

# **Dynamics of the Standard Model**

**JOHN F. DONOGHUE  
EUGENE GOLOWICH  
BARRY R. HOLSTEIN**

## **标准模型动力学**

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**To our families**

## Preface

The Standard Model lagrangian  $\mathcal{L}_{\text{SM}}$  embodies our knowledge of the strong and electroweak interactions. It contains as fundamental degrees of freedom the spin one-half quarks and leptons, the spin one gauge bosons, and the spin zero Higgs fields. Symmetry plays the central role in determining its dynamical structure. The lagrangian exhibits invariance under  $SU(3)$  gauge transformations for the strong interactions and under  $SU(2) \times U(1)$  gauge transformations for the electroweak interactions. Despite the presence of (all too) many input parameters, it is a mathematical construction of considerable predictive power.

There are several books available which describe in detail the construction of  $\mathcal{L}_{\text{SM}}$  and its quantization, and which deal with aspects of symmetry breaking. We felt the need for a book describing the next steps, how  $\mathcal{L}_{\text{SM}}$  is connected to the observable physics of the real world. There are a considerable variety of techniques, of differing rigor, which are used by particle physicists to accomplish this. We present here those which have become indispensable tools. In addition, we attempt to convey the insights and ‘conventional wisdom’ which have been developed throughout the field. This book can only be an introduction to the riches contained in the subject, hopefully providing a foundation and a motivation for further exploration by its readers.

In writing the book, we have become all too painfully aware that each topic, indeed each specific reaction, has an extensive literature and phenomenology, and that there is a limitation to the depth that can be presented compactly. We emphasize applications, not fundamentals, of quantum field theory. Proofs of formal topics like renormalizability or the quantization of gauge fields are left to books such as Bjorken and Drell, Cheng and Li, Itzykson and Zuber, Pokorski, and Ramond. We include no analysis of parton phenomenology, and refer the reader, for example, to the books by Barger and Phillips or Field. In addition, the study by computer of lattice field theory is an extensive and rapidly changing dis-

cipline which we do not attempt to cover. Although it would be tempting to discuss some of the many stimulating ideas, among them supersymmetry, grand unification and string theory, which attempt to describe physics beyond the Standard Model, limitations of space prevent us from doing so.

Although this book begins gently, we do assume that the reader already has some familiarity with quantum field theory. As an aid to those who lack familiarity with path integral methods, we include a presentation, in Appendix A, which treats this subject in an introductory manner. In addition, we assume a knowledge of the basic phenomenology of particle physics, say, at the level of the books by Perkins or Halzen and Martin.

We have constructed the material to be of use to a wide spectrum of readers who are involved with the physics of elementary particles. Certainly it contains material of interest to both theorist and experimentalist alike. Given the trend to incorporate the Standard Model in the study of nuclei, we expect the book to be of use to the nuclear physics community as well. Even the student being trained in the mathematics of string theory would be well advised to learn the role that sigma models play in particle theory.

This is a good place to stress some conventions employed in this book. Chapters are identified with roman numerals. In cross-referencing equations, we include the chapter number if the referenced equation is in a chapter different from the point of citation. The Minkowski metric is  $g_{\mu\nu} = \text{diag} \{1, -1, -1, -1\}$ . Throughout, we use the natural units  $\hbar = c = 1$ , and choose  $e > 0$  so that the electron has electric charge  $-e$ . We employ rationalized Heaviside-Lorentz units, and the fine structure constant is related to the charge via  $\alpha = e^2/4\pi$ . The coupling constants for the  $SU(3)_c \times SU(2)_L \times U(1)$  gauge structure of the Standard Model are denoted respectively as  $g_3$ ,  $g_2$ ,  $g_1$ , and we employ coupling constant phase conventions analogous to electromagnetism for the other abelian and nonabelian covariant derivatives of the Standard Model. The chiral projection operator for left-handed massless spin one-half particles is  $(1 + \gamma_5)/2$ , and in analyzing systems in  $d$  dimensions, we employ the parameter  $\epsilon \equiv 4 - d$ . What is meant by the ‘Fermi constant’ is discussed in Sect. V-2.

In preparing this book, we have benefitted from the kind advice of many colleagues. We wish to especially thank A. Andrianov, N.G. Deshpande, P.B. Pal, K.S. Sateesh, T. Sotirelis, S. Stone, G. Valencia, and D. Wyler for physics input, and R. Gilson and M. McDonald for technical assistance.

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# I

## Inputs to the Standard Model

This book is about the Standard Model of elementary particle physics. If we set the beginning of the modern era of particle physics in 1947, the year the pion was discovered, then the ensuing years of research have revealed the existence of a consistent, self-contained layer of reality. The energy range which defines this layer of reality extends up to several hundred GeV, or in terms of length, down to distances of order  $10^{-16}$  cm. The Standard Model is a field-theoretic description of strong and electroweak interactions at these energies. It requires as input nineteen independent parameters.\* These parameters are not explained by the Standard Model; their presence implies the need for an understanding of Nature at an even deeper level. Nonetheless, processes described by the Standard Model possess a remarkable insulation from signals of such new physics. Although the strong interactions remain a calculational challenge, the Standard Model would appear to have sufficient content to describe all existing data. Thus far, it is a theoretical structure which has worked splendidly.

### I.1 Quarks and leptons

In the Standard Model, the fundamental fermionic constituents of matter are the quarks and the leptons. Quarks, but not leptons, engage in the strong interactions as a consequence of their color-charge. Each quark and lepton has spin one-half. Collectively, they display conventional Fermi-Dirac statistics. No attempt is made in the Standard Model either to explain the variety and number of quarks and leptons or to compute any of their properties. That is, these particles are taken at this level as truly elementary. This is not unreasonable. There is no experimental evidence

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\* There are three charged-lepton masses, six quark masses, three gauge coupling constants, three quark-mixing angles and one complex phase, a Higgs mass and quartic coupling constant, and the *QCD* vacuum angle.



for quark or lepton compositeness, such as excited states or form factors associated with intrinsic structure.

In Table I-1 we enumerate the quarks and leptons, and display each particle's mass and electric charge. Mass values are in units of  $\text{GeV}/c^2$  and electric charges are given as multiples of the proton charge  $e$ . There are three lepton types: electron ( $\nu_e, e$ ), muon ( $\nu_\mu, \mu$ ), and tau ( $\nu_\tau, \tau$ ). The leptons fall into two classes according to electric charge, the neutral neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ , and the negatively charged  $e^-, \mu^-, \tau^-$ . Although future work may produce unambiguous evidence for neutrino mass (*cf.* Sects. V-4, XVI-6), in this book we shall posit that each neutrino is massless. Lepton mass values range from the present neutrino upper limits to the tau mass of about twice the proton mass. They fit no evident pattern.

Quarks are believed to be permanently confined entities. This forces some thought as how to properly define 'quark mass', and indeed, several distinct definitions are commonly used. Let us defer a discussion of this issue and simply note that the values in Table I-1 (called *current quark masses*) provide an overview of the quark mass spectrum. Each quark appearing in Table I-1 is said to constitute a separate *flavor*, *i.e.*, six quark flavors exist in Nature. Like the leptons the quarks fall into two classes according to their electrical charge. The  $u, c, t$  quarks have charge  $2e/3$  and the  $d, s, b$  quarks have charge  $-e/3$ . Unlike the leptons, there are no neutral quarks and quark electrical charge is fractional. As this is being written, the set of quarks is not yet complete since the top quark has not been detected experimentally; the current lower bound on its mass is displayed in Table I-1. Phenomenological analyses of electroweak radiative corrections (*cf.* Sect. XVI-5) have tended to place the top quark mass in the range  $100 \rightarrow 190 \text{ GeV}$ .

Table I-1. The basic fermions

Quarks			Leptons		
Flavor	Mass <sup>a</sup>	Charge	Flavor	Mass <sup>a</sup>	Charge
$u$	$0.003 \rightarrow 0.007$	$2e/3$	$\nu_e$	$< 0.8 \times 10^{-8}$	0
$d$	$0.007 \rightarrow 0.015$	$-e/3$	$e$	$5.110 \times 10^{-4}$	$-e$
$c$	$1.3 \rightarrow 1.7$	$2e/3$	$\nu_\mu$	$< 2.7 \times 10^{-4}$	0
$s$	$0.15 \rightarrow 0.3$	$-e/3$	$\mu$	0.1057	$-e$
$t$	$> 113$	$2e/3$	$\nu_\tau$	$< 0.035$	0
$b$	$4.8 \rightarrow 5.2$	$-e/3$	$\tau$	1.777	$-e$

<sup>a</sup>Expressed in units of  $\text{GeV}/c^2$ .