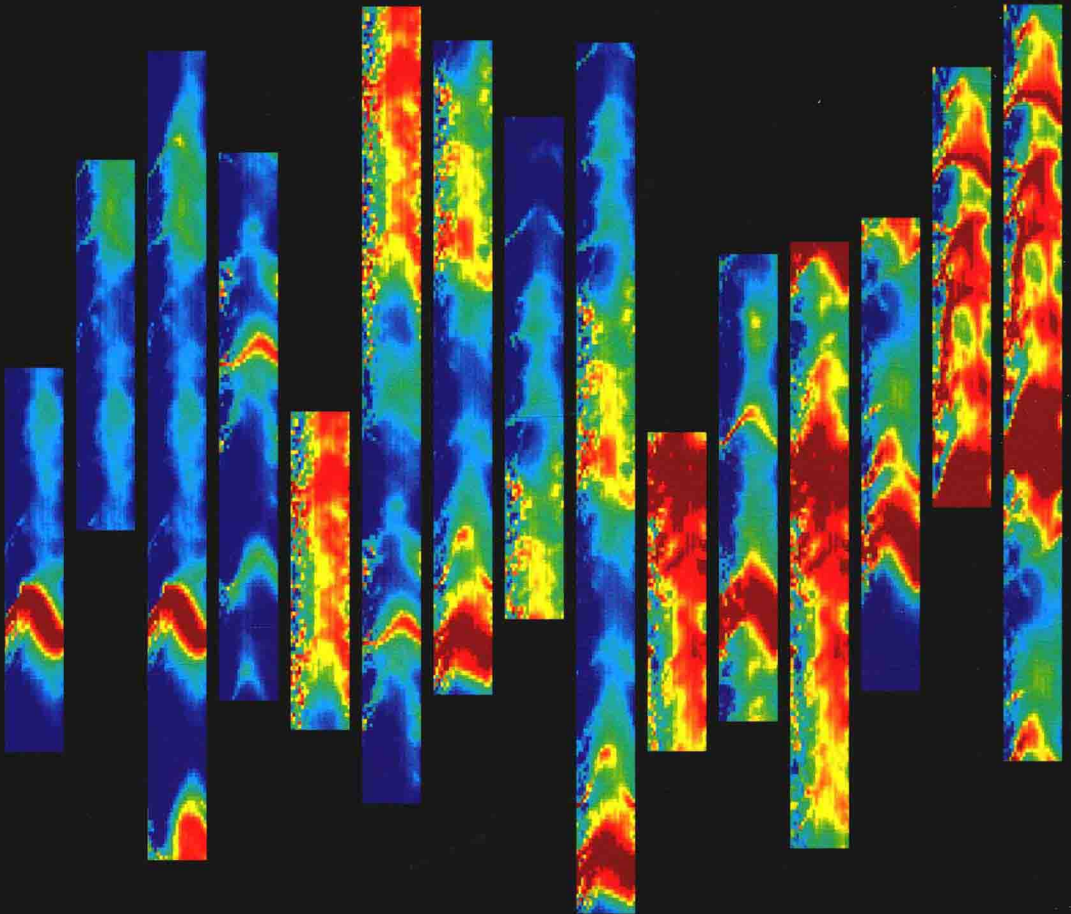


# Elements of Crustal Geomechanics

François Henri Cornet



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# Elements of Crustal Geomechanics

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This key textbook considers the mechanics of geomaterials at a wide range of scales, both in time and space. It provides detailed introduction to the study of crustal geomechanics, focusing specifically on the seismogenic crust.

Following an introduction to the necessary fundamentals of structural geology and material science, the book demonstrates how the application of continuum mechanics principles can provide efficient solutions to geomechanics problems at various scales, taking into account the multiphase characteristics of the geomaterials as well as discontinuities such as fractures and faults. It shows how field and laboratory observations can be combined with basic mathematical theory to build solutions with known levels of uncertainty. Particular consideration is given to the use of microseismicity in constraining geomechanical models – especially those involving fluid–rock interactions. Case studies are provided that illustrate how *in situ* stress determinations at very different scales provide unique constraints on the rheological characteristics of the seismogenic crust, and practical results from numerical modeling are used to illustrate the applicability and limitations of current theories.

*Elements of Crustal Geomechanics* introduces students to the common basic principles used in solving geomechanics problems ranging from exploitation of geothermal energy and long-term storage of nuclear waste to mitigating the impacts of volcanic eruptions. Accessible explanations of the mathematical formulations, convenient summaries of the key equations, and exercises that encourage students to put their learning into practice make this a valuable reference for students and researchers in geomechanics, geophysics, structural geology and engineering.

**François Henri Cornet** is a Professor at the Institut de Physique du Globe de Strasbourg. Prior to this he worked in the Department of Seismology at the Institut de Physique du Globe de Paris, and was also Visiting Scientist at Stanford University and at The Lawrence Berkeley National Laboratory. His main research interests are in rock mechanics, specializing in the measurement and modeling of stress fields; in rock–fluid interactions, including induced seismicity and applications to geothermal energy development; and in the development of large-scale, *in situ*, geophysical laboratories. Professor Cornet has extensive experience of teaching geomechanics courses at undergraduate and graduate levels and has also consulted internationally on stress field evaluations.

L'observation scientifique est toujours une  
observation polémique; elle confirme ou  
infirme une thèse antérieure, un schéma  
préalable.

Gaston Bachelard, *Le nouvel esprit scientifique*

(Scientific observation is always polemical;  
it confirms or contradicts a previous thesis,  
an earlier sketch.)

# Preface

Geomechanics refers to the mechanics of geomaterials, i.e. to the deformation and flow processes that affect the materials which make up the planet earth.

Geomechanics issues are encountered in a great variety of situations with very different scales, both in space and time. Generally, in engineering applications, time scales vary from a few days to a few tens of years and the volumes under consideration vary from a few hundreds of cubic meters to a few cubic kilometers. In earth science, however, time scales range from seconds to tens of millions of years and volumes vary from a few cubic kilometers to that of the entire planet. Accordingly, each domain of application has developed its own appropriation of the geomechanics concept, given that engineers have to deal mostly with perturbations of an existing system, with particular concern for safety issues and production or construction efficiency, while earth scientists are trying to understand natural phenomena such as fault motion, mountain building and sedimentary basin evolution.

For the last 30 years engineers have been confronted with much longer time scales and much greater volumes. For example the development of a repository for nuclear waste must be proved to be safe for up to a million years. The exploitation of geothermal energy or the filling of dams must not reactivate large faults and so trigger destructive earthquakes. Similarly, earth scientists must come up with precise seismic risk analysis, which requires an accurate description of the expected ground motion at specific locations. They must analyze, in real time, deformation fields on volcanoes in order to mitigate the hazards associated with eruption.

Today, geoenigneers and geoscientists dealing with the mechanics of earth materials need to speak the same language. The objective of this text book is to introduce the basic principles of mechanics that earth scientists and mining, petroleum, civil and environmental engineers need to apply for solving problems in geomechanics. The only materials which are considered here are crustal geomaterials. The only paradigm considered for describing the deformation and flow processes of these geomaterials is that of continuum mechanics, but the limits of this paradigm are pointed out occasionally.

The aim of this book is to introduce the material for a two-semester class on geomechanics for upper undergraduate and first-year graduate students in earth sciences. It is based on notes prepared for my classes and inspired by notes from P. R. Fosdick's continuum mechanics classes at the University of Minnesota.

In the first part of the book (chapters 1 to 7) the basic concepts of solid and fluid mechanics necessary for understanding the mechanical behavior of geomaterials are introduced. The second part of the book (chapters 8 to 12) discusses various specificities of geomechanics that result from the complexity of geomaterials. Special attention is given

to dynamic phenomena (such as microseismicity) as well as to solid–fluid interactions. In the last part of the book (chapters 13 and 14) various *in situ* stress determination methods are introduced and practical examples at various scales illustrate how a sound evaluation of the stress field helps a better understanding of the various mechanical processes at work in the seismogenic crust.

The first chapter introduces the concept of equivalent geomaterials and a description of their discontinuities (fractures and faults). The second chapter presents various unidirectional rheological models that help one to understand the basic concepts of elasticity, viscosity, plasticity and friction. The third and fourth chapters discuss the concepts of stress, strain and deformation. In the fifth chapter the behavior of linearly elastic solids is discussed and problems frequently encountered in geomechanics are solved. The sixth chapter introduces some basic elements of continuum mechanics with application to the laminar flow of incompressible materials. The seventh chapter presents basic principles of linear fracture mechanics. With chapter 8, our attention turns more specifically to geomaterials, and the results of laboratory investigations are presented. Chapter 9 addresses the application of continuum mechanics principles to geomechanics, and chapter 10 introduces specific characteristics of fractures and faults. In chapter 11 we describe the various types of wave observed in seismology and then we discuss more specifically seismic sources. Chapter 12 addresses various aspects of solid–fluid interactions, including linear poroelasticity, thermoelasticity and the nonlinear effects associated with failure processes (hydraulic fracturing and fluid induced shear fractures). Chapter 13, on *in situ* stress determination methods, gives practical applications of the various concepts that have been introduced throughout the book. In the final chapter these methods are illustrated through examples that concern the design of an underground hydroelectric power scheme ( $\text{km}^3$  scale), the design of a nuclear waste repository ( $100 \text{ km}^3$  scale) and the stress fields in the upper Rhine graben ( $1000 \text{ km}^3$  scale) and the west-central European lithosphere ( $10^6 \text{ km}^3$  scale).

I would like to thank very sincerely Susan Francis from Cambridge University Press, who suggested that I should take the time to write up my lecture notes. She did not anticipate that I would be so slow in doing so, however! I also thank her two assistants, Laura Clark and Zoe Pruce, for their help during the various preparatory phases, as well as Susan Parkinson for her thorough copyediting of the manuscript.

My sincere gratitude goes to Marco Calo, and to my son Jan, for their help in preparing most of the figures. The manuscript has also greatly benefitted from the help of my colleagues Patrick Baud, Daniel Billaux, Dominique Bruel, Michel Cara, Mai Linh Doan, Emmanuel Detrounay, Emmanuel Gaucher, Georges Jobert, Sophie Lambotte, Olivier Langline, Vincent Magnenet, Romain Prioul, Daniel Quesada and Jean Schmittbuhl for reading early versions of some chapters. They pointed out a multitude of typing errors and contributed significant improvements. But I bear the entire responsibility for all the errors that are still left in the present document.

Finally my sincere gratitude to my wife, Basia, who has helped me through all these years and kept my morale up especially during the last, never-ending, phase of this project.

F. H. Cornet

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- worked solutions to the exercises
- field-based datasets
- MATLAB codes

Geomechanics is concerned with the deformation and flow of geomaterials. A specific aspect of geomaterials, i.e. the materials that make up the planet earth, is their complex combination of solid and fluid phases.

In engineering geomechanics it is customary to talk of rock masses in a way that refers to both the rocks and the fracture systems that affect the volumes of concern. This concept, however, is often too vague for efficient mechanical modeling, and specific attention must be given to both the geomaterials and their discontinuities.

In the two first sections of this chapter we define more precisely the notion of a geomaterial and the related concept of a representative elementary volume (REV), with a brief reference to the various methods available for identifying geomaterials. In the third section we discuss the concepts of fracture sets and faults, with special attention to scaling laws. Finally, in the fourth section our attention turns to the various loading processes that may be encountered in geomechanics, whether of human or natural origin.

## 1.1 Rocks, soils and other geomaterials

Three kinds of rock can be identified: igneous, metamorphic and sedimentary. This characterization refers to the origin as well as to the past thermal and loading history of the rock. It implies strong consequences for the rock fabric, namely, the structure of its constitutive (solid) grains and of the complementary pore space, which is generally filled with fluid. And this introduces immediately the fact that materials of geological origin are most often multiphasic in their natural environment, i.e. they include solid, liquid and gas phases.

First, we introduce definitions that are used to describe the relative volumes occupied by the various phases. Then we introduce Goodman's classification of rocks according to their texture (Goodman, 1989), since in this book attention is given to the behavioral rather than to the genetic attributes of rocks. Such a nomenclature is helpful as a starting point for defining the material properties of import for a given mechanical problem.

### 1.1.1 Porosity, phase relationships, density

#### The representative elementary volume (REV) concept

Geomaterials always include some solid parts and some voids, the voids usually being filled with fluids, whether liquid or gas or both. The porosity of a geomaterial describes

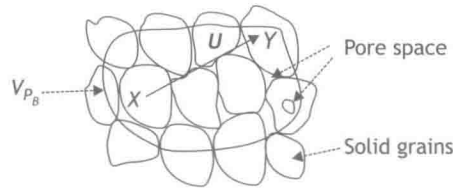


Fig. 1.1 Linear porosity.

the relative percentage of solids and voids. Let us consider a body  $B$  made up of two components, a solid component  $M$  and a fluid component  $F$ . Component  $M$  is made up of many grains that touch each other (fig. 1.1) and fluid  $F$  fills completely the voids in between the solid grains, i.e. the pore space is fully interconnected. The pore space is said to be fully interconnected when any point of it may be related to any other point of it by a continuous line, all the points of which remain within the pore space.

A small part  $P_B$  of body  $B$ , with volume  $V_{P_B}$ , includes a part  $P_S$  consisting of solid  $S$  with volume  $V_{P_S}$  and a part  $P_F$  consisting of fluid  $F$  with volume  $V_{P_F}$ , such that  $V_{P_B} = V_{P_S} + V_{P_F}$ . The volume  $V_{P_F}$  defines the pore space of part  $P_B$ . The *volume porosity*  $n$  of  $P_B$  is defined as  $n = V_{P_F}/V_{P_B}$  while the ratio  $e = V_{P_F}/V_{P_S}$  is called the *void ratio*.

The void ratio and the porosity are interrelated:

$$e = \frac{n}{1 - n}, \quad n = \frac{e}{1 + e} \quad (1.1)$$

In soil mechanics, the specific volume  $v$  is defined as the total volume of soil that contains a unit volume of solid ( $v = 1 + e$ ).

If the pore space is not fully interconnected, the volume  $V_{P_F}$  includes only the interconnected part of the total pore volume  $V_{P_P}$ . Then the fluid within the non-interconnected pore space may be different from that in the interconnected pore space, as e.g. in volcanic rocks. Furthermore, as will be discussed in section 9.3, the physical properties of the fluid that fills up the pores depend on the distance to the contact with the solid phase. For example, for water, for very small distances to the solid interface (in the micrometer range), a thin film exists that cannot flow and that can be removed only by heating up the material. It is called adsorbed water. For the purpose of defining the interconnected pore space, the adsorbed water is “assimilated” to the solid.

Let us now consider two points  $X$  and  $Y$  that define the vector  $\mathbf{U}$  with origin at  $X$  and extremity at  $Y$  (fig. 1.1). The linear porosity  $l(X, \mathbf{U})$  associated with the vector  $\mathbf{U}$  at  $X$  is defined by the ratio

$$l(X, \mathbf{U}) = \frac{|\mathbf{U}_P|}{|\mathbf{U}|} \quad (1.2)$$

where  $|\mathbf{U}|$  is the modulus of vector  $\mathbf{U}$  while  $|\mathbf{U}_P|$  is the modulus of that part of  $|\mathbf{U}|$  that intersects the pore space. When point  $Y$  is the same as point  $X$ , so that  $|\mathbf{U}| = 0$ , the linear porosity is set equal to 1 if point  $X$  is in a void and equal to 0 if it is located in a grain. As the modulus  $|\mathbf{U}|$  gets larger and larger, the variation in the linear porosity gets smaller and smaller (fig. 1.2) so that, for a length  $|\mathbf{U}|$  larger than, say,  $U^T$ , variations in linear porosity with increasing reference length  $|\mathbf{U}|$  may be neglected. The definition of this critical length



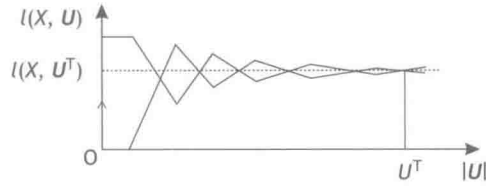


Fig. 1.2

Variation of linear porosity with the length of the defining vector  $U$ .

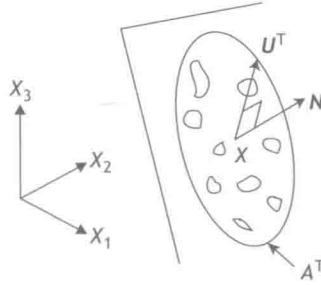


Fig. 1.3

Areal porosity at point  $X$  in a plane with normal  $N$ .

$U^T$  is somewhat arbitrary and may be chosen so that the variations in linear porosity are smaller than, say, 0.005 or 0.001 when  $|U|$  is larger than  $U^T$ .

If the linear porosity does not vary with the orientation of  $U^T$  then it is isotropic and if it does not vary with the spatial position of  $X$  in the body, it is homogeneous.

A similar approach may be followed to define an areal porosity. Consider a planar surface with normal  $N$ . The envelope of the extremities of all vectors  $U^T$  in the plane normal to  $N$  defines a closed planar contour with area  $A^T$  centered at  $X$  (fig. 1.3).

Let  $A_P$  be that part of  $A^T$  that passes through the pores. The areal porosity  $f(X, N)$  at the point  $X$  in the plane normal to  $N$  is defined by the ratio

$$f(X, N) = \frac{A_P}{A^T} \quad (1.3)$$

If  $f(X, N)$  does not vary with the orientation of  $N$ , the areal porosity is isotropic and equal to  $l(X, U^T)$ .

Finally, when the dip (see fig. 1.9) of the normal  $N$  varies from 0 to  $\pi$ , the envelope of all surfaces with area  $A^T$  normal to  $N$  defines a volume. This volume corresponds to the smallest part  $P$  of  $B$  for which the volume porosity  $n(X)$  may be defined. It is called the *representative elementary volume* (REV). For bodies with isotropic porosity,

$$l(X, U^T) = f(X, A^T) = n(X) \quad (1.4)$$

As already mentioned, when the porosity does not depend on  $X$  it is said to be homogeneous. If, however, the porosity varies in space, it is said to be heterogeneous. In general, a geomaterial is said to be heterogeneous with respect to a given property when this property varies with position in the volume under consideration.