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（影印版）

Path Integrals in Field Theory
An Introduction

场论中的路径积分导引

U. Mosel

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《国外物理名著系列》(影印版)专家委员会名单

（按姓氏笔画排序）

国外物理名著(影印版)系列序言

对于国内的物理学工作者和青年学生来讲，研读国外优秀的物理学著作是系统掌握物理学知识的一个重要手段。但是，在国内并不能及时、方便地买到国外的图书，且国外图书不菲的价格往往令国内的读者却步，因此，把国外的优秀物理原著引进到国内，让国内的读者能够方便地以较低的价格购买是一项意义深远的工作，将有助于国内物理学工作者和青年学生掌握国际物理学的前沿知识，进而推动我国物理学科研和教学的发展。

为了满足国内读者对国外优秀物理学著作的需求，科学出版社启动了引进国外优秀著作的工作，出版社的这一举措得到了国内物理学界的积极响应和支持，很快成立了专家委员会，开展了选题的推荐和筛选工作，在出版社初选的书单基础上确定了第一批引进的项目，这些图书几乎涉及了近代物理学的所有领域，既有阐述学科基本理论的经典名著，也有反映某一学科专题前沿的专著。在选择图书时，专家委员会遵循了以下原则：基础理论方面的图书强调“经典”，选择了那些经得起时间检验、对物理学的发展产生重要影响、现在还不“过时”的著作（如：狄拉克的《量子力学原理》）。反映物理学某一领域进展的著作强调“前沿”和“热点”，根据国内物理学研究发展的实际情况，选择了能够体现相关学科最新进展，对有关方向的科研人员和研究生有重要参考价值的图书。这些图书都是最新版的，多数图书都是 2000 年以后出版的，还有相当一部分是 2006 年出版的新书。因此，这套丛书具有权威性、前瞻性和应用性强的特点。由于国外出版社的要求，科学出版社对部分图书进行了少量的翻译和注释（主要是目录标题和练习题），但这并不会影响图书“原汁原味”的感觉，可能还会方便国内读者的阅读和理解。

“他山之石，可以攻玉”，希望这套丛书的出版能够为国内物理学工作者和青年学生的工作和学习提供参考，也希望国内更多专家参与到这一工作中来，推荐更多的好书。

杨国桢

中国科学院院士

中国物理学会理事长

2007 年 3 月 20 日

To all my students

Preface

This is an introductory book to path integral methods in field theories. It is aimed at graduate students and physicists who need a working knowledge of field theory and its methods for applications in hadron, particle and nuclear physics. While teaching field theory courses over the years I have found that many books on field theory present the path integral methods used in only a very recipe-like way. On the other hand, specialized books on path integrals often contain many more details than are actually needed by non-specialists. I hope that this book here fills the gap. It provides enough information to actually follow all the arguments necessary for field theoretical developments without, however, elaborating on the method as such and its mathematical intricacies. This book is – in a way – a technical companion to *Fields, Symmetries, and Quarks* by the present author.

The reader of this book should have some knowledge of the relativistic equations of motion of 'classical' quantum theory, but no prior knowledge of field theory is assumed. The material in this book can be covered in a one-semester course with 3 hrs/week. It has evolved in many years of teaching this subject. I am grateful to my students for many helpful questions and comments and, in particular, to Frank Froemel for his help in preparing the figures in this book.

Giessen
June 2003

Ulrich Mosel

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Part I

Non-Relativistic Quantum Theory

1 The Path Integral in Quantum Theory

In this starting chapter we introduce the concepts of propagators and path integrals in the framework of nonrelativistic quantum theory. In all these discussions, and the following chapters on nonrelativistic quantum theory, we work with one coordinate only, but all the results can be easily generalized to the case of d dimensions.

1.1 Propagator of the Schrödinger Equation

We start by considering a nonrelativistic particle in a one-dimensional potential $V(x)$. The Schrödinger equation reads

$$H\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = \mathrm{i}\hbar\frac{\partial\psi(x,t)}{\partial t} \ . \tag{1.1}$$

This equation allows us to calculate the wavefunction $\psi(x,t)$ at a later time, if we know $\psi(x,t_0)$ at the earlier time $t_0 < t$. For further calculations we rewrite this equation into the following form

$$\left(\mathrm{i}\hbar\frac{\partial}{\partial t} - H\right)\psi(x,t) = 0 \ . \tag{1.2}$$

Next, we consider the function $K(x,t;x_i,t_i)$ which is defined as a solution of the equation

$$\left(\mathrm{i}\hbar\frac{\partial}{\partial t} - H\right)K(x,t;x_i,t_i) = \mathrm{i}\hbar\delta(x-x_i)\delta(t-t_i) \ . \tag{1.3}$$

K is the "Green's function" of the Schrödinger equation (K is also often called the "propagator") with the initial condition

$$K(x,t_i+0;x_i,t_i) = \delta(x-x_i) \ . \tag{1.4}$$

The solution of the Schrödinger equation (1.2) can be written as

$$\psi(x,t) = \int K(x,t;x_i,t_i)\,\psi(x_i,t_i)\,\mathrm{d}x_i \tag{1.5}$$

for $t > t_i$ (Huygen's principle). Relation (1.5) can be proven by inserting the lhs into the Schrödinger equation

$$\begin{aligned}\left(\mathrm{i}\hbar\frac{\partial}{\partial t} - H\right) \int K\left(x,t;x_i,t_i\right)\psi(x_i,t_i)\,\mathrm{d}x_i \\ = \mathrm{i}\hbar\int \delta\left(t-t_i\right)\delta\left(x-x_i\right)\psi(x_i,t_i)\,\mathrm{d}x_i \\ = \mathrm{i}\hbar\delta\left(t-t_i\right)\psi(x,t_i) = 0 \qquad \text{for} \quad t > t_i\ . \end{aligned} \tag{1.6}$$

Thus the ψ defined by (1.5) is indeed a solution of the Schrödinger equation for all times $t > t_i$. $K\left(x,t;x_i,t_i\right)$ is the probability amplitude for a transition from x_i, at time t_i, to the position x, at the later time t. The restriction to later times preserves causality.

We can find an explicit form for the propagator, if the solutions of the stationary Schrödinger equation, $\varphi_n(x)$, and the corresponding eigenvalues, E_n, are known. Since the φ_n form a complete system, K can certainly be expanded in this basis (for $t \geq t_i$)

$$K\left(x,t;x_i,t_i\right) = \sum_n a_n\varphi_n(x)\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}E_n t}\Theta\left(t-t_i\right)\ . \tag{1.7}$$

Here the stepfunction $\Theta(t) = 0$ for $t < 0$ and $\Theta(t) = 1$ for $t \geq 0$ takes explicitly into account that we only propagate the wavefunction forward in time. The expansion coefficients obviously depend on x_i, t_i

$$a_n = a_n(x_i,t_i)\ . \tag{1.8}$$

Because of the initial condition $K(x,t_i+0;x_i,t_i) = \delta\left(x-x_i\right)$ we have

$$\delta\left(x-x_i\right) = \sum_n a_n(x_i,t_i)\varphi_n(x)\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}E_n t_i}\ . \tag{1.9}$$

The lhs is time-independent; thus we must have

$$a_n\left(x_i,t_i\right) = a_n(x_i)\mathrm{e}^{+\frac{\mathrm{i}}{\hbar}E_n t_i}\ , \tag{1.10}$$

and consequently

$$\delta\left(x-x_i\right) = \sum_n a_n\left(x_i\right)\varphi_n(x)\ . \tag{1.11}$$

This can be fulfilled by

$$a_n\left(x_i\right) = \varphi_n^*\left(x_i\right) \tag{1.12}$$

(closure relation). Thus we have a representation of $K\left(x,t;x_i,t_i\right)$ in terms of the eigenfunctions and eigenvalues of the underlying Hamiltonian

$$K\left(x,t;x_i,t_i\right) = \Theta(t-t_i)\sum_n \varphi_n^*\left(x_i\right)\varphi_n(x)\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}E_n(t-t_i)}\ . \tag{1.13}$$

It is easy to show that this propagator fulfills (1.3).

In Dirac's bra and ket notation this result can also be written as (for $t > t_i$)

$$\begin{aligned} K(x,t;x_i,t_i) &= \sum_n \varphi_n^*(x_i)\,\varphi_n(x)\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}E_n(t-t_i)} \\ &= \sum_n \langle n|x_i\rangle \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}E_n(t-t_i)}\langle x|n\rangle \\ &= \sum_n \langle n|\mathrm{e}^{+\frac{\mathrm{i}}{\hbar}\hat{H}t_i}|x_i\rangle\langle x|\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}t}|n\rangle \\ &= \langle x|\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}(t-t_i)}|x_i\rangle \;\equiv\; \langle x|\hat{U}(t,t_i)\,|x_i\rangle\;. \end{aligned} \tag{1.14}$$

Thus the propagator is nothing other than the time development operator

$$\hat{U}(t,t_i) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}(t-t_i)} \tag{1.15}$$

for $t > t_i$ in the x representation. It is also often written as

$$K(x,t;x_i,t_i) = \langle x|\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}(t-t_i)}|x_i\rangle \equiv \langle xt|x_it_i\rangle \tag{1.16}$$

for $t > t_i$; for $t < t_i$ it vanishes.

The notation here is that of the Heisenberg representation of quantum mechanics. In this representation the physical state vectors are time-independent and the operators themselves carry all the time-dependence, whereas this is just the opposite for the Schrödinger representation. For example, for the position operator $\hat{x}$ in the Schrödinger representation with $\hat{x}|x\rangle = x|x\rangle$ we obtain the time-dependent operator in the Heisenberg representation

$$\hat{x}_{\mathrm{H}}(t) = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\hat{H}t}\,\hat{x}\,\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}t} \tag{1.17}$$

and

$$\hat{x}_{\mathrm{H}}(t)|xt\rangle = x|xt\rangle \tag{1.18}$$

with

$$|xt\rangle = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\hat{H}t}|x\rangle\;. \tag{1.19}$$

The state $|xt\rangle$ is thus the eigenstate of the operator $\hat{x}_{\mathrm{H}}(t)$ with eigenvalue x and not the state that evolves with time out of $|x\rangle$; this explains the sign of the frequency in the exponent.

1.2 Propagator as Path Integral

We start by dividing the time-interval between t_i and t by inserting the time t_1. The wavefunction is first propagated until t_1 and then, in a second step, until t

$$\psi(x_1, t_1) = \int K(x_1, t_1; x_i, t_i)\, \psi(x_i, t_i)\, \mathrm{d}x_i \tag{1.20}$$
$$\psi(x, t) = \int K(x, t; x_1, t_1) \psi(x_1, t_1)\, \mathrm{d}x_1 \, .$$

Taking these two equations together we get

$$\psi(x, t) = \int \int K(x, t; x_1, t_1)\, K(x_1, t_1; x_i, t_i)\, \psi(x_i, t_i)\, \mathrm{d}x_i\, \mathrm{d}x_1 \, . \tag{1.21}$$

Comparing this result with (1.5) yields

$$K(x, t; x_i, t_i) = \int K(x, t; x_1, t_1)\, K(x_1, t_1; x_i, t_i)\, \mathrm{d}x_1 \, . \tag{1.22}$$

We can thus view the transition from (x_i, t_i) to (x, t) as the result of a transition first from (x, t) to all possible intermediate points (x_1, t_1), which is then followed by a transition from these intermediate points to the endpoint. We could also say that the integration in (1.22) is performed over all possible paths between the points (x_i, t_i) and (x, t), which consist of two straight line segments with a bend at t_1. This is illustrated in Fig. 1.1.

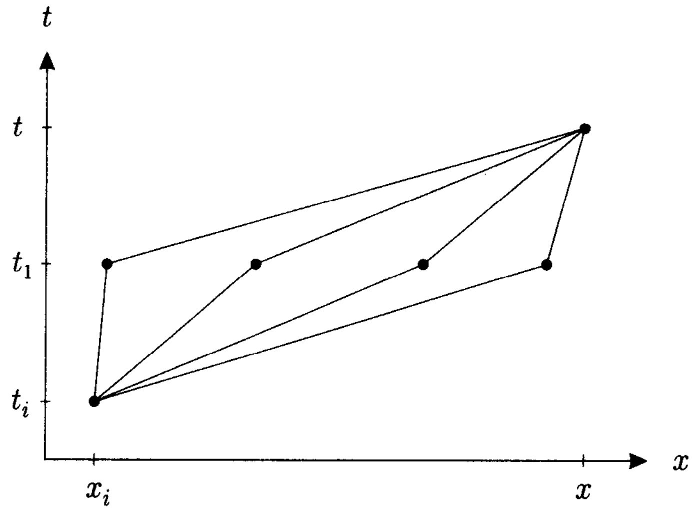

Fig. 1.1. Possible paths from x_i to x, corresponding to (1.22)

We now subdivide the time interval further into $(n + 1)$ equal parts of length $\Delta t = \eta$. We then have in direct generalization of the previous result

$$K(x, t; x_i, t_i) = \int \dots \int \mathrm{d}x_1\, \mathrm{d}x_2 \, \dots \, \mathrm{d}x_n \tag{1.23}$$
$$\times \left[K(x, t; x_n, t_n)\, K(x_n, t_n; x_{n-1}, t_{n-1}) \dots K(x_1, t_1; x_i, t_i) \right] \, .$$

The integrals run here over all possible paths between (x_i, t_i) and (x, t) which consist of $(n+1)$ segments with boundaries that are determined by the time steps $t_i, t_1, \ldots, t_n, t$.

We now calculate the propagator for a small time interval $\Delta t = \eta$ from t_j to t_{j+1}. For this propagation we have according to (1.16)

$$\begin{aligned} K(x_{j+1}, t_{j+1}; x_j, t_j) &= \langle x_{j+1}|\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}\eta}|x_j\rangle \\ &\cong \langle x_{j+1}|1 - \frac{\mathrm{i}}{\hbar}\hat{H}\eta|x_j\rangle \\ &= \delta(x_{j+1} - x_j) - \frac{\mathrm{i}}{\hbar}\eta\langle x_{j+1}|\hat{H}|x_j\rangle \\ &= \frac{1}{2\pi\hbar}\int \mathrm{e}^{\frac{\mathrm{i}}{\hbar}p(x_{j+1}-x_j)}\,\mathrm{d}p - \frac{\mathrm{i}\eta}{\hbar}\langle x_{j+1}|\hat{H}|x_j\rangle \end{aligned} \tag{1.24}$$

with the representation for the δ-function

$$\delta(x - x') = \frac{1}{2\pi}\int \mathrm{e}^{\mathrm{i}k(x-x')}\mathrm{d}k \ . \tag{1.25}$$

We now assume that $\hat{H}$ is given by

$$\hat{H} = \hat{T}(\hat{p}) + \hat{V}(\hat{x}) \ . \tag{1.26}$$

Here $\hat{T}, \hat{p}, \hat{V}, \hat{x}$ are all operators; we assume that $T(\hat{p})$ and $V(\hat{x})$ are Taylor-expandable. In this case, where the p- and x-dependencies separate, we can also bring the last term in (1.24) into an integral form. We have

$$\langle x_{j+1}|\hat{H}|x_j\rangle = \langle x_{j+1}|\hat{T} + \hat{V}|x_j\rangle \ . \tag{1.27}$$

First, we consider the first summand

$$\begin{aligned} \langle x_{j+1}|\hat{T}|x_j\rangle &= \int \mathrm{d}p'\,\mathrm{d}p\langle x_{j+1}|p'\rangle\langle p'|\hat{T}(\hat{p})|p\rangle\langle p|x_j\rangle \\ &= \int \mathrm{d}p'\,\mathrm{d}p\langle x_{j+1}|p'\rangle\delta(p' - p)T(p)\langle p|x_j\rangle \\ &= \int \mathrm{d}p\langle x_{j+1}|p\rangle T(p)\langle p|x_j\rangle \ . \end{aligned} \tag{1.28}$$

With the normalized momentum eigenfunctions

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}\mathrm{e}^{\frac{\mathrm{i}}{\hbar}px} \tag{1.29}$$

we thus obtain

$$\langle x_{j+1}|\hat{T}(\hat{p})|x_j\rangle = \frac{1}{2\pi\hbar}\int \mathrm{e}^{\frac{\mathrm{i}}{\hbar}p(x_{j+1}-x_j)}T(p)\,\mathrm{d}p \ . \tag{1.30}$$