教育科学"十五"国家规划课题研究成果

Calculus of One Variable

刘金宪　仇计清　韩骁兵

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一元函数微积分

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内容提要

本书是教育科学"十五"国家规划课题研究成果，对高等数学一元微积分部分做了较准确的、深入浅出的英文表达。内容包括函数与极限、导数与微分、中值定理与导数的应用、不定积分、定积分及其应用。数学专业知识与国内高校公共数学课程现行教学内容相当，专业技术符号系统与国内现行数学规范一致。分节配备了习题并附有答案。本书适合作为高等院校数学课程双语数学的配套教材，也可以作为科技英语专业数学课程的教科书，以及数学专业、信息与计算科学专业学科英语的阅读读物。

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总　序

为了更好地适应当前我国高等教育跨越式发展需要,满足我国高校从精英教育向大众化教育的重大转移阶段中社会对高校应用型人才培养的各类要求,探索和建立我国高等学校应用型人才培养体系,全国高等学校教学研究中心(以下简称"教研中心")在承担全国教育科学"十五"国家规划课题——"21世纪中国高等教育人才培养体系的创新与实践"研究工作的基础上,组织全国100余所以培养应用型人才为主的高等院校,进行其子项目课题——"21世纪中国高等学校应用型人才培养体系的创新与实践"的研究与探索,在高等院校应用型人才培养的教学内容、课程体系研究等方面取得了标志性成果,并在高等教育出版社的支持和配合下,推出了一批适应应用型人才培养需要的立体化教材,冠以"教育科学'十五'国家规划课题研究成果"。

2002年11月,教研中心在南京工程学院组织召开了"21世纪中国高等学校应用型人才培养体系的创新与实践"课题立项研讨会。会议确定由教研中心组织国家级课题立项,为参加立项研究的高等院校搭建高起点的研究平台,整体设计立项研究计划,明确目标。课题立项采用整体规划、分步实施、滚动立项的方式,分期分批启动立项研究计划。为了确保课题立项目标的实现,组建了"21世纪中国高等学校应用型人才培养体系的创新与实践"课题领导小组(亦为高校应用型人才立体化教材建设领导小组)。会后,教研中心组织了首批课题立项申报,有63所高校申报了近450项课题。2003年1月,在黑龙江工程学院进行了项目评审,经过课题领导小组严格的把关,确定了首批9项子课题的牵头学校、主持学校和参加学校。2003年3月至4月,各子课题相继召开了工作会议,交流了各校教学改革的情况和面临的具体问题,确定了项目分工,并全面开始研究工作。计划先集中力量,用两年时间形成一批有关人才培养模式、培养目标、教学内容和课程体系等理论研究成果报告和在研究报告基础上同步组织建设的反映应用型人才培养特色的立体化系列教材。

与过去立项研究不同的是,"21世纪中国高等学校应用型人才培养体系的创新与实践"课题研究在审视、选择、消化与吸收多年来已有应用型人才培养探索与实践成果基础上,紧密结合经济全球化时代高校应用型人才培养工作的实际需要,努力实践,大胆创新,采取边研究、边探索、边实践的方式,推进高校应用型人才培养工作,突出重点目标,并不断取得标志性的阶段成果。

教材建设作为保证和提高教学质量的重要支柱和基础,作为体现教学内

容和教学方法的知识载体，在当前培养应用型人才中的作用是显而易见的。探索、建设适应新世纪我国高校应用型人才培养体系需要的教材体系已成为当前我国高校教学改革和教材建设工作面临的十分重要的任务。因此，在课题研究过程中，各课题组充分吸收已有的优秀教学改革成果，并和教学实际结合起来，认真讨论和研究教学内容和课程体系的改革，组织一批学术水平较高、教学经验较丰富、实践能力较强的教师，编写出一批以公共基础课和专业、技术基础课为主的有特色、适用性强的教材及相应的教学辅导书、电子教案，以满足高等学校应用型人才培养的需要。

我们相信，随着我国高等教育的发展和高校教学改革的不断深入，特别是随着教育部“高等学校教学质量和教学改革工程”的启动和实施，具有示范性和适应应用型人才培养的精品课程教材必将进一步促进我国高校教学质量的提高。

全国高等学校教学研究中心

2003 年 4 月

前　言

随着对外开放的日益深入以及全球一体化的快速推进，外语作为信息交流的重要工具正在全方位地向日常工作渗透。双语教学使技术教学与外语有机结合，有利于学生综合素质的全面提高，顺应时代发展的方向。适合双语教学的、同时又与国内专业技术教学内容相适应的外文版教材，正在为教学第一线所急需。本书就是考虑到这种需要而做的一种尝试。本书在数学知识的深广度上，与国内高校数学公共课程现行教学内容相当，专业技术符号系统与国内现行教学规范一致，内容包括函数与极限、导数与微分、中值定理与导数的应用、不定积分、定积分及其应用等。有些内容打了星号，这是考虑到本书有多种不同的用途。比如作为科技英语专业的数学课教科书，受学时数量的限制可以不讲带星号的内容。

双语教学使专业技术知识的学习与语言方面的困难交织在一起，增加了课程的难度。由于快速向高等教育大众化进军，近几年学生状况也有了一些新的变化。考虑到这两方面的情况，编写过程中特别注意把知识点的难度台阶分解拆细，使得表述深入浅出，便于自学。本书作者们来自数学与英语两个专业，编写过程中力求数学概念严谨清晰、语言表达规范顺畅。本书适合作为高等院校数学课程双语教学的配套教材，也可以作为科技英语专业数学课程的教科书，以及数学专业、信息与计算专业学科英语的阅读读物。

伴随着河北科技大学数学课程双语教学试验的进程，这一组配套教材的编写已进行了五个年头。本书是集体努力的结果，先后参加编写工作的有(以姓名笔划为序)：仇计清、王荣欣、王群、江卫华、刘秀君、刘金宪、苏连青、李秀敏、李法朝、李艳、周长杰、陈红霞、郑克旺、骆舒心、索秀云、郭彦平、董丽霞、韩骁兵、蔡习宁等。其中刘金宪、仇计清、韩骁兵任主编，郑克旺任审稿。

自 98 年以来，本书在我校数学课双语教学实践中已使用过 5 届，其间做过两次全面的修改。由于编者水平所限，疏漏甚至错误在所难免，恳请业界同仁不吝赐教。校领导、教务处、理学院、外语学院对数学课双语教学以及本书的编写一直给予大力的支持，数学系领导与老师们也给予多方面的帮助，在此一并表示由衷的谢意。最后，我们要特别感谢的是高等教育出版社的李艳馥和天津大学熊洪允教授，他们宝贵的修改意见、严谨且高效率的工作作风使编者们受益匪浅。

编　者

2003 年 5 月 26 日

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CHAPTER I
FUNCTIONS AND LIMITS

The object investigated in elementary mathematics is basically constant quantities, while it is changeable quantities that we investigate in higher mathematics, whose most important contents is calculus. What is called a function is a correspondence relation between variables. As for a limit, it is a fundamental method to study variables.

The present chapter will introduce some fundamental notions such as functions, limits, continuity with their some properties.

§ 1 Functions

1.1 Sets

Set is a basic concept in mathematics.

A set is a *collection* of objects with some certain property. The individual objects in the collection are called *elements* or *members* of the set, and they are said to belong to or be contained in the set. The set, in turn, is said to contain or be composed of its elements.

For instance, all the students in a classroom form a set, and everybody is an element of the set. We have also the set of all real numbers, the set of all points in a plane, etc.

Sets usually are denoted by capital letters: $A, B, C, \cdots, X, Y, Z$; elements are designated by lower-case letters: $a, b, c, \cdots, x, y, z$. We use the special notation:

$$x \in S$$

to mean that "x is an element of S" or "x belongs to S". If x does not belong to S, we write $x \notin S$.

A set A is composed of its elements $a_1, a_2, \cdots, a_n$, we write

$$A = \{a_1, a_2, \cdots, a_n\}.$$

M is a set that is composed of all elements x, and those elements have the same property, we write

$$M = \{x \mid \text{the property that } x \text{ has}\}.$$

Two sets A and B are said to be *equal* if they consist of exactly the same elements, in which case we write $A = B$.

A set A is said to be a *subset* of a set B, and we write $A \subseteq B$, whenever every element of A belongs to B. We also say that A is contained in B or that B contains A.

The statement $A \subseteq B$ does not rule out the possibility that $B \subseteq A$. In fact, we may have both $A \subseteq B$ and $B \subseteq A$, but this happens only if A and B have the same elements. In other words, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

It is possible for a set to contain no element whatever. This set is called the *empty set*, and will be denoted by the symbol $\varnothing$.

An *interval* is a kind of number sets that is used very frequently, where the word number means a real number throughout this book.

If $a < b$, we denote by (a, b) the set of all numbers x satisfying the inequalities $a < x < b$ and refer to this set as the *open interval* from a to b. We write

$$(a, b) = \{x \mid a < x < b\},$$

read as: x is greater than a and less than b. The corresponding *closed interval*, written $[a, b]$, is the set of all numbers x satisfying $a \leqslant x \leqslant b$, that is $[a, b] = \{x \mid a \leqslant x \leqslant b\}$, read as: x is greater than or equal to a, and less than or equal to b. Similarly, the sets $[a, b) = \{x \mid a \leqslant x < b\}$, $(a, b] = \{x \mid a < x \leqslant b\}$ are called *half - open intervals*.

These intervals above are called *finite intervals*, the number $b - a$ is called the *length of the interval*. Seen from the number axis, the length of a finite interval is finite. In addition, there exist *infinite intervals*. We introduce the symbol $+\infty$ (read as plus infinity) and $-\infty$ (read as minus infinity) to express the infinite interval. For example.

$$[a, +\infty) = \{x \mid a \leqslant x\},$$
$$(-\infty, b) = \{x \mid x < b\}.$$

The graphes of the two kinds of intervals are shown in Figure 1 - 1.

Any open interval containing a point a as its midpoint is called a *neighborhood* of a. We denote by $U(a, \delta) = \{x \mid a - \delta < x < a + \delta\}$ (see Figure 1 - 2). The positive number δ is called the radius of the neighborhood.

The inequalities $a - \delta < x < a + \delta$ are equivalent to $-\delta < x - a < \delta$, and to

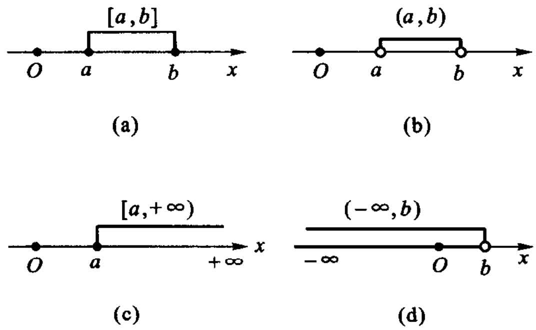

Figure 1 − 1

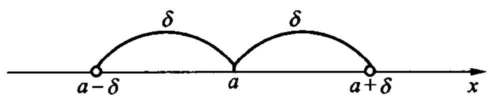

Figure 1 − 2

$|x-a|<\delta$. Thus, $U(a,\delta)$ consists of all points x whose distance from a is less than δ.

1.2 Notion of a function

In various fields we have to deal with relationships that exist between one set and another. Mathematicians refer to certain types of those relations as *functions*.

Example 1. The volume of a cube is a function of its edge − length. If the edge has length x, the volume V is given by the formula $V = x^3$. When the length x of edge takes on its value in the range of positive numbers, the volume V has a definite value corresponding to x under the rule expressed in the formula. □

Example 2. The absolute-value function. Consider the function which assigns to each real number x the nonnegative number $|x|$. Its graph is shown in Figure 1 − 3. □

The meaning of a function is essentially this: Given two sets, say X and Y, a function is a correspondence which associates each element of X with one and only one element of Y. The set X is called the *domain* of the function. Those elements of Y associated with the elements in X form a set called the *range* of the function. Below, we describe the formal definition of a function.

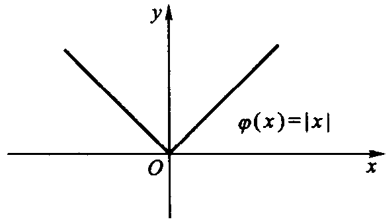

Figure 1-3

Definition 1. *The variable y is said to be a function of variable x, and we write $y=f(x)$, if there is a definite value of y corresponding by some rule to every given number $x \in D$. Where the variable x is called* ***independent variable****, the number set D is called the* ***domain of definition****, y is called* ***dependent variable****, f expresses the rule of correspondence.*

Example 3. $y=\frac{1}{\sqrt{1-x^2}} \quad (-1<x<1)$.

The function associates each number x with the number $\frac{1}{\sqrt{1-x^2}}$. If f denotes this function, then we have $f(x)=\frac{1}{\sqrt{1-x^2}}(-1<x<1)$. In particular, $f(0)=1, f(\frac{1}{2})=\frac{2}{\sqrt{3}}$. □

Example 4. $y=2x^2+x+1 \quad (-\infty<x<+\infty)$.

If f denotes this function, then we have $f(x)=2x^2+x+1$. In other words, the correspondence rule f acting on the quantity x yields $2x^2+x+1$. If f acts on another quantity $(x+1)$, this should lead to $f(x+1)=2(x+1)^2+(x+1)+1=2x^2+5x+4$. □

1.3 Properties of functions

(1) Monotonicity

Let D be the domain of $f(x)$, an interval $I \subseteq D$, the function $f(x)$ is said to be *monotone increasing* on the interval I if $f(x_1)<f(x_2)$ for every pair of points x_1 and x_2 in I with $x_1<x_2$. Similarly, $f(x)$ is said to be *monotone decreasing* on I if $f(x_1)>f(x_2)$ for all $x_1<x_2$ in I (see Figure 1-4).

Example 5. The function $f(x)=x^2$ is monotone increasing on $[0,+\infty)$ and monotone decreasing on $(-\infty, 0]$. However, $f(x)$ is not monotonic in the interval $(-\infty,+\infty)$. □