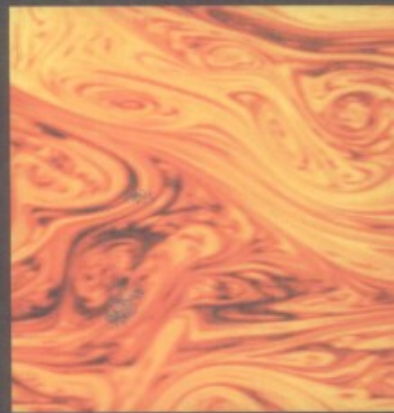
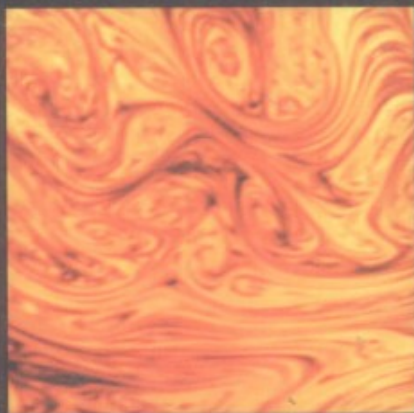


OXFORD



turbulence

AN INTRODUCTION FOR
SCIENTISTS AND ENGINEERS

P. A. DAVIDSON



turbulence

AN INTRODUCTION FOR
SCIENTISTS AND ENGINEERS



Cover image: Two-dimensional turbulence created using a soap film. Courtesy of M. Ward-Close, A. Dorn and B. R. Pearson

Based on a course taught by the author at the University of Cambridge, this comprehensive text on turbulence and fluid dynamics is aimed at fourth year undergraduates and graduates in applied mathematics, physics, and engineering, and provides an ideal reference for industry professionals and researchers. It bridges the gap between elementary accounts of turbulence found in undergraduate texts and more rigorous accounts given in monographs on the subject. Containing many examples, the author combines the maximum of physical insight with the minimum of mathematical detail where possible. The text is highly illustrated throughout, including, colour plates, and required mathematical techniques are covered in extensive appendices.

CONTENTS:

Part I: The classical picture of turbulence

- Chapter 1 The ubiquitous nature of turbulence
- Chapter 2 The equations of fluid mechanics
- Chapter 3 The origins and nature of turbulence
- Chapter 4 Turbulent shear flows and simple closure models
- Chapter 5 The phenomenology of Taylor, Richardson and Kolmogorov
- Chapter 6 Isotropic turbulence (In real space)
- Chapter 7 The role of numerical simulations
- Chapter 8 Isotropic turbulence (In spectral space)

Part III: Special topics

- Chapter 9 The influence of rotation, stratification, and magnetic fields on turbulence
- Chapter 10 Two-dimensional turbulence

Epilogue

Appendices

ALSO AVAILABLE FROM OXFORD UNIVERSITY PRESS

Elementary Fluid Dynamics

D.J. Acheson

Nonlinear Ordinary Differential Equations:

An introduction to dynamical systems (third edition)

Dominic Jordan and Peter Smith

Chaos and Nonlinear Dynamics:

An introduction for scientists and engineers

Robert C. Hilborn

Physical Hydrodynamics

Etienne Guyon, Jean-Pierre Hulin, Luc Petit, and Catalin D. Matescu

OXFORD
UNIVERSITY PRESS

www.oup.com

ISBN 0-19-852949-X



9 780198 529491

Turbulence

An Introduction for Scientists and Engineers



P.A. Davidson

University of Cambridge

OXFORD
UNIVERSITY PRESS

OXFORD

UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide in

Oxford New York

Auckland Bangkok Buenos Aires Cape Town Chennai

Dar es Salaam Delhi Hong Kong Istanbul

Karachi Kolkata Kuala Lumpur Madrid Melbourne Mexico City Mumbai

Nairobi São Paulo Shanghai Taipei Tokyo Toronto

Oxford is a registered trade mark of Oxford University Press
in the UK and in certain other countries

Published in the United States

by Oxford University Press Inc., New York

© Oxford University Press 2004

The moral rights of the author have been asserted

Database right Oxford University Press (maker)

First published 2004

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by any means,
without the prior permission in writing of Oxford University Press,
or as expressly permitted by law, or under terms agreed with the
appropriate reprographics rights organization. Enquiries concerning reproduction
outside the scope of the above should be sent to the Rights Department,
Oxford University Press, at the address above

You must not circulate this book in any other binding or cover
and you must impose this same condition on any acquirer

A catalogue record for this title is available from the British Library

Library of Congress Cataloging in Publication Data

(Data available)

ISBN 019852948 1 (Hbk)

ISBN 019852949 X (Pbk)

10 9 8 7 6 5 4 3 2 1

Typeset by Newgen Imaging Systems (P) Ltd., Chennai, India
Printed in China

For Henri and Mars

.

Preface

Turbulence is all around us. The air flowing in and out of our lungs is turbulent, as is the natural convection in the room in which you sit. Glance outside; the wind which gusts down the street is turbulent, and it is turbulence that disperses the pollutants, which belch from the rear of motor cars, saving us from asphyxiation. Turbulence controls the drag on cars, aeroplanes, and bridges, and it dictates the weather through its influence on large-scale atmospheric and oceanic flows. The liquid core of the earth is turbulent, and it is this turbulence that maintains the terrestrial magnetic field against the natural forces of decay. Even solar flares are a manifestation of turbulence, since they are triggered by vigorous motion on the surface of the sun. It is hard not to be intrigued by a subject which pervades so many aspects of our lives.

Yet curiosity can so readily give way to despair when the budding enthusiast embarks on serious study. The mathematical description of turbulence is complex and forbidding, reflecting the profound difficulties inherent in describing three-dimensional, chaotic processes.

This is a textbook and not a research monograph. Our principle aim is to bridge the gap between the elementary, heuristic accounts of turbulence to be found in undergraduate texts, and the more rigorous, if daunting, accounts given in the many excellent monographs on the subject. Throughout we seek to combine the maximum of physical insight with the minimum of mathematical detail.

Turbulence holds a unique place in the field of classical mechanics. Despite the fact that the governing equations have been known since 1845, there is still surprisingly little we can predict with relative certainty. The situation is reminiscent of the state of electromagnetism before it was transformed by Faraday and Maxwell. A myriad of tentative theories have been assembled, often centred around particular experiments, but there is not much in the way of a coherent theoretical framework.¹ The subject tends to consist of an uneasy mix of semi-empirical laws and deterministic but highly simplified cartoons,

¹ One difference between turbulence and nineteenth century electromagnetism is that the latter was eventually refined into a coherent theory, whereas it is unlikely that this will ever occur in turbulence.

bolstered by the occasional rigorous theoretical result. Of course, such a situation tends to encourage the formation of distinct camps, each with its own doctrines and beliefs. Engineers, mathematicians, and physicists tend to view turbulence in rather different ways, and even within each discipline there are many disparate groups. Occasionally religious wars break out between the different camps. Some groups emphasize the role of coherent vortices, while others downplay the importance of such structures and advocate the use of purely statistical methods of attack. Some believe in the formalism of fractals or chaos theory, others do not. Some follow the suggestion of von Neumann and try to unlock the mysteries of turbulence through massive computer simulations, others believe that this is not possible. Many engineers promote the use of semi-empirical models of turbulence; most mathematicians find that this is not to their taste. The debate is often vigorous and exciting and has exercised some of the finest twentieth century minds, such as L.D. Landau and G.I. Taylor. Any would-be author embarking on a turbulence book must carefully pick his way through this minefield, resigned to the fact that not everyone will be content with the outcome. But this is no excuse for not trying; turbulence is of immense importance in physics and engineering, and despite the enormous difficulties of the subject, significant advances have been made.

Roughly speaking, texts on turbulence fall into one of two categories. There are those that focus on the turbulence itself and address such questions as: where does turbulence come from, what are its universal features, to what extent is it deterministic? On the other hand, we have texts whose primary concern is the influence of turbulence on practical processes, such as drag, mixing, heat transfer, and combustion. Here the main objective is to parameterize the influence of turbulence on these processes. The word *modelling* appears frequently in such texts. Applied mathematicians and physicists tend to be concerned with the former category, while engineers are necessarily interested in the latter. Both are important, challenging subjects.

On balance, this text leans slightly towards the first of these categories. The intention is to provide some insight into the physics of turbulence and to introduce the mathematical apparatus which is commonly used to dissect turbulent phenomena. Practical applications, alas, take a back seat. Evidently such a strategy will not be to everyone's taste. Nevertheless, it seems natural when confronted with such a difficult subject, whose pioneers adopted both rigorous and heuristic means of attack, to step back from the practical applications and try and describe, as simply as possible, those aspects of the subject which are now thought to be reasonably well understood.

Our choice of material has been guided by the observation that the history of turbulence has, on occasions, been one of heroic initiatives which promised much yet delivered little. So we have applied the filter

of time and chosen to emphasize those theories, both rigorous and heuristic, which look like they might be a permanent feature of the turbulence landscape. There is little attempt to document the latest controversies, or those findings whose significance is still unclear. We begin, in Chapters 1–5, with a fairly traditional introduction to the subject. The topics covered include: the origins of turbulence, boundary layers, the log-law for heat and momentum, free-shear flows, turbulent heat transfer, grid turbulence, Richardson’s energy cascade, Kolmogorov’s theory of the small scales, turbulent diffusion, the closure problem, simple closure models, and so on. Mathematics is kept to a minimum and we presuppose only an elementary knowledge of fluid mechanics and statistics. (Those statistical ideas which are required, are introduced as and when they are needed in the text.) Chapters 1–5 may be appropriate as background material for an advanced undergraduate or introductory postgraduate course on turbulence.

Next, in Chapters 6–8, we tackle the somewhat refined, yet fundamental, problem of homogeneous turbulence. That is, we imagine a fluid that is vigorously stirred and then left to itself. What can we say about the evolution of such a complex system? Our discussion of homogeneous turbulence differs from that given in most texts in that we work mostly in real space (rather than Fourier space) and we pay as much attention to the behaviour of the large, energy-containing eddies, as we do to the small-scale structures.

Perhaps it is worth explaining why we have taken an unconventional approach to homogeneous turbulence, starting with our slight reluctance to embrace Fourier space. The Fourier transform is conventionally used in turbulence because it makes certain mathematical manipulations easier and because it provides a simple (though crude) means of differentiating between large and small-scale processes. However, it is important to bear in mind that the introduction of the Fourier transform produces no new information; it simply represents a transfer of information from real space to Fourier space. Moreover, there are other ways of differentiating between large and small scales, methods that do not involve the complexities of Fourier space. Given that turbulence consists of eddies (blobs of vorticity) and not waves, it is natural to ask why we must invoke the Fourier transform at all. Consider, for example, grid turbulence. We might picture this as an evolving vorticity field in which vorticity is stripped off the bars of the grid and then mixed to form a seething tangle of vortex tubes and sheets. It is hard to picture a Fourier mode being stripped off the bars of the grid! It is the view of this author that, by and large, it is preferable to work in real space, where the relationship between mathematical representation and physical reality is, perhaps, a little clearer.

The second distinguishing feature of Chapters 6–8 is that equal emphasis is given to both large and small scales. This is a deliberate

attempt to redress the current bias towards small scales in monographs on homogeneous turbulence. Of course, it is easy to see how such an imbalance developed. The spectacular success of Kolmogorov's theory of the small eddies has spurred a vast literature devoted to verifying (or picking holes in) this theory. Certainly it cannot be denied that Kolmogorov's laws represent one of the milestones of turbulence theory. However there have been other success stories too. In particular, the work of Landau, Batchelor, and Saffman on the large-scale structure of homogeneous turbulence stands out as a shining example of what can be achieved through careful, physically motivated analysis. So perhaps it is time to redress the balance, and it is with this in mind that we devote part of Chapter 6 to the dynamics of the large-scale eddies. Chapters 6–8 may be suitable as background material for an advanced postgraduate course on turbulence, or act as a reference source for professional researchers.

The final section of the book, Chapters 9 and 10, covers certain special topics rarely discussed in introductory texts. The motivation here is the observation that many geophysical and astrophysical flows are dominated by the effects of body forces, such as buoyancy, Coriolis and Lorentz forces. Moreover, certain large-scale flows are approximately two-dimensional and this has led to a concerted investigation of two-dimensional turbulence over the last few years. We touch on the influence of body forces in Chapter 9 and two-dimensional turbulence in Chapter 10.

There is no royal route to turbulence. Our understanding of it is limited and what little we do know is achieved through detailed and difficult calculation. Nevertheless, it is hoped that this book provides an introduction which is not too arduous and which allows the reader to retain at least some of that initial sense of enthusiasm and wonder.

It is a pleasure to acknowledge the assistance of many friends and colleagues. Alan Bailey, Kate Graham, and Teresa Cronin all helped in the preparation of the manuscript, Jean Delery of ONERA supplied copies of Henri Werle's beautiful photographs, while the drawing of the cigarette plume and the copy of Leonardo's sketch are the work of Fiona Davidson. I am grateful to Julian Hunt, Marcel Lesieur, Keith Moffatt, and Tim Nickels for many interesting discussions on turbulence, and to Alison Jones and Anita Petrie at OUP for their patience and professionalism. In addition, several useful suggestions were made by Ferit Boysan, Jack Herring, Jon Morrison, Mike Proctor, Mark Saville, Christos Vassilicos, and John Young. Finally, I would like to thank Stephen Davidson who painstakingly read the entire manuscript, exposing the many inconsistencies in the original text.

P.A. Davidson
Cambridge, 2003

It remains to call attention to the chief outstanding difficulty of our subject.

HORACE LAMB 1895

Part I: The classical picture of turbulence	1
1 The ubiquitous nature of turbulence	3
1.1 The experiments of Taylor and Bénard	4
1.2 Flow over a cylinder	8
1.3 Reynolds' experiment	9
1.4 Common themes	10
1.5 The ubiquitous nature of turbulence	14
1.6 Different scales in a turbulent flow: a glimpse at the energy cascade of Kolmogorov and Richardson	17
1.7 The closure problem of turbulence	21
1.8 Is there a 'theory of turbulence'?	23
1.9 The interaction of theory, computation, and experiment	24
2 The equations of fluid mechanics	29
2.1 The Navier–Stokes equation	30
2.1.1 Newton's second law applied to a fluid	30
2.1.2 The convective derivative	33
2.1.3 Integral versions of the momentum equation	34
2.1.4 The rate of dissipation of energy in a viscous fluid	35
2.2 Relating pressure to velocity	38
2.3 Vorticity dynamics	39
2.3.1 Vorticity and angular momentum	39
2.3.2 The vorticity equation	43
2.3.3 Kelvin's theorem	48
2.3.4 Tracking vorticity distributions	50
2.4 A definition of turbulence	52
3 The origins and nature of turbulence	57
3.1 The nature of chaos	58
3.1.1 From non-linearity to chaos	59
3.1.2 More on bifurcations	63
3.1.3 The arrow of time	66
3.2 Some elementary properties of freely evolving turbulence	70
3.2.1 Various stages of development	72
3.2.2 The rate of destruction of energy in fully developed turbulence	76
3.2.3 How much does the turbulence remember?	80

3.2.4	The need for a statistical approach and different methods of taking averages	84
3.2.5	Velocity correlations, structure functions and the energy spectrum	88
3.2.6	Is the asymptotic state universal? Kolmogorov's theory	94
3.2.7	The probability distribution of the velocity field	98
4	Turbulent shear flows and simple closure models	107
4.1	The exchange of energy between the mean flow and the turbulence	109
4.1.1	Reynolds stresses and the closure problem of turbulence	110
4.1.2	The eddy-viscosity theories of Boussinesq and Prandtl	113
4.1.3	The transfer of energy from the mean flow to the turbulence	117
4.1.4	A glimpse at the k - ϵ model	122
4.2	Wall-bounded shear flows and the log-law of the wall	126
4.2.1	Turbulent flow in a channel and the log-law of the wall	126
4.2.2	Inactive motion—a problem for the log-law?	131
4.2.3	Turbulence profiles in channel flow	135
4.2.4	The log-law for a rough wall	136
4.2.5	The structure of a turbulent boundary layer	137
4.2.6	Coherent structures	139
4.2.7	Spectra and structure functions near the wall	145
4.3	Free shear flows	147
4.3.1	Planar jets and wakes	147
4.3.2	The round jet	153
4.4	Homogeneous shear flow	157
4.4.1	The governing equations	157
4.4.2	The asymptotic state	161
4.5	Heat transfer in wall-bounded shear flows—the log-law revisited	162
4.5.1	Turbulent heat transfer near a surface and the log-law for temperature	162
4.5.2	The effect of stratification on the log-law—the atmospheric boundary layer	170

4.6	More on one-point closure models	176
4.6.1	A second look at the k - ϵ model	176
4.6.2	The Reynolds stress model	186
4.6.3	Large eddy simulation: a rival for one-point closures?	191
5	The phenomenology of Taylor, Richardson, and Kolmogorov	199
5.1	Richardson revisited	202
5.1.1	Time and length-scales in turbulence	202
5.1.2	The energy cascade pictured as the stretching of turbulent eddies	206
5.1.3	The dynamic properties of turbulent eddies	214
5.2	Kolmogorov revisited	223
5.2.1	Dynamics of the small scales	223
5.2.2	Turbulence induced fluctuations of a passive scalar	234
5.3	The intensification of vorticity and the stretching of material lines	242
5.3.1	Enstrophy production, the skewness factor, and scale invariance	242
5.3.2	Sheets or tubes?	246
5.3.3	Examples of concentrated vortex sheets and tubes	248
5.3.4	Are there singularities in the vorticity field?	251
5.3.5	The stretching of material line elements	256
5.3.6	The interplay of the strain and vorticity fields	260
5.4	Turbulent diffusion by continuous movements	271
5.4.1	Taylor diffusion of a single particle	273
5.4.2	Richardson's law for the relative diffusion of two particles	275
5.4.3	The influence of mean shear on turbulent dispersion	280
5.5	Why turbulence is never Gaussian	283
5.5.1	The experimental evidence and its interpretation	284
5.5.2	A glimpse at closure schemes which assume near-Gaussian statistics	288
5.6	Closure	289
Appendix: The statistical equations for a passive scalar in isotropic turbulence: Yaglom's four-thirds Law and Corrsin's integral		291

Part II: Freely decaying, homogeneous turbulence	297
6 Isotropic turbulence (In real space)	299
6.1 Introduction: exploring isotropic turbulence in real space	299
6.1.1 Deterministic cartoons versus statistical phenomenology	300
6.1.2 The strengths and weaknesses of Fourier space	304
6.1.3 An overview of this chapter	306
6.2 The governing equations of isotropic turbulence	318
6.2.1 Kinematics	318
6.2.2 Dynamics	331
6.2.3 Overcoming the closure problem	338
6.3 The dynamics of the large scales	343
6.3.1 Loitsyansky's integral	345
6.3.2 Kolmogorov's decay laws	346
6.3.3 Landau's angular momentum	347
6.3.4 Batchelor's pressure forces	351
6.3.5 The Saffman-Birkhoff spectrum	355
6.3.6 A reappraisal of the long-range pressure forces in $E \sim k^4$ turbulence	364
6.4 The characteristic signature of eddies of different shape	369
6.4.1 Townsend's model eddy	370
6.4.2 Other model eddies	375
6.5 Intermittency in the inertial-range eddies	376
6.5.1 A problem for Kolmogorov's theory?	377
6.5.2 The $\hat{\beta}$ -model of intermittency	380
6.5.3 The log-normal model of intermittency	382
6.6 The distribution of energy and vorticity across the different eddy sizes	386
6.6.1 A 'real-space' function which represents, approximately, the distribution of energy	387
6.6.2 Cascade dynamics in real space	400
6.6.3 A 'real-space' function which represents, approximately, the distribution of enstrophy	410
6.6.4 A footnote: can we capture Richardson's vision with our mathematical analysis?	412
Appendix: Turbulence composed of Townsend's model eddy	417
7 The role of numerical simulations	423
7.1 What is DNS or LES?	423
7.1.1 Direct numerical simulation	423

7.1.2	Large eddy simulations	427
7.2	On the dangers of periodicity	433
7.3	Structure in chaos	435
7.3.1	Tubes, sheets, and cascades	436
7.3.2	On the taxonomy of worms	438
7.3.3	Structure and intermittency	441
7.3.4	Shear flows	443
7.4	Postscript	445
8	Isotropic turbulence (in spectral space)	449
8.1	Kinematics in spectral space	449
8.1.1	The Fourier transform and its properties	451
8.1.2	The Fourier transform as a filter	454
8.1.3	The autocorrelation function	456
8.1.4	The transform of the correlation tensor and the three-dimensional energy spectrum	460
8.1.5	One-dimensional energy spectra	463
8.1.6	Relating the energy spectrum to the second-order structure function	467
8.1.7	A footnote: singularities in the spectrum arising from anisotropy	468
8.1.8	Another footnote: the transform of the velocity field	470
8.1.9	Definitely the last footnote: what do $E(k)$ and $E_1(k)$ really represent?	471
8.2	Dynamics in spectral space	474
8.2.1	An evolution equation for $E(k)$	474
8.2.2	Closure in spectral space	477
8.2.3	Quasi-Normal type closure schemes (Part 2)	483
Part III: Special topics 495		
9	The influence of rotation, stratification, and magnetic fields on turbulence	497
9.1	The importance of body forces in geophysics and astrophysics	497
9.2	The influence of rapid rotation and stable stratification	499
9.2.1	The Coriolis force	499
9.2.2	The Taylor–Proudman theorem	502
9.2.3	Properties of inertial waves	504
9.2.4	Turbulence in rapidly rotating systems	506
9.2.5	Turbulence with moderate rotation	510
9.2.6	From rotation to stratification (or from cigars to pancakes)	511

9.3	The influence of magnetic fields I—the MHD equations	515
9.3.1	The interaction of moving conductors and magnetic fields: a qualitative overview	515
9.3.2	From Maxwell's equations to the governing equations of MHD	521
9.3.3	Simplifying features of low magnetic Reynolds number MHD	525
9.3.4	Simple properties of high magnetic Reynolds number MHD	527
9.4	The influence of magnetic fields II—MHD turbulence	531
9.4.1	The growth of anisotropy in MHD turbulence	532
9.4.2	The evolution of eddies at low magnetic Reynolds number	534
9.4.3	The Landau invariant for homogeneous MHD turbulence	541
9.4.4	Decay laws at low magnetic Reynolds number	543
9.4.5	Turbulence at high magnetic Reynolds number	545
9.5	The combined effects of Coriolis and Lorentz forces	548
9.5.1	The shaping of eddies by Coriolis and magnetic forces	548
9.5.2	Turbulence in the core of the earth	551
9.5.3	Turbulence near the surface of the sun	563
10	Two-dimensional turbulence	569
10.1	The classical picture of two-dimensional turbulence: Batchelor's self-similar spectrum	570
10.1.1	What is two-dimensional turbulence?	571
10.1.2	What does the turbulence remember?	574
10.1.3	Batchelor's self-similar spectrum	575
10.1.4	The inverse energy cascade of Batchelor and Kraichnan	577
10.1.5	Different scales in two-dimensional turbulence	581
10.1.6	The shape of the energy spectrum: the k^{-3} law	581
10.1.7	Problems with the k^{-3} law	585
10.1.8	A Richardson-type law for the inertial range	586

10.2	Coherent vortices: a problem for the classical theory	589
10.2.1	The evidence	589
10.2.2	The significance	592
10.3	The governing equations in statistical form	593
10.3.1	Correlation functions, structure functions, and the energy spectrum	594
10.3.2	The two-dimensional Karman–Howarth equation and its consequences	598
10.3.3	Loitsyansky’s integral in two dimensions	604
10.4	Variational principles for predicting the final state in confined domains	607
10.4.1	Minimum enstrophy	608
10.4.2	Maximum entropy	610
10.5	Quasi-two-dimensional turbulence: bridging the gap with reality	611
10.5.1	The governing equations for shallow-water, rapidly rotating flow	611
10.5.2	Karman–Howarth equation for shallow-water, rapidly rotating turbulence	614
Epilogue		619
Appendices		623
Appendix 1	Vector identities and an introduction to tensor notation	623
A1.1	Vector identities and theorems	623
A1.2	An introduction to tensor notation	625
Appendix 2	The properties of isolated vortices: invariants, far-field properties, and long-range interactions	632
A2.1	The far-field velocity induced by an isolated eddy	632
A2.2	The pressure distribution in the far field	634
A2.3	Integral invariants of an isolated eddy	635
A2.4	Long-range interactions between eddies	638
Appendix 3	Long-range pressure forces in isotropic turbulence	641
A3.1	A dynamic equation for the pressure-induced, long-range correlations	641
A3.2	Experimental evidence for the strength of long-range pressure forces	643