FINITE ELEMENTS IN ELECTRICAL AND MAGNETIC FIELD PROBLEMS

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Finite Elements in Electrical and Magnetic Field Problems

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A Wiley – Interscience Publication

JOHN WILEY & SONS

Chichester · New York · Brisbane · Toronto

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Library of Congress Cataloging in Publication Data:
Main entry under title:

Finite elements in electrical and magnetic field problems.

(Wiley series in numerical methods in engineering)
'A Wiley–Interscience publication.'
Includes index.
1. Electric engineering—Mathematics. 2. Finite element method. 3. Electric fields. 4. Magentic fields. I. Chari, M. V. K. II. Silvester, Peet.
TK153.F48 621.3 79-1037
ISBN 0 471 27578 6

Photoset and printed in Malta by Interprint Limited

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Editors' Preface

Ever since the early days of development of the theory of electricity and magnetism, accurate solution of initial and boundary value problems in electrical engineering has been a major objective. Until the sixties, efforts were mainly directed towards obtaining closed form analytical and analog solutions based on simplified modelling of the boundary value problems and the associated differential equations.

With the advent of digital computers, numerical solutions such as finite difference schemes, finite elements, and integral equations have gained currency. However, the development and application of these techniques have been sporadic and generally problem oriented. Although nearly fifteen years have passed since the first efforts, many queries have remained unanswered and many more have been identified.

An International Conference on Numerical Methods in Electric and Magnetic Field Problems was held in Sta. Margherita, Italy (June 1–4, 1976) under the joint auspices of the International Center for Computer aided design of the University of Genoa and the *International Journal for Numerical Methods in Engineering*. The invited papers given at that conference, together with three additional chapters, form the basis of this book.

Within the limits of steady state finite element and integral equation solutions and linear time varying solutions of electric and magnetic field problems, the reader is here presented with a broad picture of current thought and research in this area.

The chapters are arranged into three principal categories, namely introductory and concept development; applications and advanced techniques; and specific methods. These are described in some detail in the following.

The introductory chapter reviews a number of practical situations arising in electrical engineering which can be formulated as initial and boundary value problems. Different methods of solving the associated partial differential equations by analytical, analog, and numerical methods are surveyed. These range from the classical separation of variables technique and its variants as applied to transformers and inductors by Roth and Rogowski, conformal mapping techniques, conducting paper plots, and finally computer based numerical methods such as finite difference schemes and finite elements. A brief discussion of the relative merits of the respective methods is presented.

In Chapter 1, the concept of the finite element method and its relative merits vis-à-vis other established methods such as finite differences and boundary integral methods are discussed. Different finite element approximations arising from variational principles, weighted integral expressions, Lagrangian multipliers and penalty functions, virtual work principles, and others are surveyed. General principles underlying the finite element approximation for two and three dimensional electric and magnetic field analysis are described. An application of the method for analysing transformer magnetic fields using a scalar potential approximation is presented.

The electromagnetic field is basically a tensor quantity, which may be described in a variety of ways. In Chapter 2, criteria for choosing the field representations in finite element analysis are discussed. First, any field representable by a linear combination of the finite element basis functions should be physically realizable. Secondly, the describing equations should lend themselves to a variational or projective solution which will include natural boundary conditions occurring at source-free surfaces or material interfaces. Unfortunately, no single fully satisfactory description of the electromagnetic field is known. Skilful choice of field representations, different for every new class of problems, is thus essential, so as to achieve simplifications and computational savings. Some of the more usual representation of fields are reviewed.

The finite element method involves the subdivision of the field region into subdomains or finite elements and approximation of the field in each element in terms of a limited number of parameters. Polynomial expansions are the customary choice. In Chaper 3, the different methods of selecting shape functions for elements are reviewed and the criteria for their choice are stated. The properties of polynomial functions are discussed as relevant to finite element approximations. The concept of isoparametric mapping which enables the construction of elements with curved or distorted boundaries is described.

Chapter 4 describes the various aspects of software engineering required in finite element analysis. In a typical program package, the mathematical software occupies only a small portion, while most of the code is devoted to problem definition and data handling; control of programme flow sequence and error checking; and post-processing the solutions into forms useful for engineering purposes. Software engineering seeks to ensure that programme packages serve their intended mathematical purpose while communicating with the user in a fairly problem-oriented manner. It strives to maintain reasonable programme portability and flexibility consistent with economical use of the computing hardware systems for which the programmes are intended. This chapter reviews the current trends in both hardware and software design and suggests that future software packages should be designed with a high degree of modularity and standardization of file structure.

In Chapter 5, the development of the finite element method for solving two and three dimensional electromagnetic fields in electric machines and devices is presented. Various applications of the technique for linear and nonlinear problems are discussed. Some of the areas surveyed are magnetic fields in electrical machinery cross-sections and the end-region; transformers; diffusion problems and eddy-current analysis in conducting media and electrostatic applications.

Eddy currents, which are often viewed as only a harmful phenomenon, have their uses in industrial applications such as in induction heating, magnetic propulsion and suspension, and others. In Chapter 6, different analytical methods for accurately predicting eddy currents in various practical situations are reviewed. These include series solutions for finite regions (eigenvalue problems) and infinitely extending structures (Fourier analysis). Analysis of three-dimensional problems by orthogonal function methods is also discussed. For nonlinear problems, the Galerkin projection method is recommended. It is concluded that for the general eddy current problem, no single all-comprehensive technique exists other than a careful analysis of each individual problem.

Chapter 7 summarizes the application of the high-order polynomial finite element method to electromagnetic field calculations. It provides a basic review of the development of the method indicating the motivation for its construction and outlines its algebraic development. Problems encountered in the computational implementation of the method are described and a bibliography of the published applications of the high-order polynomial finite element method in electromagnetics is provided.

Chapter 8 shows how the Fast Fourier Transform technique can be used with advantage to solve transient electromagnetic diffusion fields. Attention is focussed on the considerations underlying the application of the method to solve practical engineering problems. The method is effective because the space and time solutions to the field problem can be separately handled. This technique requires evaluation of a single frequency-response function, which is then used repeatedly with the FFT for each time function, yielding utmost computational economy.

There are many problems encountered in practice which do not clearly lend themselves to formulation in either integral or differential equations. Chapter 9 indicates one possible avenue of approach in such cases: part of the problem is dealt with in integral, part in differential form. Requiring the partial solutions to match, imposes mutual constraints on the two systems of equations, which are usually best solved by variational techniques.

In Chapter 10, various integral equation methods are described. For magnetostatic problems, three formulations are considered in detail: (a) the direct solution method for the magnetic field distribution in permeable materials, (b) a method based on a scalar potential, and (c) the use of an integral equation derived from Green's theorem, i.e. the so-called Boundary Integral Method (BIM). In case (a), results are presented for both two- and three-dimensional nonlinear problems and comparisons are made with measurements. Methods (b) and (c) lead to a more economical use of the computer than (a); for these, preliminary results for simple cases are included. Techniques for solving the eddy current problem are discussed, and computed results based on a vector potential formulation are presented.

The finite element art has had a marked impact on electromagnetic field analysis in the past decade, and will no doubt continue to do so. While most of the early work dealt with scalar, two-dimensional, static fields, the chapters presented here clearly point the way to broader problems. No doubt many new methods and many new problems will appear in the next decade. The trend to increased use of finite element methods will surely continue, fuelled by a rapidly broadening range of available computing resources, and motivated by increased acquaintance with their power—*avec le manger bient l'appetit.*

The editors wish to express their appreciation for the opportunity the authors have granted them to engage in this most rewarding in-breadth study of the finite element field. They wish also to thank the editorial staff of John Wiley & Sons Limited for their extensive counsel and assistance.

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25 March 1979 P. P. Silvester

Contents

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Introduction

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Boundary-value problems of mathematical physics occur in practically every engineering application. Different aspects are present, for instance, in structural analysis, heat transfer, fluid flow, electromagnetic fields, and they are indeed of great interest in all practical design problems.

In the design process, even in its roughest phases, the designer tries to define, by successive hypotheses and approximations, suitable boundary-value problems and to find acceptably accurate solutions to them.

In the past, and still today in design problems involving only minor economic and technological difficulties, it was generally assumed that no interaction took place among the various fields, for instance between the electromagnetic field and the thermal one. Single-field problems were solved by means of approximate procedures, generally requiring considerable simplifications of geometries and materials involved. This procedure, which largely makes use of solutions previously determined, such as uniform field solutions, originated the lumped parameter approach, so widely employed especially in electrical engineering.

The development of advanced technologies and the increase in dimensions and costs of many engineering systems have made necessary a parallel development of more general and accurate computation techniques. It has become increasingly important to obtain a deeper knowledge of the spatial distribution of vector and tensor fields, either for improved accuracy in evaluating integral parameters, or to determine and localize maximum values which generally denote critical stress conditions in materials.

In the last decades, an impressive growth of the rating of electrical systems, and consequently of their dimensions and costs, has taken place. About fifteen years ago, the maximum rating of turbo-alternators was in the range of 200 MVA, while machines in the range of 1 to 1.5 GVA have recently been built, and even larger ones are under consideration. Similar increases have taken place in the rating of transformers, cables, and many other electrical devices.

This situation poses new problems to the designer. For instance, in the past, valuable help was forthcoming from experimental data obtained from working systems. However, the increase in ratings and related costs has greatly reduced the availability of experimental data.

On the other hand, it is impractical to increase the volume of electrical devices in proportion to their rating, for obvious economic reasons. It is, therefore, necessary to increase stresses in materials, and to adapt new design criteria to the changed requirements. As a consequence, new design solutions become possible; for instance, the use of superconductors has been proposed in the case of electrical machines and cables. Frequently, new devices are also notably different in shape from previous ones; consequently, old computation procedures become inapplicable.

Therefore, an increase in accuracy of theoretical performance prediction is necessary, especially in view of the complexity of the geometry of problems involved and the characteristics of materials used.

For instance, a better knowledge of space and time distributions of both electric current and magnetic flux in an electric machine is necessary in order to obtain a reliable description of power losses and electromechanical stresses, from which the thermal and mechanical design of the machine must be derived. But this may require a generalization of steps of the computation procedures, such as taking into account anisotropy, saturation, and hysteresis of magnetic materials, the laminar structure of magnetic cores, the presence of slots and air gaps, the spatial distribution of conductors and dielectric materials, and the influence of frame materials, which can no longer be regarded as electrically and magnetically passive. Besides, the computation must be performed under time-varying conditions, which are generally non-sinusoidal on account of the nonlinearities of materials or due to the transient conditions of applied voltages or torques. Furthermore, the general hypothesis according to which the whole machine is isothermal can no longer be accepted. This hypothesis allows us to treat the electric and the magnetic fields independently, by using Maxwell's equations and constitutive relations in a form independent of temperature. If the hypothesis is no longer valid, constitutive relations must also contain temperature, and thermal field equations must be added to Maxwell's equations. Similar problems arise when determining stresses in dielectric materials.

The above considerations are applicable to high power traditional machines, such as transformers, turbo-generators, or salient pole alternators. They also apply equally well to nontraditional versions of the machines, as well as to other electrical apparatus, such as high-voltage equipment, high-field magnets, direct-current generators, and motors with limit performances. They can be extended to the implementation of interesting nontraditional electrical devices, such as linear motors, levitation systems and, possibly, special magnets for nuclear fusion, MHD generators, and energy storage. In these cases, we have to deal with problems in which geometries involved are generally not suitable for traditional approaches, and, therefore, require powerful calculation procedures.

Another electrical engineering area requiring efficient computation techniques is electronics, especially in the high-frequency range. The technologies of semiconductor devices and integrated circuits, which have produced the rapid growth of the electronics industry, have also made necessary specific requirements for solving problems with general forms of nonlinearities in semiconductor transport equations, and for taking into account, as far as possible, two- or three-dimensional effects. Other types of electronic device (for instance, in the area of wave propagation systems, or in the presence of plasmas or of special materials, such as piezo-electric ones) also require new computational techniques.

The conditions outlined above have strongly influenced the historical development of computational methods for boundary-value problems, and explain the present interest and activities in such a field, particularly in numerical procedures.

The methods used from Maxwell onwards for the solution of boundary-value problems can be divided into four categories: analogue, graphical, analytical, and numerical.

Analogue procedures consist in obtaining the unknown field by experimental measurements on an analogue of the field region, i.e. on a field region governed by the same equations and with the same boundary and interface conditions. These procedures have generally been used only for Laplace's equation under two- or three-dimensional conditions. In fact, it has been practically impossible to model inhomogeneities, nonlinearities, and so on, by means of media different from the ones involved in the real problem. Besides, in its three-dimensional version (such as, for instance, electrolytic tank or resistance network), this method is rather expensive and cumbersome, whereas the more convenient versions (such as, for example, graphitic paper or elastic membrane) are restricted to two-dimensional fields.

Graphical procedures have long been used, but they are restricted to Laplace's equation for two-dimensional geometries because they are generally based on the properties of analytic functions (see, for instance, the Lehmann method). It should be added that their accuracy is limited even when they are carefully applied.

The development of analytical methods advanced a good deal while numerical methods were still in their infancy. These methods are still widely used and include series solution and conformal mapping techniques. Other methods in vogue are integral equations, variational formulations, or approaches specific to various problems. The last mentioned ones, such as the method of images or the inversion method, are generally applicable to simple geometries and materials. In such cases, solutions are found by inspection and are based on known solutions to analogous problems, or by the use of symmetry conditions, etc.

Series solutions are generally obtained by the so-called method of separation of variables. This method can be mainly applied to Laplace or Helmholz equations in two- or three-dimensional problems, and can also be applied in time-dependent problems, for instance in the ones governed by diffusion or wave equations. Even if such a method can be considered as being more general than the previous ones, it suffers from severe restrictions essentially related to the treatment of boundary and interface conditions. In practice, its use requires the existence of a suitable coordinate system, which must fulfil two conditions: (i) every boundary or interface surface must coincide with an equi-coordinate surface; (ii) the coordinate system must allow the separation of variables. Both these conditions are rarely satisfied in complex problems. The calculations require the use of special functions (such as Bessel, Legendre, elliptic) which often are not easily handled.

The separation of variables method can also be applied to inhomogeneous equations like the Poisson equation. In this case, it is necessary to add a particular integral of the inhomogeneous equation to the general integral of the corresponding homogeneous one: the particular integral can generally be computed by volume or surface integration. However, it is often quite difficult to perform such an integration analytically, and special techniques have been introduced to solve problems, even for very simple geometries. We recall here Rogowski's and Roth's methods, developed for the solution of magnetic fields in transformers and inductors.

The solution of field problems by conformal mapping is another analytical approach which has been extensively used. It is based on the properties of analytic complex functions, so that it can only be applied to problems which can be reduced to Laplace's equation in a two-dimensional region. In such a domain, conformal mapping may often be more powerful than the series method because it can yield closed-form solutions for more complicated regions. Severe limitations have, however, to be placed on the geometry of problems to avoid difficulties in the integration of complex functions. Analytic functions can also be used to generate coordinate transformations preliminary to the handling of equations by other methods, but this application is also limited.

The aforesaid is a brief description of analytical methods. This category is very broad and includes algorithms that generalize and extend the above procedures, even if laboriously. The major deficiency of the analytical methods is the lack of generality. The classes of problems for which analytical solutions exist that can be considered somewhat general are by far the simplest ones and many algorithms are applicable only to two-dimensional and steady-state problems. Besides, algorithms for inhomogeneous and nonlinear problems are practically non-existent excepting for some extremely simple and special problems. Another notable deficiency of analytical methods lies in the effort required in obtaining the field solutions, such as developing special algorithms and discovering artifices.

The deficiencies of analytical methods are to a large extent eliminated in numerical methods, which have come to the fore with the advent of large digital computers. The principal numerical methods that are in vogue can be subdivided into finite difference schemes, image methods, integral equation techniques, and variational formulations.

Finite difference schemes were the first to be widely used, in practice since about 1940, even if they have been traced back to Gauss.[1] For one-dimensional problems a complete finite-difference approach can already e found in the graphical string polygon method for finding deflections of beams developed by Mohr in 1868.[2] The first application to two-dimensional problems was made by Runge in 1908.[3] There was a parallel development of methods of solution for the large algebraic equations resulting from finite-difference schemes; it was based on the fundamental contributions by Gauss,[4] Jacobi,[5] and Seidel,[6] and on the results obtained by Richardson[7] and Liebmann[8] in the area of iterative methods, and on the works by Gauss,[9] Doolittle,[10] and Choleski[11] in the area of direct methods.

As we can see, numerical methods had been defined long before their wide use, which began with the advent of high-speed computers.

The basis of finite-difference schemes is the replacement of a continuous domain with a grid of discrete points ('nodes'), the only ones at which the value of unknown quantities are computed. The reduction of the equations and of the boundary or interface conditions, defined in the continuous domain, to the discretized equations valid for the nodes is performed by means of various algorithms, which replace derivatives and integrals with 'divided-difference' approximations obtained as functions of the nodal values. This can be accomplished, for instance, by using interpolating functions, which are not defined in specific subdomains but simply in the neighbourhood of a node.

In its traditional versions, the grid is a regular one: that is, a rectangular grid with nodes at the intersections of orthogonal straight lines or a polar grid with nodes at the intersections of orthogonal circles and radii. This restrictive approach, which simplifies the discretization algorithms, is not necessary. However, the use of general curvilinear grids (or of irregular ones, seldom proposed in the past) has not been successful, so that regular grids are the only ones in practice to date. As a consequence, severe difficulties are encountered in solving many problems using finite-difference schemes, and therefore their efficiency is considerably limited. This is essentially due, apart from other less important problems, to geometrical reasons related to the fitting of the grid to the shapes of boundaries and interfaces involved. In fact, a regular grid is not suitable for problems with very steep variations of fields. The grid must indeed be denser in regions of high field gradients, and this requires either a very large number of nodes, so that computation time and memory requirements are significantly increased, or a complex algorithm to increase the density of the grid artfully. Moreover, a regular grid is not suitable for curved boundaries or interfaces, because they intersect gridlines obliquely at points other than nodes. This may not be a problem under Dirichlet boundary conditions, but poses difficulties under Neumann boundary conditions or under interface conditions involving normal derivatives. These situations require sophisticated interpolation schemes which are difficult to implement in an automatic form, and complicate the solution of algebraic equations resulting from the discretization.

In spite of these shortcomings, finite-difference schemes, in their traditional version, monopolized the area of numerical methods in electrical engineering practically up to 1970. In any case, they led to valuable results: we may recall here the work of Erdélyi's group at the University of Colorado,[12] and the activities of Fritz, Müller, and Wolff in the area of alternators and d.c. machines.[13]

Alternative methods have also been proposed in the area of image methods and integral equation techniques which have intensive but limited application in special areas. We mention here particularly, in the area of image methods, the work of Prinz and Singer's group.[14] They developed the charge method, in which the values of image charges are computed to simulate three-dimensional high voltage fields. Likewise, in the area of integral equations, we recall the activities of Trowbridge's group at the Rutherford Laboratory in the computation of three-dimensional structures of magnets.[15] Both methods seem mainly oriented towards three-dimensional nonsymmetrical problems, and, in this connection, they could provide interesting developments.

In other engineering areas, particularly in civil engineering, the drawbacks of traditional finite-difference schemes have been recognized at an early stage and alternative methods have been developed, such as variational procedures. This process, which can also be traced back to the past century, has led to the modern form of the finite-element method, and can be considered to have been completely established in the late fifties. Actually, the term 'finite-element method' was first used in a paper by Clough in 1960.[16]

Variational methods consist in formulating the equations of boundary-value problems in terms of variational expressions called 'energy functionals', which, in electrical applications, often coincide with the energy stored in the field. The Euler equation of this functional will generally coincide with the original partial differential equation. In the finite-element method, the field region is subdivided into elements, that is, into subregions where the unknown quantities, such as, for instance, a scalar or a vector potential, are represented by suitable interpolation functions that contain, as unknowns, the values of the potential at the respective nodes of each element. The minimization of the energy functional by the use of such interpolation functions generates an algebraic system of equations, as in the finite-difference methods, and the potential values at the nodes can be determined by direct or iterative methods.

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