Unsolved Problems in Intuitive Mathematics
Volume I

Richard K. Guy

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Preface

To many laymen, mathematicians appear to be problem solvers, people who do "hard sums". Even inside the profession we classify ourselves as either theorists or problem solvers. Mathematics is kept alive, much more than by the activities of either class, by the appearance of a succession of unsolved problems, both from within mathematics itself and from the increasing number of disciplines where it is applied. Mathematics often owes more to those who ask questions than to those who answer them. The solution of a problem may stifle interest in the area around it. But "Fermat's Last Theorem", because it is not yet a theorem, has generated a great deal of "good" mathematics, whether goodness is judged by beauty, by depth or by applicability.

To pose good unsolved problems is a difficult art. The balance between triviality and hopeless unsolvability is delicate. There are many simply stated problems which experts tell us are unlikely to be solved in the next generation. But we have seen the Four Color Conjecture settled, even if we don't live long enough to learn the status of the Riemann and Goldbach hypotheses, of twin primes or Mersenne primes, or of odd perfect numbers. On the other hand, "unsolved" problems may not be unsolved at all, or may be much more tractable than was at first thought.

Among the many contributions made by Hungarian mathematician Erdös Pál, not least is the steady flow of well-posed problems. As if these were not incentive enough, he offers rewards for the first solution of many of them, at the same time giving his estimate of their difficulty. He has made many payments, from $1.00 to $1000.00.

One purpose of this book is to provide beginning researchers, and others who are more mature, but isolated from adequate mathematical stimulus, with a supply of easily understood, if not easily solved, problems which they can consider in varying depth, and by making occasional partial progress, gradually acquire the interest, confidence and persistence that are essential to successful research.

But the book has a much wider purpose. It is important for students and teachers of mathematics at all levels to realize that although they are not yet capable of research and may have no hopes or ambitions in that direction, there are plenty of unsolved problems that are well within their comprehension, some of which will be solved in their lifetime. Many amateurs have been attracted to the subject and many successful researchers first gained their confidence by examining problems in euclidean geometry, in number theory, and more recently in combinatorics and graph theory, where it is possible to understand questions and even to formulate them and obtain original results without a deep prior theoretical knowledge.

The idea for the book goes back some twenty years, when I was impressed by the circulation of lists of problems by the late Leo Moser and co-author Hallard Croft, and by the articles of Erdös. Croft agreed to let me help him amplify his collection into a book, and Erdös has repeatedly encouraged and prodded us. After some time, the Number Theory chapter swelled into a volume of its own, part of a series which will contain a volume on Geometry, Convexity and Analysis, written by Hallard T. Croft, and one on Combinatorics, Graphs and Games by the present writer.

References, sometimes extensive bibliographies, are collected at the end of each problem or article surveying a group of problems, to save the reader from turning pages. In order not to lose the advantage of having all references collected in one alphabetical list, we give an Index of Authors, from which particular papers can easily be located provided the author is not too prolific. Entries in this index and in the General Index and Glossary of Symbols are to problem numbers instead of page numbers.

Many people have looked at parts of drafts, corresponded and made helpful comments. Some of these were personal friends who are no longer with us: Harold Davenport, Hans Heilbronn, Louis Mordell, Leo Moser, Theodor Motzkin, Alfred Rényi and Paul Turán. Others are H. L. Abbott, J. W. S. Cassels, J. H. Conway, P. Erdös, Martin Gardner, R. L. Graham, H. Halberstam, D. H. and Emma Lehmer, A. M. Odlyzko, Carl Pomerance, A. Schinzel, J. L. Selfridge, N. J. A. Sloane, E. G. Straus, H. P. F. Swinnerton-Dyer and Hugh Williams. A grant from the National (Science and Engineering) Research Council of Canada has facilitated contact with these and many others. The award of a Killam Resident Fellowship at The University of Calgary was especially helpful during the writing of a final draft. The technical typing was done by Karen McDermid, by Betty Teare and by Louise Guy, who also helped with proof-reading. The staff of Springer-Verlag in New York has been courteous, competent and helpful.

In spite of all this help, many errors remain, for which I assume reluctant responsibility. In any case, if the book is to serve its purpose it will start becoming out of date from the moment it appears; it has been becoming out

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of date ever since its writing began. I would be glad to hear from readers. There must be many solutions and references and problems which I don't know about. I hope that people will avail themselves of this clearing house. A few good researchers thrive by rediscovering results for themselves, but many of us are disappointed when we find that our discoveries have been anticipated.

Calgary 81:08:13 Richard K. Guy

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Introduction

Number theory has fascinated both the amateur and the professional for a longer time than any other branch of mathematics; so that much of it is now of considerable technical difficulty. However, there are more unsolved problems than ever before, and though many of these are unlikely to be solved in the next generation, this probably won't deter people from trying. They are so numerous that they have already filled more than one volume so that the present book is just a personal sample.

Erdös recalls that Landau, at the International Congress in Cambridge in 1912, gave a talk about primes and mentioned four problems (see A1, A7, C1 below) which were unattackable in the present state of science, and says that in 1980 they still are.

Here are some good sources of problems in number theory.

P. Erdös, Some unsolved problems, *Michigan Math. J.* **4** (1957) 291–300.

P. Erdös, On unsolved problems, *Publ. Math. Inst. Hungar. Acad. Sci.* **6** (1961) 221–254.

P. Erdös, *Quelques Problèmes de la Théorie des Nombres*, Monographies de l'Enseignment Math. #6, Geneva, 1963, 81–135.

P. Erdös, Extremal problems in number theory, *Proc. Symp. Pure Math.* **8**, Amer. Math. Soc., Providence, 1965, 181–189.

P. Erdös, Some recent advances and current problems in number theory, in *Lectures on Modern Mathematics* **3**, Wiley, New York, 1965, 196–244.

P. Erdös, Résultats et problèmes en théorie des nombres, *Seminar Delange-Pisot-Poitou* **24**, 1972–73.

P. Erdös, Problems and results in combinatorial number theory, in *A Survey of Combinatorial Theory*, North-Holland, 1973, 117–138.

P. Erdös, *Problems and Results in Combinatorial Number Theory*, Bordeaux, 1974.

Paul Erdös, Problems and results in combinatorial number theory III, *Springer Lecture Notes in Math.* **626** (1977) 43–72; *MR* **57** #12442.

P. Erdös, Combinatorial problems in geometry and number theory, *Amer. Math. Soc. Proc. Sympos. Pure Math.* **34** (1979) 149–162.

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Paul Erdös, A survey of problems in combinatorial number theory, in *Combinatorial Mathematics, Optimal Designs and their Applications* (Proc. Symp. Colo. State Univ. 1978) Ann. Discrete Math. **6** (1980) 89–115.
Paul Erdös, Problems and results in number theory and graph theory, *Congressus Numerantium* XXVII (Proc. 9th Manitoba Conf. Num. Math. Comput. 1979) Utilitas Math., Winnipeg, 1980, 3–21.
P. Erdös and R. L. Graham, *Old and New Problems and Results in Combinatorial Number Theory*, Monographies de l'Enseignment Math. No. 28, Geneva, 1980.
Pál Erdös and András Sárközy, Some solved and unsolved problems in combinatorial number theory, *Math. Slovaca*, **28** (1978) 407–421; *MR* **80i**:10001.
H. Fast and S. Świerczkowski, *The New Scottish Book*, Wrocław, 1946–1958.
Heini Halberstam, Some unsolved problems in higher arithmetic, in Ronald Duncan and Miranda Weston-Smith (eds.) *The Encyclopaedia of Ignorance*, Pergamon, Oxford and New York, 1977, 191–203.
Proceedings of Number Theory Conference, Univ. of Colorado, Boulder, 1963.
Report of Institute in the Theory of Numbers, Univ. of Colorado, Boulder, 1959.
Daniel Shanks, *Solved and Unsolved Problems in Number Theory*, Chelsea, New York, 2nd ed. 1978; *MR* **80e**:10003.
W. Sierpiński, *A Selection of Problems in the Theory of Numbers*, Pergamon, 1964.
S. Ulam, *A Collection of Mathematical Problems*, Interscience, New York, 1960.

Throughout this volume, "number" means natural number, c is an absolute positive constant, not necessarily the same each time it appears, and ε is an arbitrarily small positive constant. We use Donald Knuth's "floor" ($\lfloor\ \rfloor$) and "ceiling ($\lceil\ \rceil$) symbols for "the greatest integer not greater than" and "the least integer not less than."

The notation $f(x) = O(g(x))$ and $f(x) \ll g(x)$ mean that there are constants c_1, c_2 such that $c_1 g(x) < f(x) < c_2 g(x)$ for all sufficiently large x; while $f(x) \sim g(x)$ means that $f/g \to 1$, and $f(x) = o(g(x))$ means that $f/g \to 0$, as $x \to \infty$.

The book has been partitioned, somewhat arbitrarily at times, into six sections:

A. Prime numbers
B. Divisiblity
C. Additive number theory
D. Diophantine equations
E. Sequences of integers
F. None of the above.

A. Prime Numbers

We can partition the positive integers into three classes:

the unit, 1
the primes, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, . . .
the composite numbers, 4, 6, 8, 9, 10, . . .

A number greater than 1 is **prime** if its only positive divisors are 1 and itself; otherwise it's **composite**. Primes have interested mathematicians at least since Euclid, who showed that there were infinitely many.

Denote the nth prime by p_n, e.g., $p_1 = 2, p_2 = 3, p_{99} = 523$; and the number of primes not greater than x by $\pi(x)$, e.g., $\pi(2) = 1, \pi(3\frac{1}{2}) = 2, \pi(1000) = 168$. The greatest common divisor (g.c.d.) of m and n is denoted by (m, n). If $(m, n) = 1$, we say that m and n are **coprime**; for example, $(14, 15) = 1$.

Dirichlet's theorem tells us that there are infinitely many primes in any arithmetic progression,

$$a,\ a+b,\ a+2b,\ a+3b, \ldots$$

provided $(a, b) = 1$. An article, giving a survey of problems about primes and a number of further references, is

A. Schinzel and W. Sierpiński, Sur certains hypothèses concernant les nombres premiers, *Acta Arith.* **4** (1958) 185–208 (erratum **5** (1959) 259); *MR* **21** #4936.

Table 7 (D27) can be used as a table of primes < 1000.

The general problem of determining whether a large number is prime or composite, and in the latter case of determining its factors, has fascinated number theorists down the ages. With the advent of high speed computers, considerable advances have been made, and a special stimulus has recently been provided by the application to cryptanalysis. Some other references appear after Problem A3.

Leonard Adleman and Frank Thomson Leighton, An $O(n^{1/10.89})$ primality testing algorithm, *Math. Comput.* **36** (1981) 261–266.
Leonard M. Adleman, Carl Pomerance and Robert S. Rumely, On distinguishing prime numbers from composite numbers (to appear)
R. P. Brent, An improved Monte Carlo factorization algorithm, *BIT*, **20** (1980), 176–184.
John D. Dixon, Asymptotically fast factorization of integers, *Math. Comput.* **36** (1981) 255–260.
Richard K. Guy, How to factor a number, *Congressus Numerantium XVI* Proc. 5th Manitoba Conf. Numer. Math., Winnipeg, 1975, 49–89.
H. W. Lenstra, Primality testing, *Studieweek Getaltheorie en Computers*, Stichting Mathematisch Centrum, Amsterdam, 1980, 41–60.
G. L. Miller, Riemann's hypothesis and tests for primality, *J. Comput. System Sci.*, **13** (1976) 300–317.
J. M. Pollard, Theorems on factorization and primality testing, *Proc. Cambridge Philos. Soc.* **76** (1974) 521–528.
J. M. Pollard, A Monte Carlo method for factorization, *BIT* **15** (1975) 331–334; *MR* **50** #6992.
R. Rivest, A. Shamir and L. Adleman, A method for obtaining digital signatures and public key cryptosystems, *Communications A.C.M.*, Feb. 1978.
R. Solovay and V. Strassen, A fast Monte-Carlo test for primality, *SIAM J. Comput.* **6** (1977) 84–85; erratum **7** (1978) 118; *MR* **57** #5885.
H. C. Williams, Primality testing on a computer, *Ars Combin.* **5** (1978) 127–185. *MR* **80d**:10002.
H. C. Williams and R. Holte, Some observations on primality testing, *Math. Comput.* **32** (1978) 905–917; *MR* **57** #16184.
H. C. Williams and J. S. Judd, Some algorithms for prime testing using generalized Lehmer functions, *Math. Comput.* **30** (1976) 867–886.

A1. Are there infinitely many primes of the form $a^2 + 1$? Probably so, and in fact Hardy and Littlewood (their Conjecture E) guessed that the number, $P(n)$, of such primes less than n, was asymptotic to $c\sqrt{n}/\ln n$,

$$¿ \quad P(n) \sim c\sqrt{n}/\ln n \quad ?$$

i.e., that the ratio of $P(n)$ to $\sqrt{n}/\ln n$ tends to c as n tends to infinity. The constant c is

$$c = \prod \left\{ 1 - \frac{\left(\frac{-1}{p}\right)}{p-1} \right\} = \prod \left\{ 1 - \frac{(-1)^{(p-1)/2}}{p-1} \right\} \approx 1.3727$$

where $\left(\frac{-1}{p}\right)$ is the Legendre symbol (see F5) and the product is taken over all odd primes. They make similar conjectures, differing only in the value of c, for the number of primes represented by more general quadratic expressions. But we don't know of any integer polynomial, of degree greater than one, for which it has been proved that it takes an infinity of prime values. Is there even one prime $a^2 + b$ for each $b > 0$?

Iwaniec has shown that there are infinitely many n for which $n^2 + 1$ is the product of at most two primes, and his result extends to other irreducible quadratics.

Ulam and others noticed that the patterns formed by the prime numbers when the sequence of numbers is written in a "square spiral" seems to favor diagonals which correspond to certain "prime-rich" quadratic polynomials. For example, the main diagonal of Figure 1 corresponds to Euler's famous formula $n^2 + n + 41$.

421	420	**419**	418	417	416	415	414	413	412	411	410	**409**	408	407	406	405	404	403	402
422	**347**	346	345	344	343	342	341	340	339	338	**337**	336	335	334	333	332	**331**	330	**401**
423	348	**281**	280	279	278	**277**	276	275	274	273	272	**271**	270	**269**	268	267	266	329	400
424	**349**	282	**223**	222	221	220	219	218	217	216	215	214	213	212	**211**	210	265	328	399
425	350	**283**	224	**173**	172	171	170	169	168	**167**	166	165	164	**163**	162	209	264	327	398
426	351	284	225	174	**131**	130	129	128	**127**	126	125	124	123	122	161	208	**263**	326	**397**
427	352	285	226	175	132	**97**	96	95	94	93	92	91	90	121	160	207	262	325	396
428	**353**	286	**227**	176	133	98	**71**	70	69	68	**67**	66	**89**	120	159	206	261	324	395
429	354	287	228	177	134	99	72	**53**	52	51	50	65	88	119	158	205	260	323	394
430	355	288	**229**	178	135	100	**73**	54	**43**	42	49	64	87	118	**157**	204	259	322	393
431	356	289	230	**179**	136	**101**	74	55	44	**41**	48	63	86	117	156	203	258	321	392
432	357	290	231	180	**137**	102	75	56	45	46	**47**	62	85	116	155	202	**257**	320	391
433	358	291	232	**181**	138	**103**	76	57	58	**59**	60	**61**	84	115	154	201	256	319	390
434	**359**	292	**233**	182	**139**	104	77	78	**79**	80	81	82	**83**	114	153	200	255	318	**389**
435	360	**293**	234	183	140	105	106	**107**	108	**109**	110	111	112	**113**	152	**199**	254	**317**	388
436	361	294	235	184	141	142	143	144	145	146	147	148	**49**	150	**151**	198	253	316	387
437	362	295	236	185	186	187	188	189	190	**191**	192	**193**	194	195	196	**197**	252	315	386
438	363	296	237	238	**239**	240	**241**	242	243	244	245	246	247	248	249	250	**251**	314	385
439	364	297	298	299	300	301	302	303	304	305	306	**307**	308	309	310	**311**	312	**313**	384
440	365	366	**367**	368	369	370	371	372	**373**	374	375	376	377	378	**379**	380	381	382	**383**

Figure 1. Primes (in **bold**) Form Diagonal Patterns.

The only result for expressions (*not* polynomials!) of degree greater than 1 is due to Pyateckii-Šapiro, who proved that the number of primes of the form $\lfloor n^c \rfloor$ in the range $1 < n < x$ is $(1 + o(1))x/(1 + c)\ln x$ if $1 \le c \le 12/11$.

Martin Gardner, The remarkable lore of prime numbers, *Scientific Amer*. **210** #3 (Mar. 1964) 120–128.

G. H. Hardy and J. E. Littlewood, Some problems of 'partitio numerorum' III: on the expression of a number as a sum of primes, *Acta Math.* **44** (1922) 1–70.

Henryk Iwaniec, Almost-primes represented by quadratic polynomials, *Invent. Math.* **47** (1978) 171–188; *MR* **58** #5553.

Carl Pomerance, A note on the least prime in an arithmetic progression, *J. Number Theory* **12** (1980) 218–223.

I. I. Pyateckii-Šapiro, On the distribution in sequences of the form $[f(n)]$, *Mat. Sbornik N.S.* **33** (1953) 559–566; *MR* **15**, 507.

A2. Are there infinitely many primes of the form $n! + 1$? The only values of $n \le 230$ which give primes are 1, 2, 3, 11, 27, 37, 41, 73, 77, 116, and 154. It is not known if $n! - 1$ or

$$X = 1 + \prod_{i=1}^{k} p_i$$

is prime infinitely often. The only values of $p_k \le 1031$ for which X is prime are $p_k = 2, 3, 5, 7, 11, 31, 379, 1019$, and 1021.

Let q be the least prime greater than X. Then R. F. Fortune conjectures that $q - X + 1$ is prime for all k. It is clear that it is not divisible by the first k primes, and Selfridge observes that the truth of the conjecture would follow from one of Schinzel, that for $x > 8$ there is always a prime between x and $x + (\ln x)^2$. The first few fortunate primes are 3, 5, 7, 13, 23, 17, 19, 23, 37, 61, 67, 61, 71, 47, 107, 59, 61, 109, 89, 103, 79, ... The answers to the questions are probably "yes," but it does not seem conceivable that such conjectures will come within reach either of computers or of analytical tools in the foreseeable future.

More hopeful, but still difficult, is the following conjecture of Erdös and Stewart: are $1! + 1 = 2$, $2! + 1 = 3$, $3! + 1 = 7$, $4! + 1 = 5^2$, $5! + 1 = 11^2$ the only cases where $n! + 1 = p_k^a p_{k+1}^b$ and $p_{k-1} \le n < p_k$? [Note that $(a, b) = (1, 0), (1, 0), (0, 1), (2, 0)$, and $(0, 2)$ in these five cases.]

Erdös also asks if there are infinitely many primes p for which $p - k!$ is composite for each k such that $1 \le k! < p$; for example, $p = 101$ and $p = 211$. He suggests that it may be easier to show that there are infinitely many integers n $(l! < n \le (l+1)!)$ all of whose prime factors are greater than l, and for which all the numbers $n - k!$ $(1 \le k \le l)$ are composite.

David Silverman noticed that the product

$$\prod_{i=1}^{m} \frac{p_i + 1}{p_i - 1}$$

is an integer for $m = 1, 2, 3, 4$ and 8 and asked if it ever is again.

I. O. Angell and H. J. Godwin, Some factorizations of $10^n \pm 1$, *Math. Comput.* **28** (1974) 307–308.

Alan Borning, Some results for $k! \pm 1$ and $2 \cdot 3 \cdot 5 \cdots p \pm 1$, *Math. Comput.* **26** (1972) 567–570.

Martin Gardner, Mathematical Games, *Sci. Amer.* **243** #6 (Dec. 1980) 18–28.

Solomon W. Golomb, On Fortune's conjecture, *Math. Mag.* (to appear)

S. Kravitz and D. E. Penney, An extension of Trigg's table, *Math. Mag.* **48** (1975) 92–96.

Mark Templer, On the primality of $k! + 1$ and $2 * 3 * 5 * \cdots * p + 1$, *Math. Comput.* **34** (1980) 303–304.

A3. Primes of special form have been of perennial interest, especially the **Mersenne primes** $2^p - 1$ (p is necessarily prime, but that is *not* sufficient! $2^{11} - 1 = 2047 = 23 \times 89$) in connexion with perfect numbers (see B1) and **repunits**, $(10^p - 1)/9$.

The powerful Lucas-Lehmer test, in conjunction with successive generations of computers, and more sophisticated techniques in using them, continues to add to the list of primes for which $2^p - 1$ is also prime:

$$2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203,$$
$$2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, \ldots.$$

Their number is undoubtedly infinite, but proof is again hopelessly beyond reach. Suppose $M(x)$ is the number of primes $p \le x$ for which $2^p - 1$ is prime. Find a convincing heuristic argument for the size of $M(x)$. Gillies gave one suggesting that $M(x) \sim c \ln x$, but some people do not believe this. Pomerance has an argument for $M(x) \sim c(\ln \ln x)^2$ but he says this doesn't agree with the facts.

D. H. Lehmer puts $S_1 = 4$, $S_{k+1} = S_k^2 - 2$, supposes that $2^p - 1$ is a Mersenne prime, notes that $S_{p-2} \equiv 2^{(p+1)/2}$ or $-2^{(p+1)/2} \pmod{2^p - 1}$ and asks: which?

Selfridge conjectures that if n is a prime of the form $2^k \pm 1$ or $2^{2k} \pm 3$, then $2^n - 1$ and $(2^n + 1)/3$ are either both prime or neither of them are. Moreover if both are prime, then n is of one of those forms. Is this an example of "the strong law of small numbers"?

If p is a prime, is $2^p - 1$ always **squarefree** (does it never contain a repeated factor)? This seems to be another unanswerable question. It is safe to conjecture that the answer is "No!" This *could* be settled by computer if you were lucky. As D. H. Lehmer has said about various factorization methods, "Happiness is just around the corner." Selfridge puts the computational difficulties in perspective by proposing the problem: find fifty more numbers like 1093 and 3511. (Fermat's theorem tells us that if p is prime, then p divides $2^p - 2$; the primes 1093 and 3511 are the only ones less than 3×10^9 for which p^2 divides $2^p - 2$.)

The corresponding primes for $(10^p - 1)/9$ are 2, 19, 23, 317, 1031, the last two of which were found by Hugh Williams quite recently, subject to final tests being completed in the last case. Repunits >1 are known never to be squares. Are they ever cubes? When are they squarefree?

The **Fermat numbers**, $F_n = 2^{2^n} + 1$, are also of continuing interest; they are prime for $0 \le n \le 4$ and composite for $5 \le n \le 19$ and for many larger values of n. Hardy and Wright give a heuristic argument which suggests that only a finite number of them are prime. Selfridge would like to see this strengthened to support the conjecture that all the rest are composite.

Because of their special interest as potential factors of Fermat numbers, and because proofs of their primality are comparatively easy, numbers of the form $k \cdot 2^n + 1$ have received special attention, at least for small values

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