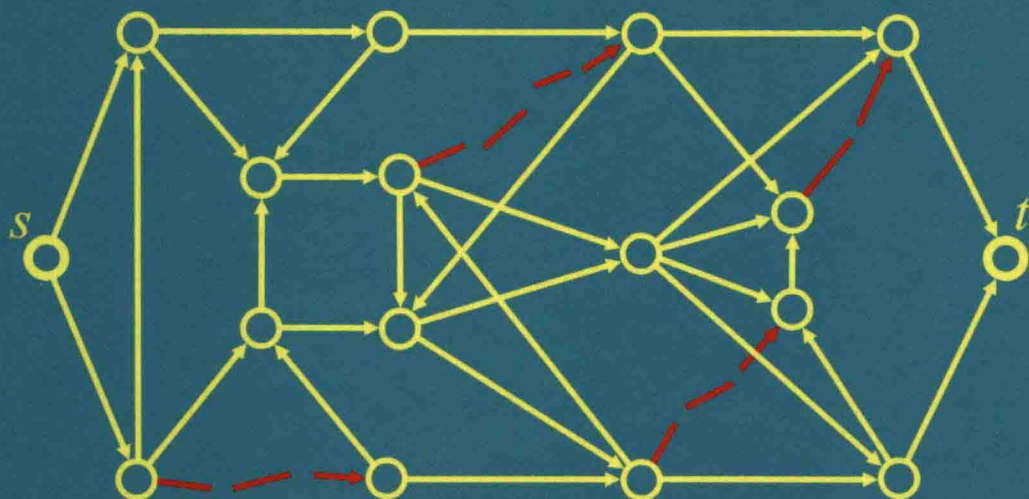




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FLOW NETWORKS

ANALYSIS AND OPTIMIZATION OF REPAIRABLE
FLOW NETWORKS, NETWORKS WITH DISTURBED FLOWS,
STATIC FLOW NETWORKS AND RELIABILITY NETWORKS

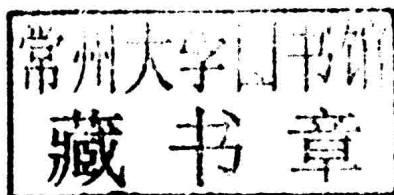
MICHAEL T. TODINOV

Flow Networks

Analysis and Optimization of Repairable Flow Networks, Networks with Disturbed Flows, Static Flow Networks and Reliability Networks

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Elsevier
32 Jamestown Road, London NW1 7BY
225 Wyman Street, Waltham, MA 02451, USA

First edition 2013

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British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

ISBN: 978-0-12-398396-1

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Flow Networks

To Prolet, Marina and Marin

Preface

A lot has been written about static flow networks, where repairs and flow disruption caused by failures of components are not considered. Static flow networks have numerous applications and currently they are an inextricable part of the modern education in computer algorithms and operational research. However, despite the years of intensive research on static flow networks, a common fundamental flaw of the classical methods for maximising the throughput flow has been identified. *The classical algorithms for maximising the throughput flow leave undesirable directed loops of flow in the optimised networks.* These parasitic flow loops are associated with wastage of energy and resources and increased levels of congestion in the optimised networks and are associated with big financial losses for the affected sectors of the economy.

Furthermore, an important aspect of real flow networks is that their flows are often disturbed by particular contingency events, for example, overloading, congestion, failures and sudden fluctuations in flow generation and demand. Indeed, in many real flow networks (electrical, production, transportation, manufacturing and computer networks, etc.) components fail or sections of the network suffer congestion or overloading. These events disturb the network flows and in this sense, it can be stated that almost all real networks are, in fact, networks with disturbed flows.

After a contingency event, for example, a congestion or failure, it is important to redirect the flows immediately through alternative paths, so that a new maximum of the throughput flow is quickly reached.

At the same time, on a failure of a component from a production line, computer network, power supply system or water supply system, a repair is initiated and after a particular delay, the failed component is returned to operation. Similarly, after an accident on a road section, a clearing operation is initiated and after a certain delay, the road section is returned to operation at its full capacity. Consequently, it can be stated that real flow networks are almost always repairable networks. The research on networks with disturbed flows and repairable flow networks is an important emerging research area, part of the modern network science.

The potential application of repairable flow networks is huge: oil and gas production networks, computer networks, power networks, telecommunication networks, transportation networks, water supply networks, emergency evacuation networks, supply networks, etc. Repairable flow networks can even be used for conducting reliability analysis of large and complex systems.

An important problem associated with repairable flow networks delivering a particular commodity (gas, oil, electrical power, data, goods, etc.) is the adverse effect of component failures and unavailability of generation sources on the network performance. The extent of this adverse effect is measured by the probability of

having a particular level of the throughput flow on demand or by the probability of having a particular expected level of total throughput flow, during a specified time interval. These performance measures are commonly specified in contracts and are a key for evaluating the performance and quality of service of flow networks. They are used for comparing alternative solutions and for making an informed selection among competing network topologies. This is the main reason why oil and gas production companies, for example, have already invested in software for simulating the performance of their repairable oil and gas production systems and determining their production availability. There is also an emerging demand for such software tools from the telecom sector, power distribution sector and distribution logistics sector.

As a result, failures, repairs, fluctuating supply and demand and the flow reoptimisation after these events are an essential part of the network analysis, optimisation and real-time management. This important aspect of the network flow analysis has not yet received the attention it deserves.

Although the analysis and optimisation of networks with disturbed flows and repairable flow networks is extremely important, currently no books are available to provide the much-needed support for researchers and practitioners. By introducing the subject 'networks with disturbed flows and repairable flow networks', this book takes the flow networks a step further. The book has been written with the intent to fill the existing gaps, by developing the theory, algorithms and applications related to networks with disturbed flows and repairable flow networks. The research area *Networks with disturbed flows and repairable flow networks* is new and this is the first book covering the subject.

The theoretical results presented in this book form the foundations of a new generation of ultra-fast algorithms for optimising networks with flows disturbed by failures, congestion or sudden change in demand or flow generation. The high computational speed creates the possibility of optimal control of very large and complex networks in real time. This is of particular importance to large and complex power distribution networks, where the optimisation of the power flows after overloading or failure of a power line needs to be done within the range of milliseconds. Reoptimising the network flows in real time significantly increases the yield from real production networks and reduces to a minimum the lost flow and disruption caused by failures.

The initial chapters introduce basic concepts, conventions and techniques related to static flow networks, where no contingency events disturbing the edge flows exist. The concepts, results and techniques discussed are necessary for introducing the theory of repairable flow networks and networks with disturbed flows.

In order to correct the common flaw of existing classical methods for maximising the throughput flow, an efficient algorithm for identifying and removing directed loops of flow has been developed.

New efficient algorithms for maximising the flow in static, single commodity and multi-commodity networks have also been proposed. In this respect, a new fundamental theorem referred to as '*dual network theorem for static networks*' has been stated and proved. The theorem states that the maximum throughput flow in

any static network is equal to the sum of capacities of the edges coming out of the source minus the total excess flow at all excess nodes plus the maximum throughput flow in the dual network. Consequently, a new algorithm for maximising the throughput flow in a network has been proposed. For networks with few imbalanced nodes, the proposed algorithm outperforms all classical algorithms for maximising the throughput flow in a network. An additional stage in the proposed new algorithm guarantees that no parasitic directed loops of flow are left after the throughput flow maximisation.

It is also shown that the computational speed related to determining the maximum throughput flow in a network with merging flows can be improved enormously if the tree topology of the network is exploited directly. Accordingly, for networks with merging flows, an efficient algorithm with linear running time in the size of the network has been proposed for maximising the throughput flow. A theorem which provides the theoretical justification for the proposed algorithm has also been stated and proved.

Chapter 5 introduces the theory of networks with disturbed flows and discusses several fundamental results referred to as 'dual network theorems for networks with disturbed flows'. One of the results states that *in any network with maximised throughput flow, the new maximum throughput flow after choking the flows along several edges is equal to the maximum throughput flow in the original network, minus the total amount of excess flow at the excess nodes, plus the maximum throughput flow in the dual network.*

On the basis of this result, very efficient augmentation algorithms have been proposed for restoring the maximum possible throughput flow in a network with disturbed flows after an edge failure. The proposed algorithms are the fastest available methods for reoptimising the throughput flow after edge failures. In many cases, the average running time of the proposed algorithms is constant, independent of the size of the network or varies linearly with the size of the network. It is shown how the developed algorithms can be applied for optimising the performance of gas production networks.

The high computational speed of the proposed reoptimisation algorithm makes it suitable for optimising the performance of large and complex repairable flow networks in real time. Essentially, the proposed algorithm is a very efficient decentralised algorithm for achieving a global maximum of the throughput flow in a network by independent distributed agents, which possess local knowledge about the network topology but do not necessarily possess knowledge about the entire network topology. Consequently, a very important application of the proposed algorithm has been found in the high-speed control of active power distribution networks after a failure or congestion of power lines or after a sudden change in demand and power generation.

In Chapter 9, an important result has been stated regarding the average production availability of repairable flow networks composed of independently working edges, whose times to failure follow the negative exponential distribution. The average production availability is the ratio of (i) the average of the maximum throughput flow on demand calculated after removing the separate edges with probabilities equal to their unavailabilities, to (ii) the maximum throughput flow in the

absence of failures. For the first time, the new algorithm for determining the production availability, created the basis of extremely fast solvers for the production availability of large and complex repairable networks, whose running time is independent of the length of the operational interval, the failure frequencies of the edges or the lengths of their repair times.

A discrete-event solver for determining the production availability of repairable flow networks with complex topology has also been constructed, where the times to failure of the edges could follow any specified distribution. The proposed discrete-event solver maximises the throughput flow rate in the network upon each component failure and return from repair. Maximising the flow rate upon each component failure and return from repair, ensures a larger total throughput flow during a specified time interval.

Methods for assessing the reliability of the throughput flow in a network with complex topology occupy a central space in the book. These methods use Monte Carlo simulation techniques to estimate the probability that the throughput flow will be equal to or greater than a specified threshold value and are superior to alternative methods based on minimal paths and cut sets. They can be applied for assessing the quality of service of computer networks and telecommunication networks. Analytical methods for determining the probability of a source-to-sink flow on demand, have also been introduced and discussed.

By using a specially designed software tool, a study has been presented on the link between performance, topology and size of repairable flow networks. The topology of repairable flow networks has a significant impact on their performance. Two networks built with identical type and number of components can have very different performance levels because of slight differences in their topology.

In Chapter 11, a new topology optimisation algorithm has been proposed for achieving a maximum throughput flow and a maximum production availability within a specified budget for building the network. The algorithm incorporates an efficient search in the space of available alternatives, based on combining the branch and bound method and pruning of the full-complexity network. The algorithm always determines the exact solution and is considerably faster than exact algorithms based on a full exhaustive search. At the heart of the optimisation procedure is a production availability algorithm, whose running time is independent of the length of the operational interval, the failure frequencies of the components, or the lengths of their repair times.

The topology optimisation method has also been applied to reliability networks of safety-critical systems. An important problem has been solved, related to how to build within a fixed budget a safety-critical system, characterised by the smallest risk of failure. The topology optimisation of reliability networks creates the opportunity of increasing the safety margin of common safety-critical systems without increasing the cost of current designs.

Substantial space in the book has been allocated on flow optimisation in non-reconfigurable repairable flow networks. A number of theorems related to non-reconfigurable repairable flow networks have been stated and proved. For a specified source-to-sink path, the difference between the sum of the unavailabilities along its

forward edges and the sum of the unavailabilities along its backward edges has been referred to as *path resistance*. This property plays an important role in modelling the properties of non-reconfigurable flow networks. In a non-reconfigurable flow network, the absence of augmentable cyclic paths with a negative resistance is a necessary and sufficient condition for a minimum lost flow due to edge failures.

For a given source-to-sink path, the difference between the sum of the hazard rates of its forward empty edges and the sum of the hazard rates of its backward empty edges is the flow disruption number of the path. In non-reconfigurable networks, the absence of augmentable cyclic paths with a negative flow disruption number is a necessary and sufficient condition for a minimum probability of undisturbed throughput flow by edge failures.

Reliability networks, their design and analysis also received attention. It is shown that a reliability network can be interpreted as a repairable flow network. The probability of system operation is equal to the probability that, on demand, a path with unit flow can be augmented from the source to each of the end nodes. This analogy permits reliability networks to be analysed with the tools developed for repairable flow networks.

Unlike the repairable flow networks, the reliability network does not necessarily match the functional diagram of the modelled system. This is the reason why alongside the analysis of reliability networks an extended discussion has been provided on building reliability networks. It is demonstrated that contrary to a widespread belief, complex reliability networks that cannot be represented with series and parallel arrangements are common. Even simple mechanical systems may have reliability networks that cannot be reduced to a combination of series and parallel arrangements.

It is also shown that the traditional presentation of reliability networks, based on a single start node, single end node, undirected edges and edges with two end points, is insufficient for a correct representation of the logic of operation and failure of some systems. Undirected and directed edges, multiple end nodes and 'edges' with multiple end points are all necessary to represent correctly the logic of operation and failure of these systems.

A powerful algorithm for analysis of reliability networks, which avoids the drawbacks of commonly accepted methods based on cut sets or path sets, has also been introduced.

Finally, a method has been proposed for virtual accelerated testing of complex repairable networks. As a result, the life of a complex repairable flow network under normal operating conditions can be extrapolated from the accelerated life models of its edges and nodes, at elevated levels of the acceleration stresses (temperature, humidity, pressure, vibrations, speed, concentration, corrosion activity, etc.). The proposed method makes building test rigs for complex flow networks unnecessary, which can be an expensive and a very difficult task. It also reduces drastically the amount of time and resources needed for accelerated life testing of complex flow networks.

Building a model of a complex repairable network based on the accelerated life models of its components also reveals the impact of the acceleration stresses on the availability of the network.

The algorithms in the book have been embedded in a software tool with graphics user interface through which the network is drawn on screen by the user and the parameters characterising the components are specified. Functions have been provided for quickly transferring parameters from one edge/node to another. Nodes and edges can be easily deleted and added, which permits easy modifications of an existing network. This is particularly useful for comparing quickly the performance of derivative network topologies and selecting the topology with the best performance.

In conclusion, I gratefully acknowledge the financial support received from The Leverhulme Trust, with the research grant F/00 382/J 'High-speed algorithms for the output flow in repairable flow networks'. I also acknowledge the help of Mr Paul Hansford and Mr Calvin Earp in developing the graphics user interface of the software tool; the helpful discussions with clients from oil and gas companies, the helpful comments from colleagues in the Department of Mechanical Engineering and Mathematical Sciences at Oxford Brookes University and from overseas colleagues.

I acknowledge the editing and production staff at Elsevier for their excellent work and in particular, the help of Ms Tracey Miller, Dr Erin Hill-Parks and Mr Stalin Viswanathan.

Finally, I acknowledge the immense help and support from my wife Prolet, during the preparation of this book.

I hope that the findings, algorithms and examples presented in this book will provide key knowledge, useful techniques and tools to mathematicians, computer scientists, engineers, and operators of power networks, computer networks and production networks. The book has already been used with success as a basis of the module 'Repairable flow networks and networks with disturbed flows' taught by me to final year students in mathematics at Oxford Brookes University, the United Kingdom. The chapter related to building and analysing reliability networks constitutes an essential part of the module 'Engineering Reliability and Risk Management' taught by me to postgraduate students in Oxford Brookes University. I also believe that the book will strongly complement the education in computer algorithms, operational research methods and network science.

Oxford, September 2012

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