed Form Solutions 364
Discrete Dividends 365
Synthetic Options 366
Circuit Breakers 368
The Wildcard Option 369
12.6 Options on Index Futures 369
Pricing Index Futures Options 371
12.7 Commodity Derivatives 372
Futures Prices 372
Futures Options 374
12.8 Summary 377

PART IV INTEREST RATE CONTRACTS, THE HJM MODEL, AND EXTENSIONS

13 INTEREST RATE CONTRACTS 386
13.0 Introduction 386
13.1 Zero-Coupon Bonds 386
Discount Rates 387
Simple Interest Rates 388
Continuously Compounded Interest Rates 389
13.2 Coupon Bonds 389
Pricing 390
Yield-to-Maturity 391
Quotes 393
Floating Rate Notes 396
13.3 The Term Structure of Default-Free Interest Rates 397
Forward Rates 397
Formalization 399
Par Bond Yield Curve 401
Computing the Zero-Coupon Yield Curve 403
13.4 Traditional Measures of Interest Rate Risk 404
Duration 404
Convexity 407
Limitations of Analysis 409
13.5 Treasury Bill Futures 410
13.6 Eurodollar Contracts 411
Eurodollar Deposits 411

Forward Rate Agreements (FRAs) 413
Formalization 414
Futures Contracts 416

13.7 Treasury Bond and Note Futures 418
The Delivery Process 420

13.8 Treasury Bond Futures 421
Conversion Factors 421
Cheapest to Deliver 423
Wild Card Option 424
Timing Option 425

13.9 Summary 425

Appendix: Duration and Convexity Correction for a Semiannual Coupon Bond 428

14 SWAPS 431

14.0 Introduction 431

14.1 Interest Rate Swaps 431
Pricing Schedules 433
Warehousing 434
Valuation 434
Par Swaps 437
Variants of Interest Rate Swaps 439

14.2 Foreign Currency Swaps 439
Valuation of Currency Swaps 441
Variants of Foreign Currency Swaps 444

14.3 Commodity Swaps 444
Valuation of Commodity Swaps 445
Variants of Commodity Swaps 446

14.4 Equity Swaps 447
Valuation 448
Variants of the Basic Equity Swaps 450

14.5 Summary 450

15 INTEREST RATE DERIVATIVES 455

15.0 Introduction 455

15.1 Construction of the Lattice 455

15.2 Spot Rate Process 463
Normal Distribution 463
Lognormal Distribution 468

15.3 Valuing Options on Treasury Bills 473
Put Options 474
A Replicating Portfolio 475
Call Options 477
Put-Call Parity 477
15.4 Treasury Bill Futures 478
Pricing 478
Hedging 480
15.5 Summary 482

16 PRICING TREASURY BILLS, TREASURY BONDS, TREASURY FUTURES, AND HEDGING WITH MODEL MISSPECIFICATION 487
16.0 Introduction 487
16.1 The Heath-Jarrow-Morton Model 488
The Forward Rate Evolution 488
The Spot Rate and Money Market Account Evolution 490
Zero-Coupon Bond Price Evolution 491
Arbitrage-Free Restrictions 492
Mean Reversion/Volatility Reduction 493
16.2 Hedging Treasury Bills 494
Hedge Ratio (Delta) 496
Gamma Hedging 500
Gamma and Convexity 501
16.3 Hedging Treasury Bonds 504
16.4 Treasury Bill Futures 506
Hedging 508
Hedge Ratio (Delta) 509
Gamma Hedging 512
16.5 Treasury Bond Futures 512
16.6 Summary 516
Appendix: Proof of the Results on Pages 492–493 520

17 PRICING INTEREST RATE OPTIONS AND HEDGING WITH MODEL MISSPECIFICATION 522
17.0 Introduction 522
17.1 Options on Treasury Bills 522
Pricing 523
Hedging 525