TURING 图灵原版数学·统计学系列

Visual Complex Analysis

复分析

可视化方法

（英文版）

［美］Tristan Needham 著

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内容提要

本书是复分析领域近年来较有影响的一本著作。作者用丰富的图例展示各种概念、定理和证明思路，十分便于读者理解，充分揭示了复分析的数学之美。书中讲述的内容有几何、复变函数变换、默比乌斯变换、微分、非欧几何、复积分、柯西公式、向量场、复积分、调和函数等。

本书可作为大学本科、研究生的复分析课程教材或参考书。

图灵原版数学·统计学系列

复分析——可视化方法（英文版）

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For

Roger Penrose

and

George Burnett-Stuart

推　荐　序

我非常高兴地向读者推荐Tristan Needham所写的 *Visual Complex Analysis*一书.

我是在网上查阅有关材料时偶尔看到此书的介绍的. 在网上不可能看到全书，而只看到出版者（Oxford University Press）提供的部分章节中的三节，印象很深. 它为大学本科生(二年级)讲到了很深的数学知识，把拓扑学的一些重要的概念结合复分析来讲本是常见的事，但讲得如此系统，而且深入浅出是非常少见的. 此书几何特色很突出，而国内现有的复分析教材很少有这样做的. 因为几何味重，就有可能较充分地利用计算机图形学的手段，使全书可视性很强，这在国内的教材中十分少见.

关于本书内容的选取，也有许多值得称道的地方. 首先有许多在当代数学及其应用中很重要，而现行的教材中多有省略的，在本书中得到较充分的反映. 特别是关于非欧几何，只在20世纪50年代翻译的一本前苏联复变函数教材中讲过，而其后就完全销声匿迹了. 但现在看来，它的重要性日益明显，本书就讲得比较充分. 类似的还有Moebius变换等等. 其次，还有许多材料，其他书也讲到了，但本书时常从另一角度来处理，常有一种出于意料之感. 例如用向量场讲积分理论，在我翻译的Spivak所著的《流形上的微积分》一书（已由人民邮电出版社出版）中也讲了，但本书讲法大有特色，是我未曾想到的. 而且讲得很好懂，是很不容易的.

总的说来，本书确实体现了近几十年数学教材的一个发展趋势. 把最新的成就，用浅显的方法教给低年级学生. 本书第一版于1997年出版，到2006年已第十二次印刷，并已有德文和日文译本，可见其受欢迎的程度。

有许多关于本书的评论，在一篇书评中把它与牛顿的著作联系起来，这当然不是说本书可与牛顿的原理一书相提并论，而是指现在微积分的教学过于形式化，与牛顿当年强调几何直观的精神，相去甚远. 因此，许多数学家主张回到牛顿. 这篇书评把本书与Chandrasekhar（美国大物理学家）的名著《为普通读者写的牛顿原理》一书相提并论，则比较合适. 又一篇书评把它与大物理学家Feynman的名著《物理学讲义》相比，也是这个道理.

关于本书的评论还有不少，其中希望注意两位评论者.一位是Penrose，英国大物理学家，名望似可与霍金相比较.另一位是Ian Stewart，既是一位优秀的数学家，又是著名的科普作者，前两年是著名刊物《科学美国人》的专栏作家。

教育部的数学教学指导委员会最近在上海开了一次讨论会，主题是如何在大学教材中反映数学科学的新发展，我介绍了这本书，引起了不少同行的关切.我相信及时引进此书会受到国内读者的欢迎.

齐民友

于武汉大学数学学院

Preface

Theories of the known, which are described by different physical ideas, may be equivalent in all their predictions and hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown. For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in one's attempt to understand what is not yet understood.

R. P. Feynman [1966]

A Parable

Imagine a society in which the citizens are encouraged, indeed compelled up to a certain age, to read (and sometimes write) musical scores. All quite admirable. However, this society also has a very curious—few remember how it all started—and disturbing law: *Music must never be listened to or performed!*

Though its importance is universally acknowledged, for some reason music is not widely appreciated in this society. To be sure, professors still excitedly pore over the great works of Bach, Wagner, and the rest, and they do their utmost to communicate to their students the beautiful meaning of what they find there, but they still become tongue-tied when brashly asked the question, "What's the point of all this?!"

In this parable, it was patently unfair and irrational to have a law forbidding would-be music students from experiencing and understanding the subject directly through "sonic intuition." But in our society of mathematicians we *have* such a law. It is not a written law, and those who flout it may yet prosper, but it says, *Mathematics must not be visualized!*

More likely than not, when one opens a random modern mathematics text on a random subject, one is confronted by abstract symbolic reasoning that is divorced from one's sensory experience of the world, *despite* the fact that the very phenomena one is studying were often discovered by appealing to geometric (and perhaps physical) intuition.

This reflects the fact that steadily over the last hundred years the honour of visual reasoning in mathematics has been besmirched. Although the great mathematicians have always been oblivious to such fashions, it is only recently that the "mathematician in the street" has picked up the gauntlet on behalf of geometry.

The present book openly challenges the current dominance of purely symbolic logical reasoning by using new, visually accessible arguments to explain the truths of elementary complex analysis.

Computers

In part, the resurgence of interest in geometry can be traced to the mass-availability of computers to draw mathematical objects, and perhaps also to the related, somewhat breathless, popular interest in chaos theory and in fractals. This book instead advocates the more sober use of computers as an aid to geometric *reasoning*.

I have tried to encourage the reader to think of the computer as a physicist would his laboratory—it may be used to check existing ideas about the construction of the world, or as a tool for discovering new phenomena which then demand new ideas for their explanation. Throughout the text I have suggested such uses of the computer, but I have deliberately avoided giving *detailed* instructions. The reason is simple: whereas a mathematical idea is a timeless thing, few things are more ephemeral than computer hardware and software.

Having said this, the program "$f(z)$" is currently the best tool for visually exploring the ideas in this book; a free demonstration version can be downloaded directly from Lascaux Graphics [http://www.primenet.com/~lascaux/]. On occasion it would also be helpful if one had access to an all-purpose mathematical engine such as *Maple*® or *Mathematica*®. However, I would like to stress that none of the above software is essential: the entire book can be fully understood without *any* use of a computer.

Finally, some readers may be interested in knowing how computers were used to produce this book. Perhaps five of the 501 diagrams were drawn using output from *Mathematica*®; the remainder I drew by hand (or rather "by mouse") using CorelDRAW™, occasionally guided by output from "$f(z)$". I typeset the book in LaTeX using the wonderful *Y&Y TeX System for Windows* [http://www.YandY.com/], the figures being included as EPS files. The text is Times, with Helvetica heads, and the mathematics is principally MathTime™, though nine other mathematical fonts make cameo appearances. All of these Adobe Type 1 fonts were obtained from Y&Y, Inc., with the exception of Adobe's *MathematicalPi-Six* font, which I used to represent quaternions. Having typeset the book, I used the DVIPSONE™ component of the *Y&Y TeX System for Windows* to generate a fully page-independent, DSC-compliant PostScript® file, which I transmitted to Oxford via the Internet (using FTP) in the form of a single ZIP file. Finally, OUP printed the book directly from this PostScript® file.

The Book's Newtonian Genesis

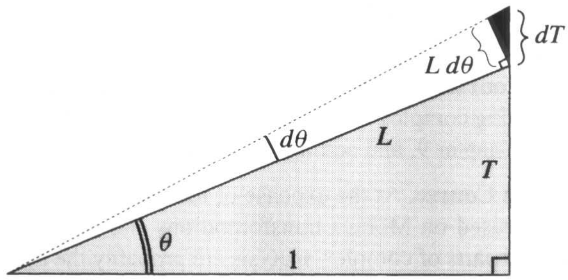
In the summer of 1982, having been inspired by Westfall's [1980] excellent biography, I made an intense study of Newton's [1687] masterpiece, *Philosophiae Naturalis Principia Mathematica*. While the Nobel physicist S. Chandrasekhar [1995] has sought to lay bare the remarkable nature of Newton's *results* in the *Principia*, the present book instead arose out of a fascination with Newton's *methods*.

It is fairly well known that Newton's original 1665 version of the calculus was different from the one we learn today: its essence was the manipulation of power series, which Newton likened to the manipulation of decimal expansions in arithmetic. The symbolic calculus—the one in every standard textbook, and the one now associated with the name of Leibniz—was also perfectly familiar to

Newton, but apparently it was of only incidental interest to him. After all, armed with his power series, Newton could evaluate an integral like $\int e^{-x^2}\,dx$ just as easily as $\int \sin x\,dx$. Let Leibniz try *that!*

It is less well known that around 1680 Newton became disenchanted with both these approaches, whereupon he proceeded to develop a *third* version of calculus, based on *geometry*. This "geometric calculus" is the mathematical engine that propels the brilliant physics of Newton's *Principia*.

Having grasped Newton's method, I immediately tried my own hand at using it to simplify my teaching of introductory calculus. An example will help to explain what I mean by this. Let us show that if $T = \tan\theta$, then $\frac{dT}{d\theta} = 1 + T^2$. If we increase θ by a small amount $d\theta$ then T will increase by the amount dT in the figure below. To obtain the result, we need only observe that in the limit as $d\theta$ tends to zero, the black triangle is ultimately similar [exercise] to the shaded triangle. Thus, in this limit,



$$\frac{dT}{L\,d\theta} = \frac{L}{1} \quad \Longrightarrow \quad \frac{dT}{d\theta} = L^2 = 1 + T^2.$$

Only gradually did I come to realize how naturally this mode of thought could be applied—almost exactly 300 years later!—to the geometry of the complex plane.

Reading This Book

In the hope of making the book fun to read, I have attempted to write as though I were explaining the ideas directly to a friend. Correspondingly, I have tried to make you, the reader, into an active participant in developing the ideas. For example, as an argument progresses, I have frequently and deliberately placed a pair of logical stepping stones sufficiently far apart that you may need to pause and stretch slightly to pass from one to the next. Such places are marked "[exercise]"; they often require nothing more than a simple calculation or a moment of reflection.

This brings me to the exercises proper, which may be found at the end of each chapter. In the belief that the essential prerequisite for finding the answer to a question is the *desire* to find it, I have made every effort to provide exercises that provoke curiosity. They are considerably more wide-ranging than is common, and they often establish important facts which are then used freely in the text itself. While problems whose be all and end all is routine calculation are thereby avoided, I believe that readers will automatically develop considerable computational skill *in the process* of seeking solutions to these problems. On the other hand, my intention in a large number of the exercises is to illustrate how geometric thinking can often *replace* lengthy calculation.

Any part of the book marked with a star ("*") may be omitted on a first reading. If you do elect to read a starred section, you may in turn choose to omit any starred *sub*sections. Please note, however, that a part of the book that is starred is not necessarily any more difficult, nor any less interesting or important, than any other part of the book.

Teaching from this Book

The entire book can probably be covered in a year, but in a single semester course one must first decide what *kind* of course to teach, then choose a corresponding path through the book. Here I offer just three such possible paths:

• **Traditional Course.** Chapters 1 to 9, *omitting all starred material* (e.g., the whole of Chapter 6).

• **Vector Field Course.** In order to take advantage of the Pólya vector field approach to visualizing complex integrals, one could follow the "Traditional Course" above, omitting Chapter 9, and adding the unstarred parts of Chapters 10 and 11.

• **Non-Euclidean Course.** At the expense of teaching any integration, one could give a course focused on Möbius transformations and non-Euclidean geometry. These two related parts of complex analysis are probably the most important ones for contemporary mathematics and physics, and yet they are also the ones that are almost entirely neglected in undergraduate-level texts. On the other hand, graduate-level works tend to assume that you have already encountered the main ideas as an undergraduate: Catch 22!

Such a course might go as follows: All of Chapter 1; the unstarred parts of Chapter 2; all of Chapter 3, including the starred sections but (possibly) omitting the starred *sub*sections; all of Chapter 4; all of Chapter 6, including the starred sections but (possibly) omitting the starred *sub*sections.

Omissions and Apologies

If one believes in the ultimate unity of mathematics and physics, as I do, then a very strong case for the necessity of complex numbers can be built on their apparently fundamental role in the quantum mechanical laws governing *matter*. Also, the work of Sir Roger Penrose has shown (with increasing force) that complex numbers play an equally central role in the relativistic laws governing the structure of *space–time*. Indeed, if the laws of matter and of space–time are ever to be reconciled, then it seems very likely that it will be through the auspices of the complex numbers. This book cannot explore these matters; instead, we refer the interested reader to Feynman [1963, 1985], to Penrose [1989, 1994], and to Penrose and Rindler [1984].

A more serious omission is the lack of discussion of Riemann surfaces, which I had originally intended to treat in a final chapter. This plan was aborted once it became clear that a serious treatment would entail expanding the book beyond reason. By this time, however, I had already erected much of the necessary scaffolding, and this material remains in the finished book. In particular, I hope that the interested reader will find the last three chapters helpful in understanding Riemann's original physical insights, as expounded by Klein [1881]. See also Springer [1957, Chap. 1], which essentially reproduces Klein's monograph, but with additional helpful commentary.

I consider the history of mathematics to be a vital tool in understanding both the current state of mathematics, and its trajectory into the future. Sadly, however, I can do no more than touch on historical matters in the present work; instead I refer you to the remarkable book, *Mathematics and Its History*, by John Stillwell [1989]. Indeed, I strongly encourage you to think of his book as a companion to mine: not only does it trace and explain the development of complex analysis, but it also explores and illuminates the connections with other areas of mathematics.

To the expert reader I would like to apologize for having invented the word "amplitwist" [Chapter 4] as a synonym (more or less) for "derivative", as well the component terms "amplification" and "twist". I can only say that the need for *some* such terminology was forced on me in the classroom: if you try teaching the ideas in this book *without* using such language, I think you will quickly discover what I mean! Incidentally, a precedence argument in defence of "amplitwist" might be that a similar term was coined by the older German school of Klein, Bieberbach, *et al*. They spoke of "eine Drehstreckung", from "drehen" (to twist) and "strecken" (to stretch).

A significant proportion of the geometric observations and arguments contained in this book are, to the best of my knowledge, new. I have not drawn attention to this in the text itself as this would have served no useful purpose: students don't need to know, and experts will know without being told. However, in cases where an idea is clearly unusual but I am aware of it having been published by someone else, I have tried to give credit where credit is due.

In attempting to rethink so much classical mathematics, I have no doubt made mistakes; the blame for these is mine alone. Corrections will be gratefully received, and then posted, at http://www.usfca.edu/vca.

My book will no doubt be flawed in many ways of which I am not yet aware, but there is one "sin" that I have intentionally committed, and for which I shall not repent: many of the arguments are not rigorous, at least as they stand. This is a serious crime if one believes that our mathematical theories are merely elaborate mental constructs, precariously hoisted aloft. Then rigour becomes the nerve-racking balancing act that prevents the entire structure from crashing down around us. But suppose one believes, as I do, that our mathematical theories are attempting to capture aspects of a robust Platonic world that is not of our making. I would then contend that an initial lack of rigour is a small price to pay if it allows the reader to see into this world more directly and pleasurably than would otherwise be possible.

San Francisco, California T. N.
June, 1996

Acknowledgements

First and foremost I wish to express my indebtedness to Dr. Stanley Nel. He is my friend, my colleague, and my Dean, and in all three of these capacities he has helped me to complete this book. As a friend he offered support when progress was slow and my spirits were low; as a mathematical colleague he read much of the book and offered helpful criticisms; as Dean he granted me a succession of increasingly powerful computers, and when the US Immigration Service sought to have my position filled by an "equally qualified" American, he successfully fought them on my behalf. For all this, and much else besides, I offer him my deep gratitude.

Next I would like to thank Prof. John Stillwell of Monash University. The great value I place on his writings should be clear from the frequency with which I refer to them in the pages that follow. Also, though I lack his gift for conciseness, I have sought to emulate elements of his approach in an attempt to give back *meaning* to mathematical concepts. Finally, my greatest and most concrete debt arises from the fact that he read each draft chapter as it was written, and this despite the fact that we had never even met! The book owes a great deal to his numerous helpful suggestions and corrections.

I consider myself very fortunate that the mathematics department here at the University of San Francisco is completely free of political intrigue, rivalry, and other assorted academic blights. I am grateful to *all* my colleagues for creating such a friendly and supportive atmosphere in which to work. In particular, however, I should like to single out the following people for thanks:

- Nancy Campagna for her diligent proof-reading of half the book;
- Allan Cruse and Millianne Lehmann, not only for granting all my software requests during their respective tenures as department chair, but also for all their kind and sage advice since my arrival in the United States;
- James Finch for his patience and expertise in helping me overcome various problems associated with my typesetting of the book in LaTeX;
- Robert Wolf for having built up a superb mathematics collection in our library;
- Paul Zeitz for his great faith in me and in the value of what I was trying to accomplish, for his concrete suggestions and corrections, and for his courage in being the first person (other than myself) to teach complex analysis using chapters of the book.

Prof. Gerald Alexanderson of Santa Clara University has my sincere thanks for the encouragement he offered me upon reading some of the earliest chapters, as well as for his many subsequent acts of kindness.

I will always be grateful for the education I received at Merton College, Oxford. It is therefore especially pleasing and fitting to have this book published by OUP, and I would particularly like to thank Dr. Martin Gilchrist, the former Senior Mathematics Editor, for his enthusiastic encouragement when I first approached him with the idea of the book.

When I first arrived at USF from England in 1989 I had barely seen a computer. The fact that OUP printed this book directly from my Internet-transmitted PostScript® files is an indication of how far I have come since then. I owe all this to James Kabage. A mere graduate student at the time we met, Jim quickly rose through the ranks to become Director of Network Services. Despite this fact, he never hesitated to spend *hours* with me in my office resolving my latest hardware or software crisis. He always took the extra time to clearly explain to me the reasoning leading to his solution, and in this way I became his student.

I also thank Dr. Benjamin Baab, the Executive Director of Information Technology Services at USF. Despite his lofty position, he too was always willing to roll up his sleeves in order to help me resolve my latest Microsoft® conundrum.

Eric Scheide (our multitalented Webmaster) has my sincere gratitude for writing an extremely nifty *Perl* program that greatly speeded my creation of the index.

I thank Prof. Berthold Horn of MIT for creating the magnificent *Y&Y TeX System for Windows* [http://www.YandY.com/], for his generous help with assorted TeXnical problems, and for his willingness to adopt my few suggestions for improving what I consider to be the Mercedes-Benz of the TeX world.

Similarly, I thank Martin Lapidus of Lascaux Graphics for incorporating many of my suggestions into his "$f(z)$" program [http://www.primenet.com/~lascaux/], thereby making it into an even better tool for doing "visual complex analysis".

This new printing of the book incorporates a great many corrections. Most of these were reported by readers, and I very much appreciate their efforts. While I cannot thank each one of these readers by name, I must acknowledge Dr. R. von Randow for single-handedly having reported more than 30 errors.

As a student of Roger Penrose I had the privilege of watching him think out loud by means of his beautiful blackboard drawings. In the process, I became convinced that if only one tried hard enough—or were clever enough!—every mathematical mystery could be resolved through geometric reasoning. George Burnett-Stuart and I became firm friends while students of Penrose. In the course of our endless discussions of music, physics, and mathematics, George helped me to refine both my conception of the nature of mathematics, and of what constitutes an acceptable explanation within that subject. My dedication of this book to these two friends scarcely repays the great debt I owe them.

The care of several friends helped me to cope with depression following the death of my beloved mother Claudia. In addition to my brother Guy and my father Rodney, I wish to express my appreciation to Peter and Ginny Pacheco, and to Amy Miller. I don't know what I would have done without their healing affection.

Lastly, I thank my dearest wife Mary. During the writing of this book she allowed me to pretend that science was the most important thing in life; now that the book is over, she is my daily proof that there is something even more important.

Contents