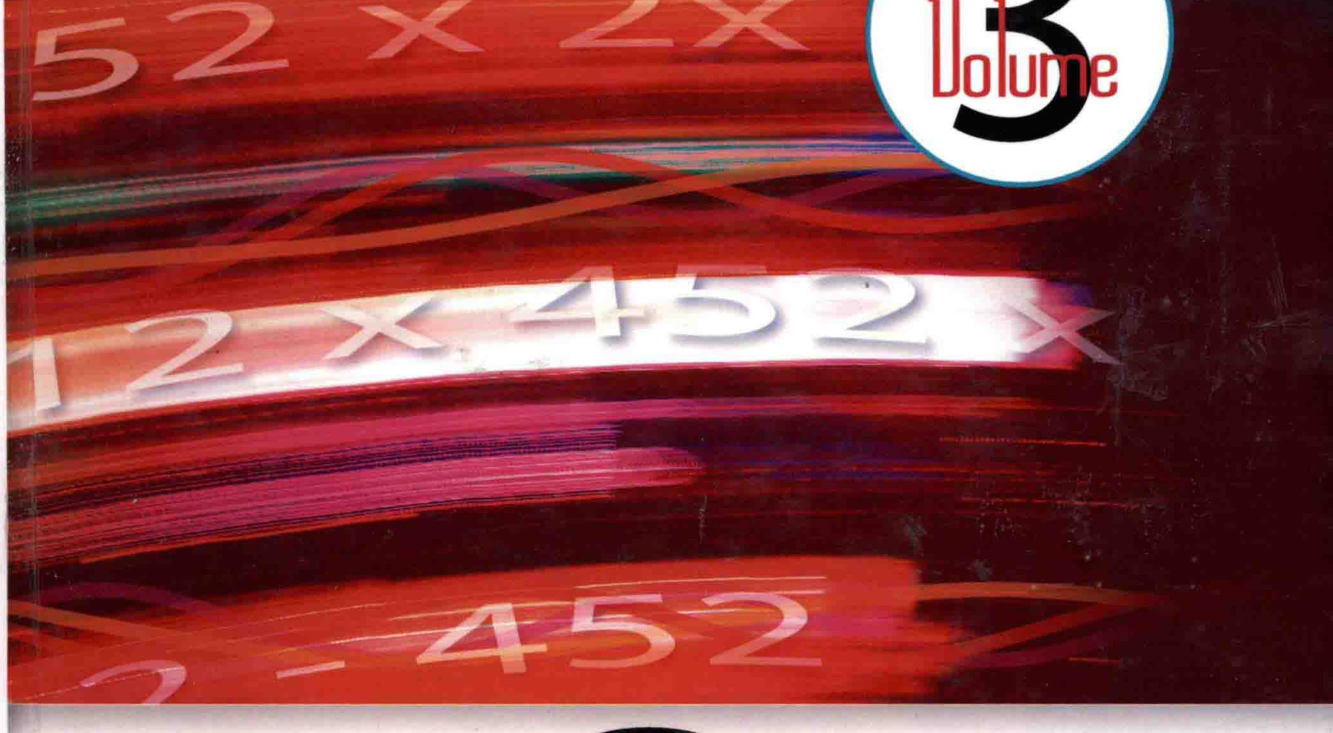
Mathematics, Game Theory and Algebra Compendium

Jacob H. Mathias
Editor

Volume 3



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MATHEMATICS RESEARCH DEVELOPMENTS

MATHEMATICS, GAME THEORY AND ALGEBRA COMPENDIUM

VOLUME 3

JACOB H. MATHIAS
EDITOR

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PREFACE

This book is devoted to new advances in all branches of mathematics, game theory and applications, and pure and applied algebra and geometry including mathematical formulation of NMR experimental parameters for diffusion magnetic resonance imaging; optimization of Kalman Filtering performance in received signal strength based mobile positioning; ORE extensions over near pseudo valuation rings; subset selection of treatments; rigorous kinetic analysis of the racket flick-motion in tennis for generating topspin and backspin and linear versus nonlinear human operator modeling.

Magnetic resonance sequences have been developed to label the position of a spin, albeit indirectly, through the Larmore frequency. This provides the basis for measuring diffusion. However, the interpretation of the data is complicated by the effects of restricted geometries and the mathematical modeling required to account for this becomes nontrivial especially in cylindrical and spherical geometries. Generally one have to resort to numerical methods and/or approximations to model diffusion within restricted geometries depending on the experimental conditions. Based on the new NMR diffusion equation, parts I and II of Chapter 1 intend to provide a very straightforward theoretical background for measuring diffusion of water protons and specific chemicals encountered in most common advanced MRI methods including diffusion MRI, perfusion MRI, functional MRI.

During the past decade, major breakthroughs in magnetic resonance imaging (MRI) quality were made by means of great improvement in scanner hardware and pulse sequences. Some advanced MRI techniques have truly revolutionized the detection of disease states and MRI can now-within a few minutes-acquire important quantitative information non-invasively from an individual in any plane or volume at comparatively high resolution. However, the very basic physics of this promising technological breakthrough is not well understood. In Chapter 2, parameters that are measured from time to time in advanced MRI seem to be logically and functionally related but the theoretical facility to optimally explore them is still missing. In a single experimental investigation, for example, few of huge amount of information available are effectively used. Parts I and II of this study intend to provide a very straightforward theoretical background for measuring diffusion of water protons and specific chemicals encountered in most common advanced MRI methods including diffusion MRI, perfusion MRI, functional MRI.

Chapter 3 considers the problem of Mobile Terminal (MT) positioning based on a time series of Received Signal Strength (RSS) measurements and provides statistical estimators of the MT- Base Station (BS) distance which substantially improve the MT positioning accuracy. The set of MT-BS distance estimators defined for optimizing RSS based positioning when a single RSS measurement sample is available [1] is used as a basis. The problem that is analyzed refers to the adaptation of these estimators to the presence of Kalman filtering in the MT position calculation process. Three different Kalman filtering options can be applied at different stages of the MT position calculation process [2]: (a) RSS Kalman filtering, (b) MT-BS distance Kalman filtering and (c) MT position coordinates Kalman filtering. To identify the optimal MT-BS distance estimators at the presence of Kalman filtering a method exploiting the characteristics of the steady state Kalman filter performance is developed. This novel method allows for the definition of refined MT-BS distance estimators matching the nature of the Kalman filter process. The results indicate that the resulting estimators provide good efficiency for all Kalman Filter options.

Let R be a commutative Noetherian ring which is also an algebra over □ , □ is the field of rational numbers). Let σ be an automorphism of R and δ a σ-derivation of R such that $\sigma(\delta(a)) = \delta(\sigma(a))$, for all $a \in R$. Chapter 4 concerns Ore extensions over near pseudo valuation rings (NPVR) and almost δ-divided rings. Towards this the authors prove:

1. If R is a near pseudo valuation ring, then $O(R) = R[x;\sigma,\delta]$ is a near pseudo valuation ring.
2. If R is an almost δ-divided ring, then $O(R) = R[x;\sigma,\delta]$ is an almost δ-divided ring.

In Chapter 5, the authors consider $p(\geq 2)$ independent treatment populations characterized by the unknown location parameters $\mu_1,\ldots,\mu_p$ and $q(\geq 2)$ independent control populations characterized by the unknown location parameters $\xi_1,\ldots,\xi_q$. The authors assume that θ is the common scale parameter of all the $p+q$ populations. Let $\xi_{[q]}$ be the largest of ξs and define a treatment population to be "good" if its location parameter exceeds $\xi_{[q]} - \varepsilon$ for a given $\varepsilon > 0$. A selection procedure is proposed to select a subset of the p treatment populations which includes all the good treatment populations with probability at least P^*, where P^* is a pre-assigned value. Moreover, for given values of ε_1 and ε_2 $(\varepsilon_2 > \varepsilon_1 > 0)$, the authors define the ith treatment population as "bad" if $\mu_i < \xi_q - \varepsilon_2$, i =1, …, p. In this case a selection procedure is proposed and a sample size is determined so that the probability of omitting a "good" treatment population or selecting a "bad" treatment population is at most $1-P^*$. Finally, the implementation of the proposed methodology is demonstrated through numerical example based on real life data.

In modern tennis, the essential part of executing high-performance shots is generating topspin and backspin on the ball. The purpose of Chapter 6 is to present a rigorous kinetic analysis of the flick-motion of the tennis racket, based on the original concept of the vector of mass/inertia moments coupled to the pole and for the corresponding axis. The forward racket flick-motion generates topspin on the ball, while the backward racket flick-motion generates backspin (also known as slice) on the ball. To describe both kinds of the racket flick-motion the authors use rigorous kinetic analysis. This advanced rigid-body analysis includes the six degrees-of-freedom (DOF) Newton-Euler dynamics, a new sophisticated form of vectors/tensors of the racket mass inertia moments, impact forces during the racket-ball contact, and mass-deviational moment vectors of the racket and ball rotation before, during and after contact.

The motivation behind mathematically modeling the *human operator* is to help explain the response characteristics of the complex dynamical system including the human manual controller. In Chapter 7, the authors present two approaches to human operator modeling: classical linear control approach and modern nonlinear control approach. The latter one is formalized using both fixed and adaptive Lie-Derivative based controllers.

Suppose that $m \geq 4$ and that R is a commutative ring with identity in which 2 is invertible. Let $N(m,R)$ be the nilpotent subalgebra of the orthogonal Lie algebra $o(2m,R)$. In Chapter 8, the authors give an explicit description of the derivation algebra of $N(m,R)$.

Historical development of algebra occurred in three stages; rhetorical or prose algebra, syncopated or abbreviated algebra and symbolic algebra - known as "school algebra". The analysis of Chapter 9 suggests that the first civilization to develop symbolic algebra was the Vedic Indians. The philosophical and religious ideas influenced the development of the decimal system and arithmetic and that led to algebra. Symbolic algebra appears to be deep rooted in Vedic philosophy. The Vedic mathematic were of a high level at an early period. The Hindus applied algebra freely creating formulas that simplified calculations. In geometry and trigonometry they developed formulas useful to understand the physical world satisfying the needs of religion (apara and para vidya). Geometrical focus, logic and proof type are features of Greek mathematics "boldness of conception, abstraction, symbolism" are evident in Indian mathematics. From history, a number of implications can be drawn. Real life, imaginative and creative problems that encourage risk should be the focus in student learning; allowing students freely move between symbols, numbers and magnitudes rather than taking a static unchanging view. Concrete, pictorial and symbolic modes are present in ancient learning. Real life practical, philosophical and religious needs in concert motivated progress to symbolic algebra. The historical analysis supports the use of rich context based problems that stimulate and motivate students to raise levels higher to transfer knowledge. The road from arithmetic to algebra was clearly in line with current emphasis in mathematics education but at an early stage in human history.

The purpose of Chapter 10 is to introduce the concept of regular fuzzy biclosure spaces and investigate some of their characterizations.

In Chapter 11 it is given the basic properties of a near-ring, which involve a set of matrix units and matrix near-rings and it is investigated the answer of the question "when the inclusion $Mat_n(R;R) \subseteq M_E(R^n)$ becomes an equality?" which posed in [1].

In Chapter 12 the authors introduce a new class of functions called contra- $\alpha\hat{g}$-continuous functions and study some of their basic properties in topological spaces.

The purpose of Chapter 13 is to introduce a new class of functions called (1,2)*generalized α-continuous functions, $(1,2)*\alpha$-generalized-continuous functions and $(1,2)*\alpha$-strongly semi-continuous functions in bitopological spaces. Also the authors obtain some decompositions of $(1,2)*\alpha$-continuous function in bitopological spaces.

Chapter 14 provides a simple exact expansion of a general function, while using constant coefficients. In addition, the point of expansion is not arbitrarily chosen.

Chapter 15 synthesizes and analyzes some important current and recent contributions to the theory of the firm under uncertainty. In so doing, it examines the production and hedging decisions of the competitive firm under a single source and multiple sources of uncertainty.

In Chapter 16 several properties and a characterization of δ_M -lifting modules are proved. Also the authors investigate the interconnections between δ_M -lifting, δ_M - supplemented modules and δ_M - semiperfect modules.

In Chapter 17, the authors apply Ghoussoub-Preiss's generalized Mountain Pass Lemma with Cerami-Palais-Smale type condition to study the existence of new periodic solutions with a given period for some second order Hamiltonian systems.

Chapter 18 deals with partial actions of inductive groupoids on rings. The authors establish a one-to-one correspondence between partial actions of an inductive groupoid G on a ring R, in which the domain of each partial bijection is an ideal of R, and meet-preserving global actions of the Birget-Rhodes expansion G^{BR} of G on R. Using this correspondence the authors obtain that the Birget-Rhodes expansion of the action groupoid $R \times_\alpha G$ and the action groupoid $R \times_\beta G^{BR}$ are isomorphic, where α is a partial action of G on a ring R and β is the correspondent action of G^{BR} on R. In particular, they get the equivalence of two suitable functors from the category of the partial actions of inductive groupoids into that of the ordered groupoids.

CONTENTS

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Chapter 1

Mathematical Formulation of NMR Experimental Parameters for Diffusion Magnetic Resonance Imaging – Part I (Cylindrical Geometry)

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Abstract

Magnetic resonance sequences have been developed to label the position of a spin, albeit indirectly, through the Larmore frequency. This provides the basis for measuring diffusion. However, the interpretation of the data is complicated by the effects of restricted geometries and the mathematical modeling required to account for this becomes nontrivial especially in cylindrical and spherical geometries. Generally one have to resort to numerical methods and/or approximations to model diffusion within restricted geometries depending on the experimental conditions. Based on the new NMR diffusion equation, parts I and II of this study intend to provide a very straightforward theoretical background for measuring diffusion of water protons and specific chemicals encountered in most common advanced MRI methods including diffusion MRI, perfusion MRI, functional MRI.

Keywords: Bloch NMR flow equations, Diffusion, Brownian motion, Perfusion, fMRI, Biological flow

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Introduction

The Bloch NMR flow equations are a set of coupled differential equations that describe the behavior of the macroscopic magnetization. The equations can account for the effects of precession, relaxation, field in-homogeneity, and RF pulses. If one considers the magnetization as a function of space as well as time, we can include the effects of gradients and diffusion. The NMR transverse magnetizations, the diffusion coefficient obtained in terms of NMR flow parameters from the solution of the Bloch NMR flow equations as presented in the earlier studies [1-8] can play more fundamental role in the analysis of NMR parameters for functional magnetic resonance (fMRI). The development of MRI entailed using NMR pulse gradient sequences specifically to localize a population of spins in space [9]. As Lauterbur perspicaciously observed, it also created the potential for incorporating existing NMR measurements of relaxation properties within an MRI sequence: "The basic zeugmatographic principle permits the generation of two and three dimensional images displaying chemical compositions, diffusion coefficients, and other properties of objects measurable by spectroscopic techniques" [9]. Although the possibility of combining diffusion NMR and MRI was foreseen by Lauterbur in 1973 [9], it was not until 1984 that diffusion MRI was first realized. In a series of two papers, Wesbey et al. showed how one can account for the effects of diffusion gradients within an MRI sequence [7] and demonstrated diffusion MRI in different phantoms [6]. Unfortunately, this implementation of diffusion MRI did not allow for separate control of the imaging gradients used to perform spatial localization and of the diffusion gradients used to sensitize the mobile spins to be able to measure their displacements. However, in 1985, clinically useable diffusion MRI sequences were first proposed and realized [4, 5] that overcame this problem. Several excellent books and review articles have been written describing many important aspects of diffusion MRI [10, 11]. Diffusion weighted images (DWIs) are MRIs obtained by incorporating diffusion gradient pulses within a conventional MRI pulse sequence [10-14]. The intensity of a DWI is "weighted" or attenuated by the effective diffusivity of the spin-labeled species in each voxel, just as the diffusion NMR signal is weighted or attenuated by the diffusivity within the excited volume.

Mathematical Formulation

For this investigation, Based on the new NMR diffusion equations [15-23] we derive analytical expressions in Cartesian and cylindrical polar coordinates for the NMR transverse magnetization which can be detected by the recovery unit in the MRI scanner based on the Bloch NMR flow equations with the assumption that resonance condition exists at Larmor frequency $f_o = \gamma B - \omega = 0$ The x, y, z components (in the rotating frame) of the magnetization of a particle may be given by the Bloch NMR flow equations which may be written as follows [15]:

$$V^2\frac{\partial^2 M_y}{\partial x^2}+2V\frac{\partial^2 M_y}{\partial x\partial t}+\frac{\partial^2 M_y}{\partial t^2}+\left(\frac{1}{T_1}+\frac{1}{T_2}\right)V\frac{\partial M_y}{\partial x}+\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\frac{\partial M_y}{\partial t}+\left(\gamma^2 B_1^2(t)+\frac{1}{T_1T_2}\right)M_y=\frac{M_o\gamma B_1(x,t)}{T_1} \quad (1a)$$

where γ is the gyromagnetic ratio of the material M_o is the equilibrium magnetization T_1 and T_2 are the spin-lattice and spin-spin relaxation parameters respectively The solution presented here is subject to the following two reasonable initial boundary conditions which may conform to the real-time experimental arrangements; i) $M_o \neq M_z$, a situation which hold good in general and in particular when the rF $B_1(t)$ field is very strong such that M_y is maximum when $M_o = 0$. ii) Before entering signal detector coil, the soft particle has $M_x= 0$, $M_y = 0$ and. $\gamma^2 B_1^{\ 2} << 1/T_1T_2$.

Cylindrical Geometry

We can assume a solution to equation (1a) and write:

$$M_y(x,t) = Ae^{\mu x+\eta t} \tag{1b}$$

where A is constant. Equation (1a) becomes:

$$V^2\frac{\partial^2 M_y}{\partial x^2}+\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\frac{\partial M_y}{\partial t}=\frac{M_o\gamma B_1(x,t)}{T_1} \tag{1c}$$

provided that

$$\eta^2 = T_g \quad and \quad 2\eta = T_o$$

or

$$D\frac{\partial^2 M_y}{\partial x^2}=\frac{\partial M_y}{\partial t} \tag{1d}$$

where

$$D=\frac{V^2}{T_o}=V^2T_1 \tag{1e}$$

$$T_o = \left(\frac{1}{T_1}+\frac{1}{T_2}\right) \quad and \quad T_g = \frac{1}{T_1T_2}$$

If the geometry describing the immediate environment of the particle is cylindrical, the NMR diffusion equation is given as:

$$\frac{\partial M_y}{\partial t} = D\left(\frac{\partial^2 M_y}{\partial r^2} + \frac{1}{r}\frac{\partial M_y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 M_y}{\partial \phi^2} + \frac{\partial^2 M_y}{\partial z^2}\right) + \frac{F_o}{T_o}\gamma B_1(x,t) \tag{1f}$$

Equation (1) is applicable under the following conditions:

1) The selected slice must be homogeneous and made up of just one compartment (either of tissue or fluid compartment).
2) Rate of fluid influx and efflux should be approximately very small and equal (such as in some slow blood flow junctions) or static fluid (such as in edema).
3) The size of the chemical substance to be investigated must be significantly very small compared to the tissue compartment in which they are found.
4) The geometry of the compartment should be somehow defined and could be approximated to the very common geometrical system.

Based on the above conditions we write

$$\frac{\partial M_y}{\partial t} = D\left(\frac{\partial^2 M_y}{\partial r^2} + \frac{1}{r}\frac{\partial M_y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 M_y}{\partial \phi^2} + \frac{\partial^2 M_y}{\partial z^2}\right) \tag{2}$$

The method of separation of variables [18] is applied to solve equation (1f) with the following assumption that

$$M_y = F(r,\phi,z)U(t) \tag{3}$$

Substituting Equation (2) into (1) and then multiplying the results by the term $\frac{1}{FU}$, we have

$$\frac{1}{F}\left(\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \phi^2} + \frac{\partial^2 F}{\partial z^2}\right) = \frac{1}{D}\frac{1}{U}\frac{dU}{dt} \tag{4}$$

Since both sides of equation (4) are independent of each other, both sides must be equal to a constant $-k_1^2$, then we have

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \phi^2} + \frac{\partial^2 F}{\partial z^2} + k_1^2 F = 0 \tag{5a}$$

$$\frac{dU(t)}{dt} + k_1^2 DU(t) = 0 \tag{6}$$

A solution to equation (6) is

$$U(t) = Ae^{-k_1^2 Dt} \tag{7}$$

where A is the constant of integration

Equation (5) is the Helmholtz equation which can be solved if we assume that

$$F = Q(r,\phi)Z(z)$$

Hence, equation (6) becomes

$$Z\frac{\partial^2 Q}{\partial r^2} + \frac{Z}{r}\frac{\partial Q}{\partial r} + \frac{Z}{r^2}\frac{\partial^2 Q}{\partial \phi^2} + Q\frac{d^2 Z}{dz^2} + k_1^2 QZ = 0 \tag{8}$$

dividing both sides of equation (8) by QZ and re – arranging terms, we have

$$\frac{1}{Q}\left(\frac{\partial^2 Q}{\partial r^2} + \frac{1}{r}\frac{\partial Q}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Q}{\partial \phi^2}\right) = -\frac{1}{Z}\frac{d^2 Z}{dz^2} - k_1^2 \tag{9}$$

Since equation (9) has both sides independent of each other, they must individually be equal to a constant $-\lambda^2$, that is

$$\frac{1}{Q}\left(\frac{\partial^2 Q}{\partial r^2} + \frac{1}{r}\frac{\partial Q}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Q}{\partial \phi^2}\right) = -\frac{1}{Z}\frac{d^2 Z}{dz^2} - k_1^2 = -\lambda^2$$

we have

$$\frac{\partial^2 Q}{\partial r^2} + \frac{1}{r}\frac{\partial Q}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Q}{\partial \phi^2} + \lambda^2 Q = 0 \tag{10}$$

and

$$\frac{d^2 Z}{dz^2} + \left(k_1^2 - \lambda^2\right)Z = 0 \tag{11}$$

A solution to Equation (11) is obtained by making the assumption:

$$Z(z) = ae^{wz}$$

it follows that

$$Z(z) = A_1 \cos\sqrt{(k_1^2 - \lambda^2)}z + A_2 \sin\sqrt{(k_1^2 - \lambda^2)}z \tag{12}$$

where

$$(A_1 = a_1 + a_2, A_2 = i(a_1 - a_2))$$

If we write

$$Q(r,\phi) = R(r)G(\phi)$$

equation (10) becomes

$$G\frac{d^2R}{dr^2} + \frac{G}{r}\frac{dR}{dr} + \frac{R}{r^2}\frac{d^2G}{d\phi^2} + \lambda^2 RG = 0$$

Multiplying all through by $\frac{r^2}{RG}$, we have

$$\frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} + \lambda^2 r^2 = -\frac{1}{G}\frac{d^2G}{d\phi^2} \tag{13}$$

Both sides of (13) must be equal to a constant m^2. We obtain is,

$$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} + (\lambda^2 r^2 - m^2)R = 0 \tag{14}$$

and

$$\frac{d^2G}{d\phi^2} + m^2 G = 0 \tag{15}$$

Equation (14) is a Bessel's differential equation and the solution is given as

$$R(r) = C_1 J_m(\lambda r) + C_2 Y_m(\lambda r) \tag{16}$$

where $J_m(\lambda r)$ is the Bessel function of the first kind, of order m and $Y_m(\lambda r)$ is the Bessel function of the second kind, of other m. C_1 and C_2 are constants. Solution to equation (15) is obtained by making the assumption that

$$G(\phi) = be^{q\phi}$$

Since $q = \pm im$, it follows that

$$G(\phi) = b_1 e^{im\phi} + b_2 e^{-im\phi}$$