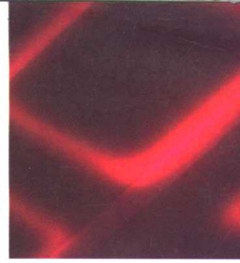
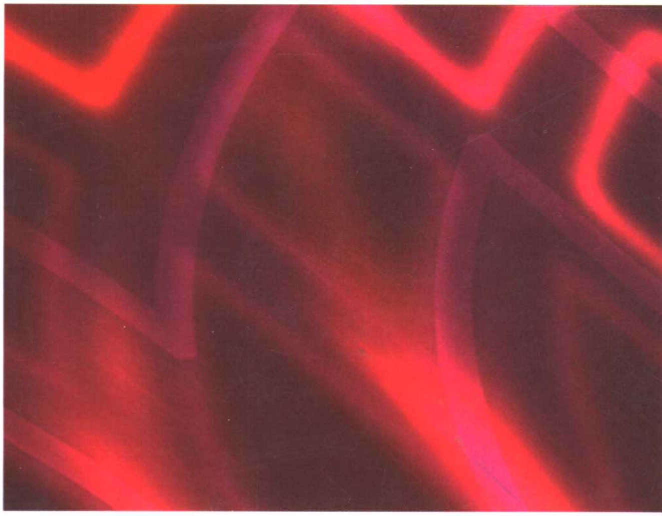
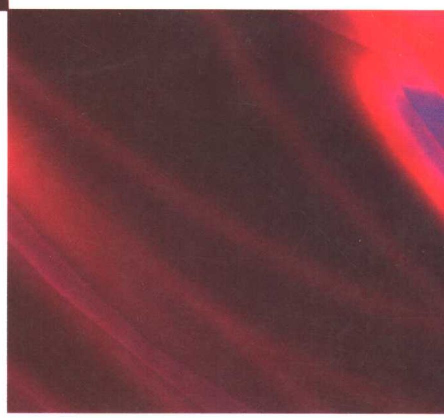
Signals and Systems

信号与系统

（影印版）







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Signals and Systems

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Preface

Analysing and designing systems with the help of suitable mathematical tools is extraordinarily important for engineers. Accordingly, systems theory is a part of the core curriculum of modern electrical engineering and serves as the foundation of a large number of subdisciplines. Indeed, access to specialised areas of electrical engineering demands a mastery of systems theory.

An introduction to systems theory logically begins with the simplest abstraction: linear, time-invariant systems. We find applications of such systems everywhere, and their theory has attained advanced maturity and elegance. For students who are confronted with the theory of linear, time-invariant systems for the first time, the subject unfortunately can prove difficult, and, if the required and deserved academic progress does not materialise, the subject might be downright unpopular. This could be due to the abstract nature of the subject area coupled with the deductive and unclear presentation in some lectures. However, since failure to learn the fundamentals of systems theory would have catastrophic repercussions for many subsequent subjects, the student must persevere.

We have written this book as an easily accessible introduction to systems theory for students of electrical engineering. The content itself is nothing new; the theory has already been described in other books. What is new is how we deliver the material. By means of small, clear explanatory steps, we aim to present the abstract concepts and interconnections of systems theory so simply as to make learning easy and fun. Naturally, only the reader can assess whether we have achieved our goal.

To aid understanding, we generally use an inductive approach, starting with an example and then generalising from it. Additional examples then illustrate further aspects of an idea. Wherever a picture or a figure can enrich the text, we provide one. Furthermore, as the text progresses, we continuously order the statements of systems theory in their overall context. Accordingly, in this book a discussion of the importance of a mathematical formula or a theorem takes precedence over its proof. While we might omit the derivation of an equation, we never neglect a discussion of its applications and consequences! The numerous exercises at the end of each chapter (with detailed solutions in the appendix) help to reinforce the reader's knowledge.

Although we have written this book primarily for students, we are convinced that it will also be useful for practitioners. An engineer who wants to brush up quickly on some subject will appreciate the easy readability of this text, its practice-oriented presentation, and its many examples.

This book evolved out of a course on systems theory and the corresponding laboratory exercises at the Friedrich Alexander University in Erlangen-Nürnberg. The course is compulsory for students of electrical engineering in the fifth semester. As such, the material in this book can be worked through completely in about 50 hours of lectures and 25 hours of exercises. We do assume knowledge of the fundamentals of engineering mathematics (differential and integral calculus, linear algebra) and basic knowledge of electrical circuits. Assuming that this mathematical knowledge has been acquired earlier, the material is also suitable for use in the third or fourth semester. An engineering curriculum often encompasses complex function theory and probability theory as well; although these fields are helpful, we do not assume familiarity with them.

This book is also suitable for self-study. Assuming full-time, concentrated work, the material can be covered in four to six weeks.

Our presentation begins with continuous signals and systems. Contrary to some other books that first introduce detailed forms of description for signals and only much later add systems, we treat signals and systems in parallel. The purpose of describing signals by means of their Laplace or Fourier transformations becomes evident only through the characteristics of linear, time-invariant systems. In our presentation we emphasise the clear concept of Eigen functions, whose form is not changed by systems. To take into account initial states, we use state space descriptions, which elegantly allow us to couple an external and an internal component of the system response. After covering sampling, we introduce time-discrete signals and systems and so extend the concepts familiar from the continuous case. Thereafter discrete and continuous signals and systems are treated together. Finally, we discuss random signals, which are very important today.

To avoid the arduous and seldom perfect step of correcting camera-ready copy, we handled the layout of the book ourselves at the university. All formulas and most of the figures were typeset in LaTeX and then transferred onto overhead slides that were used for two years in the systems theory lectures. We are most grateful to some 200 registered students whose attentive and astute criticism helped us to debug the presentation and the typeset equations. In addition, one year's students read the first version of the manuscript and suggested diverse improvements. Finally numerous readers of the German version reported typographic errors and sent comments by e-mail.

Our student assistants Lutz and Alexander Lampe, Stephan Gödde, Marion Schabert, Stefan von der Mark and Hubert Rubenbauer demonstrated tremendous commitment in typesetting and correcting the book as well as the solutions to the exercises. We thank Ingrid Bärtsch, who typed and corrected a large portion of the text, as well as Susi Koschny, who produced many figures.

For their attentive and tireless proof-reading, we especially thank Peter Eisert, Achim Hummel, Wolfgang Sörgel, Gerhard Runze and Reinhard Bernstein. For their generous availability for discussions about tricky mathematical questions, we sincerely thank Peter Steffen and Ulrich Forster. Edward Kimber has mastered the ambitious task of translating the German manuscript into English. Finally, we express our gratitude to John Wiley & Sons for their uncomplicated co-operation and their support of this project.

When the second edition of this book appears, we would like to extend our list of acknowledgements. Therefore we have the following request to our readers. Please send us your comments and suggestions. The simplest route is per e-mail to `stbuch@LNT.de`. Whatever error you might detect and however small it may be, please do not keep it to yourself. We promise that we will take to heart all serious comments.

Erlangen, Germany, October 2000

Bernd Girod Rudolf Rabenstein Alexander Stenger

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1 Introduction

Systems theory concerns signals and systems. What are signals? What are systems? Before defining these terms, let us first examine some examples.

1.1 Signals

Signals describe quantities that change. Figure 1.1 depicts the electrical voltage that a microphone produces in response to the spoken word 'car'. This voltage corresponds largely to the acoustic pressure on our ear, which reacts to the changes in this pressure over time. The curve in Figure 1.1 shows the value of microphone voltage in relation to time. Since there is a voltage value for every point in time, we term this a *continuous-time* signal. We call time the *independent* variable and the voltage changing over time the dependent variable or signal amplitude. We usually represent the independent variable horizontally (x-axis) and the dependent variable vertically (y-axis).

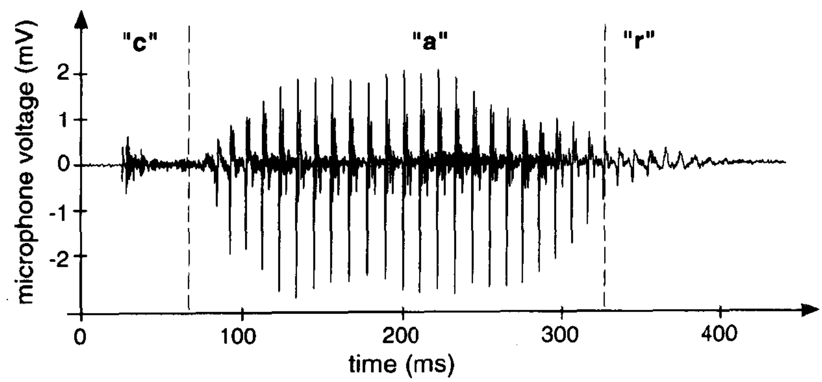

Figure 1.1: Example of a continuous-time signal: voice signal for the syllable 'car'

Figure 1.2 depicts another continuous signal. The diagram shows the temperature curves for a house wall, not over time, but in relation to the location. The curves show the temperature profile inside a 15 cm thick brick wall where the air temperature at the right side suddenly rose by 10 K. One hour later the local temperature follows the curve represented by the thick line. At another time we would have a different temperature curve. In contrast to Figure 1.1, time here is a parameter of a family of curves; the independent continuous variable is the location in the wall.

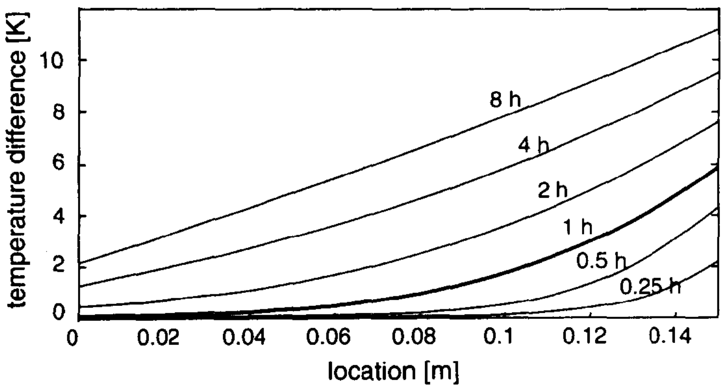

Figure 1.2: Temperature curve for a house wall

Figure 1.3 shows another kind of variable quantity, the stock market index over time. Although this index changes all the while the stock market is open, the diagram shows only the weekly average. Thus the depicted value does not change continuously, but only once a week. When the signal amplitude occurs only at certain fixed points in time (discrete times), but not for points in between, we call the signal *discrete* or, more precisely, *discrete-time*. In our example, however, the signal amplitude itself is not discrete but continuous.

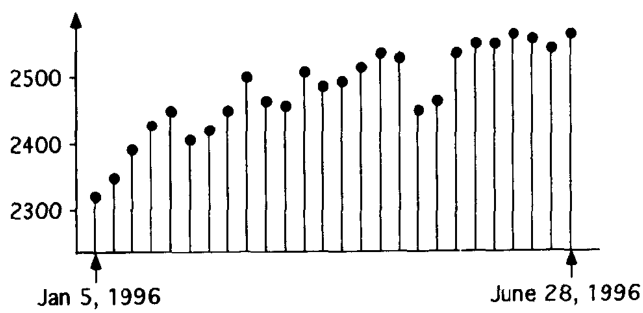

Figure 1.3: The weekly German stock market index between January 5, 1996, and June 28, 1996

In Figure 1.4 we have entered the frequency of earned marks for a test in system theory at the University of Erlangen–Nürnberg in April, 1996. The individual marks assume only discrete values (1.0 - 5.0); the frequencies (in contrast to the average stock index) are whole numbers and so likewise discrete. In this case both the independent and the dependent variables are discrete.

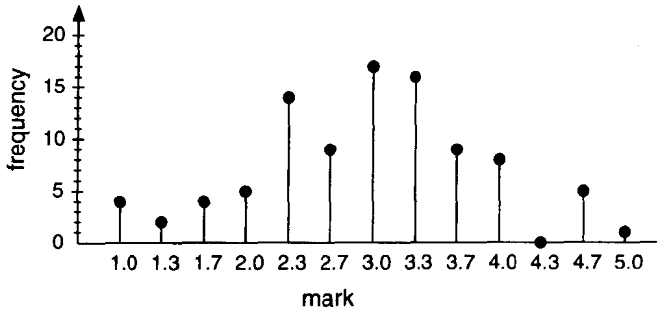

Figure 1.4: Frequency of earned marks for a test in systems theory

The signals we have considered thus far have been quantities that depend on a *single* independent variable. However, there are quantities with dependencies on two or more variables. The greyscales of Figure 1.5 depend on both the x and the y co-ordinates. Here both axes represent independent variables. The dependent variable $s(x, y)$ is entered along one axis, but is a greyscale value between the extreme values black and white.

When we add motion to pictures, we have a dependency on three independent variables (Figure 1.6): two co-ordinates and time. We call these two- or three-dimensional (or generally multidimensional) signals. When greyscale values change continuously over space or over space and time, these are continuous signals.

All our examples have shown parameters (voltage, temperature, stock index, frequencies, greyscale) that change in relation to values of the independent variables. Thereby they transmit certain information. In this book we define a signal as follows:

Definition 1: Signal

A signal *is a function or sequence of values that represents information.*

The preceding examples have shown that signals can assume different forms. Signals can be classified according to various criteria, the most important of which are summarised in Table 1.1.

Figure 1.5: A picture as a continuous two-dimensional signal

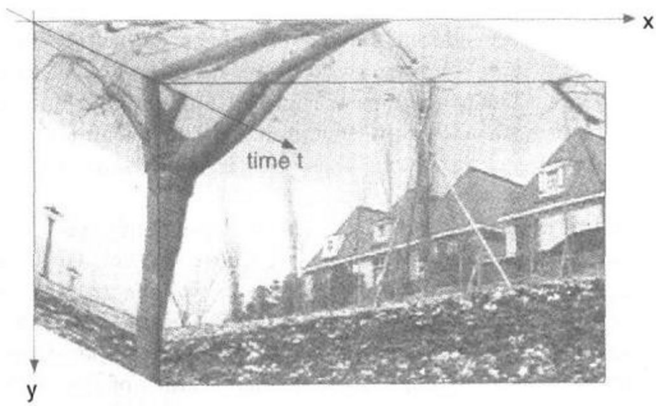

Figure 1.6: Moving picture as an example of a continuous three-dimensional signal

Table 1.1: Criteria for classifying signals

continuous(-time)	-	discrete(-time)
amplitude-continuous	-	amplitude-discrete
analogue	-	digital
real-valued	-	complex-valued
unidimensional	-	multidimensional
finite domain	-	infinite domain
deterministic	-	stochastic

We have already discussed the difference between continuous and discrete signals on the basis of Figures 1.1 and 1.3. Discrete signals are also termed discontinuous. Most of the preceding signals have been amplitude-continuous, because their dependent variable can take on any value. However, the signal in Figure 1.4 is amplitude-discrete, for the dependent variable (number of examinees) can assume only integer values. Taken precisely, the stock index in Figure 1.3 is likewise amplitude-discrete, since the stock index is specified to only a certain number of decimal places. Signals whose dependent and independent variables are continuous are called *analogue* signals. If both variables are discrete, we call the signal *digital.* The output voltage of a microphone is an analogue signal, for at any given time amplitude values can be read with any desired precision. Sequences of values stored in a computer are always digital, since the amplitude values can be stored only with finite word length in distinct (discrete) storage cells.

All of the signals we have considered so far had real amplitudes and so are classified as *real-valued.* Signals whose dependent variable assumes complex values are called *complex-valued.*

The signals in Figures 1.1 to 1.4 are unidimensional, while those in Figures 1.5 and 1.6 are multidimensional. For reasons of graphic representation, all the signals in the previous examples had finite domains of their independent variables and so are classified as *finite-domain* signals. However, if we consider the signal in Figure 1.6 as the picture of a television camera, then the domain of the location variable becomes finite again due to the restricted picture excerpt, but the domain of the time variable is infinite (neglecting the finite lifetime of the camera).

Signals are termed *deterministic* if their behaviour is known and can be represented, e.g., by a formula. The deflection voltage of an oscilloscope is a deterministic signal, for its behaviour is known and can be represented as a sawtooth wave. By contrast, we cannot define the amplitude values of a voice signal (see Figure 1.1) by means of formulae or graphical elements; furthermore, their continued behaviour is not known. Such signals are termed *stochastic.* Since it is impossible to specify their behaviour in terms of functions, such signals are described by expected values (mean, variance and many others).

1.2 Systems

1.2.1 What is a System?

We have seen that signals represent information. In many technical applications we want to do more than just view information; we want to store, transfer, or couple it with other information. This requires establishing and describing relationships between signals. This leads us to the definition of a *system*:

Definition 2: System

A system *is the abstraction of a process or object that puts a number of signals into some relationship.*

In this general form we can imagine a system as a black box that communicates with the outside world via various signals. Figure 1.7 depicts such a system that establishes a relationship among the signals x_1 to x_n.

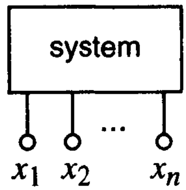

Figure 1.7: General system

In many cases we can classify a system's signals as input and output signals. Input signals exist independently of the system and are not affected by the system; instead, the system reacts to these signals. Output signals bear information generated by the system, often in response to input signals. The simple system in Figure 1.8 has one input signal x and one output signal y. We also term y the *system response* to x.

Naturally a system might contain multiple inputs and outputs. The system determines the influence of individual inputs on the output signals. In general, each output depends on all inputs. To simplify the notation, we combine input and output signals in vectors (Figure 1.8).

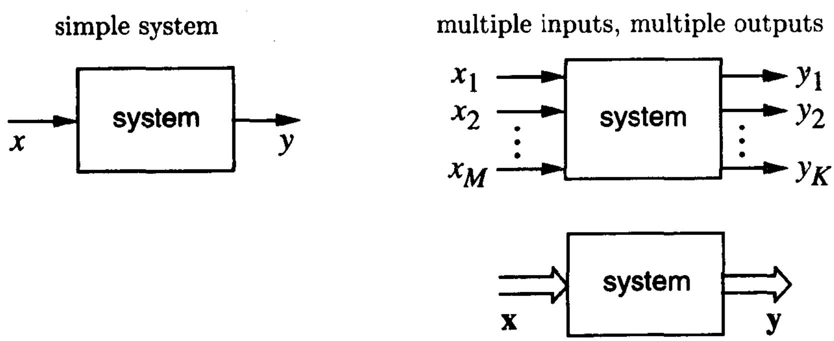

Figure 1.8: Input/output systems