ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

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Application of the Finite Element Method in Implant Dentistry

ZHEJIANG UNIVERSITY PRESS
浙江大学出版社

Springer

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With 100 figures

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图书在版编目 (CIP) 数据

有限元在植牙学的应用= Application of the Finite Element Method in Implant Dentistry: 英文 / 耿建平，严伟琪, 徐未编. —杭州：浙江大学出版社，2008.3
ISBN 978-7-308-05510-9

Ⅰ.有… Ⅱ.①耿…②严…③徐… Ⅲ.有限元法-应用-牙再植 Ⅳ.R782.12

中国版本图书馆 CIP 数据核字 (2007) 第 182269 号

有限元在植牙学的应用

耿建平　严伟琪　徐未　编

责任编辑 张　琛
出版发行 浙江大学出版社
网址：http://www.zjupress.com
Springer-Verlag GmbH
网址：http://www.springer.com
排　　版 浙江大学出版社电脑排版中心
印　　刷 杭州富春印务有限公司
开　　本 787mm×960mm　1/16
印　　张 9.5
字　　数 342 千
版 印 次 2008 年 3 月第 1 版　2008 年 3 月第 1 次印刷
书　　号 ISBN 978-7-308-05510-9 (浙江大学出版社)
ISBN 978-3-540-73763-6 (Springer-Verlag GmbH)
定　　价 60.00 元

浙江大学出版社发行部邮购电话 (0571)88072522

Foreword

There are situations in clinical reality when it would be beneficial to be able to use a structural and functional prosthesis to compensate for a congenital or acquired defect that can not be replaced by biologic material.

Mechanical stability of the connection between material and biology is a prerequisite for successful rehabilitation with the expectation of life long function without major problems.

Based on Professor Skalak's theoretical deductions of elastic deformation at/of the interface between a screw shaped element of pure titanium at the sub cellular level the procedure of osseointegration was experimentally and clinically developed and evaluated in the early nineteen-sixties.

More than four decades of clinical testing has ascertained the predictability of this treatment modality, provided the basic requirements on precision in components and procedures were respected and patients continuously followed.

The functional combination of a piece of metal with the human body and its immuno-biologic control mechanism is in itself an apparent impossibility. Within the carefully identified limits of biologic acceptability it can however be applied both in the cranio-maxillofacial skeletal as well as in long bones.

This book provides an important contribution to clinical safety when bone anchored prostheses are used because it explains the mechanism and safety margins of transfer of load at the interface with emphasis on the actual clinical anatomical situation. This makes it particularly useful for the creative clinician and unique in its field. It should also initiates some critical thinking among hard ware producers who might sometimes underestimate the short distance between function and failure when changes in clinical devices or procedures are too abruptly introduced.

An additional value of this book is that it emphasises the necessity of respect for what happens at the functional interaction at the interface between molecular biology and technology based on critical scientific exploration and deduction.

P-I Brånemark

Preface

This book provides the theoretical foundation of Finite Element Analysis(FEA) in implant dentistry and practical modelling skills that enable the new users (implant dentists and designers) to successfully carry out FEA in actual clinical situations.

The text is divided into five parts: introduction of finite element analysis and implant dentistry, applications, theory with modelling and use of commercial software for the finite element analysis. The first part introduces the background of FEA to the dentist in a simple style. The second part introduces the basic knowledge of implant dentistry that will help the engineering designers have some backgrounds in this area. The third part is a collection of dental implant applications and critical issues of using FEA in dental implants, including bone-implant interface, implant-prosthesis connection, and multiple implant prostheses. The fourth part concerns dental implant modelling, such as the assumptions of detailed geometry of bone and implant, material properties, boundary conditions, and the interface between bone and implant. Finally, in fifth part, two popular commercial finite element software ANSYS and ABAQUS are introduced for a Brånemark same-day dental implant and a GJP biomechanical optimum dental implant, respectively.

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Finite Element Method

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1.1 Introduction

The finite element method may be applied to all kinds of materials in many kinds of situations: solids, fluids, gases, and combinations thereof; static or dynamic, and, elastic, inelastic, or plastic behaviour. In this book, however, we shall restrict the treatment to the deformation and stress analysis of solids, with particular reference to dental implants.

1.2 Historical Development

Deformation and stress analysis involves the formulation of force-displacement relationships. These have been used in increasingly sophisticated forms from the 1660s, when Robert Hooke came out with his Law of the Proportionality of Force and Displacement.

The nineteenth and twentieth centuries saw a lot of applications of the force-displacement relationships for the analysis and design of large and complex structures, by manual methods using logarithmic tables, slide rules, and in due course, manually and electrically operated calculators.

Particular mention must be made of the contributions of the following scientists, relevant to modern structural analysis:

1857: Clapeyron Theorem of Three Moments
1864: Maxwell Law of Reciprocal Deflections
1873: Castigliano Theorem of Least Work
1914: Bendixen Slope-deflection Method

References for these works and others to follow are given at the end of the chapter.

These and other early methods and applications to articulated (stick-type) structures were based on formulas developed from structural mechanics principles, applied to straight, prismatic members such as axial force bars, beams, torsion rods, etc.

All these techniques yielded simultaneous equations relating components of forces and displacements at the joints of the structure. The number of simultaneous equations that could be solved by hand (between 10 and 15) set a practical limit to the size of the structure that could be analysed.

To avoid the direct solution of too many simultaneous equations, successive approximation methods were developed. Among them should be cited the following:

1932: Cross Moment Distribution Method

1940: von Karman and Biot Finite Difference Methods for Field Problems

1942: Newmark Finite Difference Methods for Structural Problems

1946: Southwell Relaxation Methods for Field Problems

These expanded the size limitations outwards by many orders of magnitude, enabling large complex articulated as well as plate-type structures to be analysed and designed.

The appearance of commercial digital computers in the 1940s revolutionised structural analysis. The simultaneous equations were not an obstacle any more. Solutions became even more efficient when the data and processing were organised in matrix form. Thus was matrix analysis of structures born.

It was the aeronautical industry that exploited this new tool to best advantage, but structural designers were quick to follow their lead. By the 1960s, not only could better and bigger aircraft be manufactured, but large bridges and buildings of unconventional design could be built.

This also resulted in the computerised revival of the somewhat abandoned earlier methods of consistent deformation and slope deflection. Not only could much larger problems be handled, but also effects formerly neglected as secondary (out of computational necessity) could be included. Pioneers in matrix computer analysis were:

1958: Argyris-Matrix Force or Flexibility Method

1959: Morice-Matrix Displacement or Stiffness Method

From matrix analysis of articulated structures to finite element analysis of continuous systems, it was a big leap, inspired and spurred on by the digital computer. However, it was not as if the entire idea was new.

Actually, the history of the Finite Element Method is the history of discretisation, the technique of dividing up a continuous region into a number of simple shapes. The progress from conceptualisation and formalisation, to implementation and application, may be summarised as follows:

1774: Concepts of Discretisation of Continua (Euler)

1864: Framework Analysis (Maxwell)

1875: Virtual Work Methods for Force-displacement Relationships (Castigliano)

1906: Lattice Analogy for Stress Analysis (Wieghardt)

1915: Stiffness Formulation of Framework Analysis (Maney)

1915: Series Solution for Rods and Plates (Galerkin)
1932: Moment Distribution Method for Frames (Hardy Cross)
1940: Relaxation (Finite Difference) Methods (Southwell)
1941: Framework Method for Elasticity Problems (Hrenikoff)
1942: Finite Difference Methods for Beams and Columns (Newmark)
1943: Concept of Discretisation of Continua with Triangular Elements (Courant)
1943: Lattice Analogy for Plane Stress Problems (McHenry)
1953: Computerisation of Matrix Structural Analysis (Levy)
1954: Matrix Formulation of Structural Problems (Argyris)
1956: Triangular Element for Finite Element Plane Stress Analysis (Turner, et al.)
1960: Computerisation of Finite Element Method (Clough)
1964: Matrix Methods of Structural Analysis (Livesley)
1963: Mathematical Formalisation of the Finite Element Method (Melosh)
1965: Plane Stress and Strain, and Axi-symmetric Finite Element Program (Wilson)
1967: Finite Element Method in Structural and Continuum Mechanics (Zienkiewicz)
1972: Finite Element Applications to Nonlinear Problems (Oden)

Old theories of solid continua were reexamined. Up to the 1950s, only continuous uniform regions of some regular shape such as square and circular plates or prisms could be analysed with closed form solutions. Some extensions were made by conformal mapping techniques. Series and finite difference solutions were developed for certain broader class of problems. But all these remained in the domain of academic pursuit of theoretical advancement, with few general applications and limited practical use.

Again, it was the aircraft industry that pioneered the idea of analysing a region as the assemblage of a number of triangular elements. The force-displacement relationships for each element were formulated on the basis of assumed displacement functions. The governing equations resulted after approximately assembly modelled the behaviour of the entire region. Once the equations were formulated, further solution followed the same steps as the matrix structural analysis.

The idea worked, and very efficiently with computers. It was also confirmed that the finer the division, the better the results. Now the aircraft designers could consider not only the airframe, but the fuselage that covered it and the bulkheads that stiffened it, as a single system of stress bearing components, resisting applied forces as an integrated unit.

This technique came to be called the "Finite Element Method" ("FEM" for short), both because a region could be only broken up into a finite number of elements, and because many of the ideas were extrapolated from an infinitesimal element of the theory to a finite sized element of practical dimensions.

Clough and his associates brought this new technique into the civil engineering profession, and soon engineers used it for better bridges and stronger shells.

Mechanical engineers exploited it for understanding component behaviour and designing new devices.

Computer programs were developed all over the Western world and Japan. The first widely accepted program was "SAP" (for Structural Analysis Package) by E.L. Wilson, which got him a Ph. D. from the University of California, USA. Most programs were in FORTRAN, the only suitable language at the time. Soon there was a veritable explosion in programs, and today, there are scores of packages in recent languages which are menu-driven and automated to the extent that with minimal (self-)training, anybody can do a finite element analysis for better or for worse!

Purists viewed the early applications with considerable reservation, pointing out the lack of mathematical rigour behind the technique. Appropriate responses were not slow in coming. Melosh and others soon connected the assumptions behind the formulation of the element with the already prevalent classical methods of interpolation functions.

Argyris in Europe, Zienkiewicz in UK, and Clough, Wilson, Oden, and numerous others in USA, pushed the frontiers of finite element knowledge and applications fast and wide. Between the 1950s and 1970s, applications of the finite element method grew enormously in variety and size, supported or triggered by fantastic developments in digital computer technology. In the last two decades, new developments have not been so many, but practical applications have become wider, easier, and more sophisticated.

Early users, the author included, considered hundred elements as a boon. A decade later, third generation computers enabled analysts to routinely use thousands of elements. By the 1970s, capacity and speed had increased ten times further. Nothing seemed to be beyond reach of finite element analysis whether it be a nuclear reactor (Fig. 1.1(a),(b),(c)), or a tooth (Fig. 1.1(d)), both of which the author has analysed.

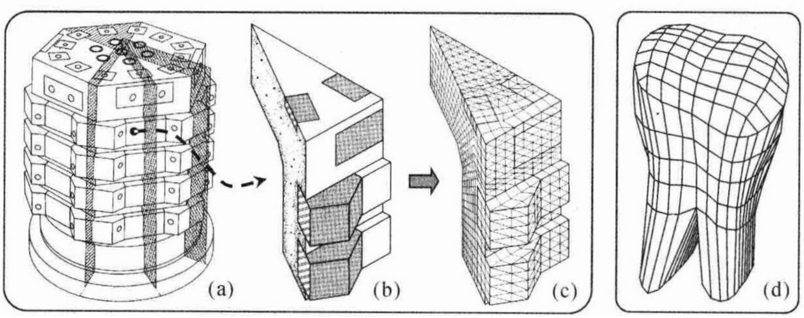

Fig. 1.1 (a) Test Model of Prestressed Concrete Nuclear Reactor; (b) One-twelfth Symmetry Segment for Analysis; (c) 3-D Finite Element Idealisation of the Analysis Segment; (d) 3-D Finite Element Idealisation of a Tooth

Now, computer packages which once demanded a mainframe have come to the desk top, and been loaded with powerful program graphics user interfaces, and interactive, online modelling and solutions.

It was just a small imaginative step to extend the applications beyond linear structural analysis, to non-linear and plastic behaviour, to fluids and gases, to dynamics and stability, to thermal and other field problems, because all of them involved the same kind of differential equations, differing only in parameters and properties, while the overall formulation, assembly, and solution techniques remained the same.

The references of historical importance, given at the end of the chapter, are merely representative, often the earliest in a series of many publications on a topic by the same or other authors. More detailed coverage of the history and further references may be found in the works by Cook, Desai, Gallagher, Huebner, Oden, Przemieniecki, and Zienkiewicz. Readers can referr to these resources for additional information on any of the topics discussed by the author in the following chapters.

Today, there is almost no field of engineering, no subject where any aspect of mechanics is involved, in which the finite element method has not made and is not continuing to make significant contributions to knowledge, leading to unprecedented advances in state of the art and its ultimate usefulness to mankind including contributions to dentistry.

1.3 Definitions and Terminology

The basic procedure for matrix analysis depends on the determination of relationships between the "Actions", namely forces, moments, torques, etc. acting on the body, and the corresponding "Displacements", namely deflections, rotations, twists, etc. of the body.

A "structure" is conventionally taken to consist of an assembly of straight "members" (as in trusses, frames, etc.) or curved lines whose shape can be mathematically evaluated, which are connected, supported, and loaded at their ends, called "joints". Fig.1.2(a) shows a two-storey structure consisting of frames in the vertical plane, grids in the horizontal plane, and trusses for the entrance canopy.

A "system" conventionally consists of a continuous membrane, plate, shell, or solid, single or in combination, each divided for analysis purposes into a finite number of "elements", connected, supported, and loaded at their vertices and other specified locations on edges or inside, called "nodes". Systems may include structures as well.

Fig. 1.2(b) shows a machine part system consisting of a solid, thin-walled shell, and a projecting plate. The suggested divisions are shown in lines of a lighter shade. Generally, the curved boundaries will be modelled as straight lines. The circular pipe in this case will be simulated as a hexagonal tube.

The principal difference between a structure and a system is this: The articulated structure is automatically, naturally, divided into straight (and certain regularly curved) members such as the truss member AB in Fig. 1.2(a), whose behaviour is well known and can be formulated theoretically. On the other hand, the

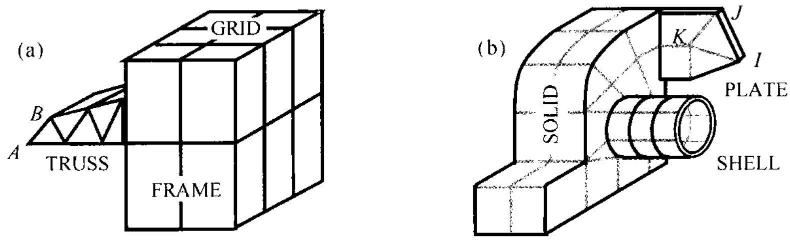

Fig. 1.2 (a) Two-storey Articulated Structure; (b) Machine Part System, Continuum

continuous system has no such theoretical basis and has to be divided into pieces of simple shape, such as the triangle IJK in Fig. 1.2(b), whose behaviour must be formulated by special methods.

Most real-life facilities involve a combination of both types described above. For instance, a building has flat plate-type walls and floors; a machine may sit atop columns and beams. In practice, "member" and "joint" usually apply to a structure, while "element" and "node" apply to a system in particular, and to a structure also in general.

Each node or joint can have a number of independent action (force or moment) or displacement (deflection or rotation) components called "Degrees Of Freedom" (DOF) along a certain direction corresponding to coordinate axies.

A plane truss member such as AB in Fig. 1.2(a) shown enlarged in Fig. 1. 3(a) has two DOF at each joint. Hence the member has a total of (2×2) or 4 DOF.

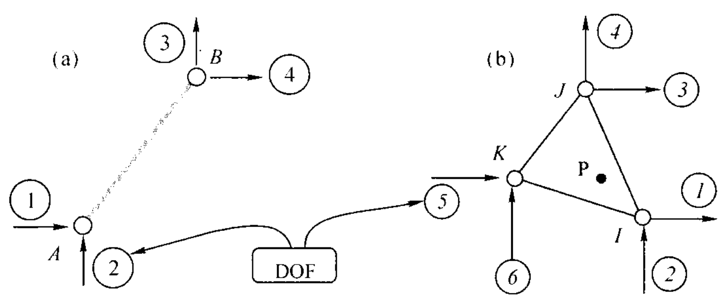

Fig. 1.3 (a) A Truss Member *AB*; (b) A Triangular Finite Element *IJK*

A triangular membrane element such as IJK in Fig. 1.2(b) shown enlarged in Fig. 1.3(b) has two DOF at each node. Hence the element has a total of (3×2) or 6 DOF.

Different types of members and elements have different numbers of DOF. For instance, a 3D frame member has two joints and six DOF (3 forces or displacements and 3 moments or rotations) per joint and 12 DOF in total. A solid "brick" element has eight nodes and three DOF (3 forces or displacements) per node and 24 DOF in total.

Additionally, in the case of finite elements, joint the same type of element may even have different number of nodes in "transition" elements.

1.4 Flexibility Approach

Fig. 1.4 shows a truss member with actions and corresponding displacements along the two DOF at each end. The sets of four actions and displacements can be represented vectorially or in terms of x, y components, as follows:

$\{A\} = \{A_1, A_2, A_3, A_4\} = \{X_i, Y_i, X_j, Y_j\}$, the "Action Vector"
$\{D\} = \{D_1, D_2, D_3, D_4\} = \{u_i, v_i, u_j, v_j\}$, the "Displacement Vector"

The displacement D at every DOF (say I) is a function of the actions A_1, A_2, at all connected DOF. Within the elastic limit, D_i is a linear combination of the effects of all actions.

Thus, their relationship may be written as:

$$D_1 = f_{11}A_1 + f_{12}A_2 + f_{13}A_3 + f_{14}A_4 \quad (1.1)$$

in which f_{ij} stands for the displacement at DOF I due to a unit action at DOF J, and is known as the "Flexibility Coefficient".

Three more such equations may be written for D_2, D_3, and D_4. The four equations may be represented in matrix form as:

$$\underset{4\times1}{\{D\}} = \underset{4\times4}{[F]} \cdot \underset{4\times1}{\{A\}} \quad (1.2)$$

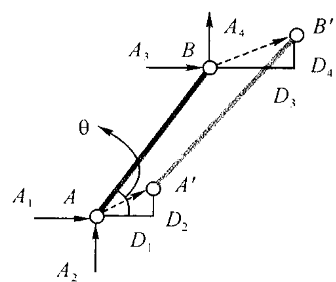

Fig. 1.4 Displaced Truss Member

in which, the [F] matrix of flexibility coefficients is known as "Flexibility Matrix".

The flexibility coefficients for prismatic bars can be determined from basic theoretical principles of strength of materials and theory of structures.

The flexibility approach was quite popular as the "Force Method" for manual analysis, the "Method of Consistent Deformation" being a typical application. With the advent of computers, it was found that this approach was not convenient to formulate or solve large and complex problems. Hence, the flexibility approach was not pursued further for practical applications.

1.5 Stiffenss Formulation

1.5.1 Stiffness Matrix

An alternative formulation, an exact opposite—in fact the inverse—of the flexibility approach, called "stiffness approach" or "displacement approach" was also in use for manual solutions. The "Slope Deflection Method" for continuous beams and the

"Moment Distribution Method" for beams and frames were very popular.

This approach was very convenient for computerisation and became the preferred method for computer solutions, especially for finite element analysis.

In general, the displacement along every DOF needs an action along that DOF and reactions at all the other connected DOFs for equilibrium. For elastic behaviour, the function is a linear combination of all the displacement effects.

Thus, the action-displacement relationships of the truss member in Fig. 1.4 is written as:

$$\begin{aligned}
A_1 &= k_{11} D_1 + k_{12} D_2 + k_{13} D_3 + k_{14} D_4 \\
A_2 &= k_{21} D_1 + k_{22} D_2 + k_{23} D_3 + k_{24} D_4 \\
A_3 &= k_{31} D_1 + k_{32} D_2 + k_{33} D_3 + k_{34} D_4 \\
A_4 &= k_{41} D_1 + k_{42} D_2 + k_{43} D_3 + k_{44} D_4
\end{aligned} \qquad (1.3)$$

in which k_{ij} stands for the action at DOF I due to a unit displacement at DOF J (with all other displacements set to zero) and is known as the "Stiffness Coefficient".

The four Eq.(1.3) may be represented in matrix form as:

$$\underset{4\times 1}{\{A\}} = \underset{4\times 4}{[k]}\ \underset{4\times 1}{\{D\}} \qquad (1.4)$$

in which the $[k]$ matrix of stiffness coefficients is known as "Stiffness Matrix".

The stiffness coefficients for prismatic bars can be determined from basic theoretical principles of strength of materials and theory of structures.

For instance, consider the truss member AB, of length L and cross-sectional area A_t from a material with Young's Modulus of elasticity E, inclined at an angle θ with the horizontal, subjected to a unit displacement along DOF 1, as shown in Fig. 1.5 (a).

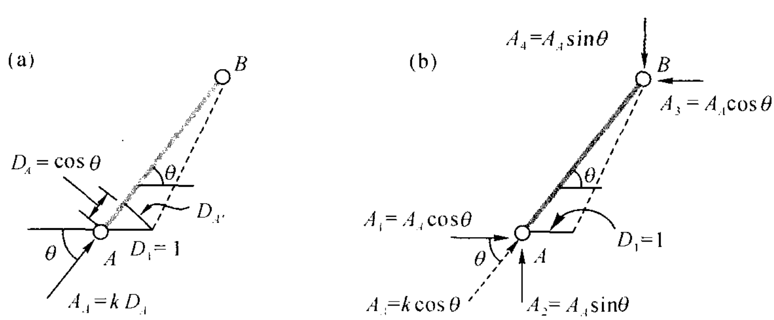

Fig. 1.5 (a) Unit Global Displacement; (b) Action Components

The unit horizontal displacement D_1 resolves into an axial displacement $D_A = 1 \cdot \cos\theta = \cos\theta$ and a transverse displacement $D_A' = 1 \cdot \sin\theta = \sin\theta$.

As the truss member ends are pinned, only the axial displacement D_A needs a force $A_A = kD_A$, or $k\cos\theta$, k being the stiffness of the axial force bar, namely (EA_t/L).

As shown in Fig. 1.5(b), this axial force A_A may now be resolved into: