MICROMAGNETICS

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Preface

To understand ferromagnetic materials, we must examine them on a smaller scale than that of ordinary observations. On one such scale we speak of domains; on another, of lattice sites. This tract analyzes them on an intermediate scale: small enough to reveal details of the transition regions between domains, yet large enough to permit the use of a continuous magnetization vector rather than of individual atomic spins.

This approach to ferromagnetic theory—"micromagnetics"—began with the well known "wall" calculation of Landau and Lifshitz in 1935. Since then there have been various contributions to it, but mostly as incidental parts of scattered and uncorrelated investigations. Only since the emergence of rigorous nucleation-field theory in 1957 has micromagnetics, as such, received conscious attention.

The object of this tract is to review the origins and underlying principles of the theory and to present it in a unified way. Dynamic theory is discussed only enough to show its relation to static problems. The point of view adopted is phenomenological: atomic concepts are used only to derive plausible mathematical forms for thermodynamic or dynamic expressions; discussions that begin in quantum mechanics and end in micromagnetics seem to me to exaggerate the dependence of the latter on the former.

Parts of this book, especially Chapter 7, contain results of my own not previously published in detail; this work was assisted by a grant from the National Science Foundation. The final work on the manuscript was completed at the Weizmann Institute of Science, Rehovot, Israel, where I spent the spring and summer of 1962 as a Fulbright scholar. I am grateful to the Institute for its hospitality and to Professor E. H. Frei, of the Electronics Department, for suggesting the visit and doing much to make it rewarding. For helpful discussions of many topics I am deeply indebted to Drs. A.

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Contents

Chapter 1

Introduction

1.1 The General Problem

A ferromagnetic material may be defined as one that possesses a spontaneous magnetization: that is, sufficiently small volumes of it have a magnetization* (magnetic moment per unit volume) M_s dependent on the temperature but independent, or at least only slightly dependent, on the presence or absence of an applied magnetic field.† The existence of this spontaneous magnetization is explained by the Weiss "molecular field" postulate, amended quantum-mechanically by Heisenberg; the amendment replaces the mysterious molecular field by exchange forces, which are less mysterious or more so according to one's feeling toward quantum mechanics. But this theory, based on exchange forces that tend to align the spins and thermal agitation that tends to disalign them, says nothing about the *direction* of the vector magnetization **M**; only that its *magnitude* must be M_s.

Experimentally, it is observed that though the *magnitude* of **M** is uniform throughout a homogeneous specimen at uniform temperature, the *direction* of **M** is in general not uniform, but varies from one region to another, on a scale corresponding to visual observations with a microscope. Uniformity of direction is attained

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* This word will be used instead of the longer term *intensity of magnetization*.

† By *applied (magnetic) field* we shall always mean the field of magnetizing coils or magnets (or both) external to the specimen, as distinguished from the field (be it the **H** field or the **B** field) produced by the magnetization of the specimen under consideration.

only by applying a field, or by choosing as a specimen a body which is itself of microscopic dimensions (a "fine particle"); the evidence of uniformity in the latter case is indirect but convincing. The tendency of a ferromagnetic specimen to break up into "domains," with their vector magnetizations oriented differently, explains the possibility of a demagnetized state; and in fact such a domain structure was postulated by Weiss in order to reconcile his theoretically predicted spontaneous magnetization with the experimental possibility of demagnetization. Today the evidences of domain structure are so many and so inescapable that its status is no longer that of a postulate, but rather that of an experimental fact.

In two respects, however, the range of validity of this fact has at times been supposed more universal than it actually is.*

First, domains were for a long time tacitly assumed to be present in *all specimens*, regardless of their geometry. This naive assumption delayed the theoretical understanding and practical application of the properties of fine particles.

Second, domains have often been discussed as if they were a phenomenon to be expected in *all* ferromagnetic *materials*. Actually, both theory and experiment indicate that domains in the usual sense—regions within which the direction of the spontaneous magnetization is uniform or at least nearly so—do not occur unless there are present strong "anisotropy" forces, which cause certain special directions of magnetization to be preferred. When such forces are absent or weak, the magnetization direction, over dimensions comparable with the usual domain dimensions, varies gradually and smoothly.

It is therefore clear that domain structure, though normal, is not universal. More generally, we should suppose merely that the direction angles Φ and Θ of the spontaneous magnetization, or equivalently its direction cosines α, β, and γ (subject to the constraint $\alpha^2 + \beta^2 + \gamma^2 = 1$), are functions of the coordinates x, y, z of the point P at which the vector magnetization $\mathbf{M}$ is being evaluated. Whether these functions are constant or variable, con-

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* See, for example, the discussions in Becker and Döring (1939).

tinuous or discontinuous, step-functions or sinusoids, need not be decided until later.

The general problem to be examined in this book is the problem of developing a theory of this magnetic microstructure, concerning which the Weiss-Heisenberg theory is noncommittal.

1.2 Forces Involved

The most basic method of solving this problem would be to use an atomic model, such as a lattice of spins, and to introduce into the model those forces that the Weiss-Heisenberg theory left out. As has already been mentioned, that theory takes account only of exchange forces and thermal agitation. The known forces that remain to be introduced are: magnetic dipole-dipole forces; forces due to spin-orbit coupling and to magnetic quadrupole and higher moments; modifications of the exchange forces that result when the directions of neighboring spins are not exactly parallel; and "magnetostrictive" forces, which are not physically distinct from the ones already enumerated but are the modifications of them that come about because of the ability of the lattice to undergo strains. Available methods of treating ferromagnetism atomically are already inadequate when only exchange forces and thermal agitation are taken into account; they become quite unmanageable when, for example, magnetic dipole-dipole forces are introduced. Accordingly, we must resort to a phenomenological type of theory.

The possibility of such a theory rests on the fact that all these new forces have only a small perturbing effect on the parallelism (or, in certain cases, antiparallelism) of neighboring spins. The spin direction, in other words, can change only by a small angle from one lattice point to the next. It therefore seems legitimate to approximate the direction angles of the spins (or more conveniently of the associated magnetic moments) with *continuous* functions of position. By this device, exactly analogous to the replacement of individual atomic masses by a continuous density in elementary mechanics, sums over lattice points are replaced by integrals over a volume, and the techniques of calculus become applicable. The basic concept in such a theory is a vector magnetization **M** whose direction angles or direction cosines vary continuously with position. Changes through appreciable angles may occur on a scale that is small in comparison with the "domain" scale, or on a scale that is comparable with the domain scale; these two cases will correspond, respectively, to the case in which "domains" in the ordinary sense are observed and to the case in which only a gradual variation is observed.

The detailed development of such a theory is the theme of this book. No claim is made that the theory has been fully developed; all that can be said is that the foundations have been laid.

1.3 Methods of Solution

In the foregoing discussion, the term "force" has been used in a general sense. If magnetostriction is ignored, our model is a rigid specimen with a vector magnetization **M** whose direction varies continuously with the coordinates x, y, z, but whose magnitude has a value M_s determined by the temperature. The "forces" are then torques (couples) that act on the magnetic moments $\mathbf{M}d\tau$ of the volume elements $d\tau$. In thermodynamic equilibrium, the orientation of **M** at each point must be such that the total torque on each moment element $\mathbf{M}d\tau$ is zero. When the field is changed, the torques in the old orientations usually cease to be zero; then the dissipative processes that tend toward thermodynamic equilibrium will ultimately establish a new equilibrium distribution of orientations.

One method of describing the orientation of **M** is by use of polar angles Φ, Θ referred to fixed x, y, z axes in the usual way. The corresponding components, $L_\Phi d\tau$ and $L_\Theta d\tau$, of the torque on a volume element $d\tau$ are, in the dynamic sense, generalized forces (for the element $d\tau$) corresponding to Φ and Θ as generalized coordinates.

In the course of our discussions we shall find it necessary to recognize both the importance of temperature (since M_s and other quantities are temperature-dependent) and the importance of dynamic effects (since ferromagnetic resonance at microwave frequencies is now a phenomenon of both theoretical and practical significance). We therefore have need of a theory that is *thermodynamic* in the literal sense of the word, i.e., capable of handling thermal and dynamic effects simultaneously. Unfortunately thermodynamics in this sense does not yet exist; the body of theory commonly known as "thermodynamics" is an equilibrium or at most a near-equilibrium theory, more properly described as *thermostatics*. Our theory is therefore perforce a patchwork affair, proceeding as follows.

In many of our discussions, the peculiarly dynamic effects characteristic of resonance may be ignored, since the changes are slow or the frequencies low. We may then pay proper attention to thermostatic requirements. In such problems the convenient independent thermal variable is absolute temperature T rather than entropy S'. Therefore for a rigid material, the appropriate basic energy function is not the internal energy U but the Helmholtz function A; for a homogeneous specimen, at uniform temperature and uniform magnetization, $A = U - TS'$. In a small reversible change, for unit volume,

$$\delta U = L_\Phi \delta\Phi + L_\Theta \delta\Theta + T\delta S', \tag{1-1}$$

whereas

$$\delta A = L_\Phi \delta\Phi + L_\Theta \delta\Theta - S'\delta T. \tag{1-2}$$

Under nonuniform conditions and in the presence of an applied field, more complicated expressions and additional transformations are needed, and these will be discussed later; but from what has been said, it is clear that the distinction between U and A is important, and that for isothermal processes ($\delta T = 0$) it is the latter that plays the role of an energy function, from which the "forces" (L_Φ, L_Θ) can be derived by differentiation with respect to the "coordinates," or from which the equilibrium values of the "coordinates" can be found by a variational procedure.

In discussions of high-frequency phenomena such as resonance, or of transient phenomena, dynamic contributions to the forces are important. In such discussions we shall treat the specimen as a dynamic system whose potential energy is of the same form as the

A of the equilibrium theory. If we actually identify this potential energy with A, we thereby assume that the dynamic effects do not appreciably perturb the thermodynamic equilibrium; in other words, that the periods of the alternating fields, or the times required for establishment of *magnetic* equilibrium, are long in comparison with the times required for establishment of *thermal* equilibrium. At the other extreme, we may identify the potential energy with U if we assume an inequality in the opposite sense, so that the conditions are approximately adiabatic rather than isothermal; the parameters that appear in the energy formula must then, like the "elastic constants" in vibration calculations, be interpreted as adiabatic rather than isothermal "constants." Use of our equations with this interpretation will be legitimate only when the spontaneous magnetization M_s varies negligibly with temperature under the conditions considered; for we shall always suppose that $|\mathbf{M}| = M_s = \text{const}$ in the processes examined, and this is strictly true only under isothermal and not under adiabatic conditions, since M_s is a function of T and since in an adiabatic change T changes.

Unless one or the other of these simplifications—constant temperature, or adiabatic conditions with negligible dependence of M_s on T—is assumed, details of the heat-flow process must be studied simultaneously with details of the magnetic process. We shall not consider this complex situation. Over a limited range of frequencies the departure from isothermal conditions, with resulting irreversibility, is taken into account by inclusion of a phenomenological damping term in the equation of motion.

Though direct use of torques is sometimes convenient, energy methods are generally more powerful. We shall therefore require, as a starting point for most of our calculations, an expression for the "free energy." By "free energy," as distinguished from "energy," we mean A or some other thermodynamic potential in which the natural independent thermal variable is T, as distinguished from U or some other thermodynamic potential in which the natural thermal variable is S.

Besides torques and free energies, still another concept is sometimes useful; this is the "effective field." If we choose as "coordinates" not the angles Φ and Θ, but the components of magnetization M_x, M_y, M_z, then the corresponding "forces" that are obtained by differentiation of a free-energy density are quantities $\mathcal{H}_x$, $\mathcal{H}_y$, $\mathcal{H}_z$ with the physical dimensions of a magnetic field intensity. We may regard the vector $\boldsymbol{\mathcal{H}} = \mathcal{H}_x\mathbf{i} + \mathcal{H}_y\mathbf{j} + \mathcal{H}_z\mathbf{k}$, the generalized vector force corresponding to $\mathbf{M}$ as vector coordinate, as an "effective field intensity." Since $\mathbf{M}$ is subject to the constraint $|\mathbf{M}| = M_s = \text{const}$, the component of $\boldsymbol{\mathcal{H}}$ in the direction of $\mathbf{M}$ is physically ineffective and in fact indeterminate. For by use of the relation $M_x^2 + M_y^2 + M_z^2 = M_s^2$, a free-energy density of the form $f(M_x, M_y, M_z)$ can be replaced by, say, $f(M_x, M_y, (M_s^2 - M_x^2 - M_y^2)^{1/2})$; this yields on differentiation an $\boldsymbol{\mathcal{H}}$ that differs from the original one by a vector in the direction of $\mathbf{M}$. From any such $\boldsymbol{\mathcal{H}}$, however, a unique vector torque per unit volume $\mathbf{L}$, normal to $\mathbf{M}$, can be found by the formula

$$\mathbf{L} = \mathbf{M} \times \boldsymbol{\mathcal{H}}, \tag{1-3}$$

just as in the case when $\boldsymbol{\mathcal{H}}$ is an actual external magnetic field intensity. Similar remarks apply when the direction cosines α, β, and γ are used as coordinates: the corresponding force $M_s\boldsymbol{\mathcal{H}}$ is indeterminate by an arbitrary vector in the direction (α, β, γ), but the torque is uniquely determined.

Whichever method—energies, torques, or effective fields—is used in particular calculations, the first requirement is a method of finding expressions for the various terms in the free energy, corresponding to the types of force already enumerated. Given such expressions, the calculation of torques or effective fields is straightforward.

In Section 1.2 we classified the forces according to their physical *origin*. In a phenomenological theory, it is more convenient to classify them according to the mathematical *form* of the free-energy expressions that describe them. In a rigid cubic crystal, the dipole-dipole forces correspond to free-energy expressions similar in form to the energy integrals of formal magnetostatic theory; spin-orbit and quadrupole forces, to free-energy densities dependent on the local direction of magnetization; and the exchange forces, as perturbed by nonuniformity of magnetization, to free-energy densities dependent on the spatial gradients of the direction cosines (or direction angles) of the magnetization. These three contributions to the free energy are usually called, in order, the *magnetic* or *magnetostatic* energy, the *anisotropy* or *crystalline-anisotropy* or *magnetocrystalline-anisotropy* energy, and the *exchange* or *exchange-stiffness* energy. The terminology is poor, for it confuses classification according to origin and classification according to form; but in cubic crystals it causes little trouble. In hexagonal crystals, on the other hand,* the dipole-dipole forces contribute, besides the formal magnetostatic-energy integral, a term in the form of a free-energy density dependent on the local magnetization direction. In a phenomenological theory, concerned with forms and not with origins, this term must be treated as part of the "anisotropy energy" and in fact cannot be distinguished from other terms of the same form but of different origin; thus an energy term of magnetic origin is included in the "anisotropy energy" and not in the "magnetic energy." We shall not try to reform the established terminology; no confusion will occur if we remember that our theory is phenomenological and that our classification is on the basis of form, not of origin.

For a crystal capable of strain, the Helmholtz function contains terms linear in the strains and terms quadratic in them. The latter occur also with nonmagnetic materials and are called the "elastic energy"; the approximation is usually made that the coefficients in them are independent of the magnetization direction. (This does not imply that the *measured* "elastic constants" are independent of magnetization.) The linear terms have coefficients that depend on the magnetization direction; these terms represent coupling between magnetic and mechanical processes and are usually called "magnetoelastic" or "magnetostrictive" energy. After a transformation to new thermodynamic variables, such as stresses rather than strains, the new thermodynamic potential will again consist of linear and quadratic terms; but the linear part of the new expres-

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* See equation (3-27) and the paragraph following it.

sion is not simply the linear part of the old expression rewritten as a function of the new variables; therefore if the linear part is again called "magnetostrictive" and the quadratic part "elastic," these terms now have new meanings. The conclusion from all this is that descriptive names for individual terms in the free energy should be used either with great caution or not at all.

1.4 Methods of Evaluating Energy Functions

If this theory were to be developed on the basis of a truly atomic model, a conceivable procedure would be to evaluate the partition function, and from it the free energy, by the methods of statistical mechanics. Though formal expressions in the form of sums over states and lattice sites might be derived without too much trouble, the reduction of these to usable forms is out of the question. Furthermore, even if this procedure could be carried out, it would not, at least without basic modification, lead to a theory of magnetic hysteresis; for standard statistical mechanics yields only states of complete thermostatic equilibrium, and magnetic remanence is not such a state.

The theory needed is a thermostatic theory based on phenomenological expressions for various contributions to the free energy, in terms of such variables as may be relevant in the particular problems considered. These variables may include not only components of magnetization and of elastic displacement, but also, as elasticity theory illustrates, of their derivatives with respect to x, y, and z.

In the attempt to derive such free-energy expressions, there are two possible approaches, which may be used singly or in conjunction.

One approach is to assume, at given temperature, a series in the relevant variables, e.g. in the direction cosines α, β, γ; truncate the series after a few terms, in the hope that these will prove sufficient; and use crystalline-symmetry considerations to decrease the number of (temperature-dependent) parameters in the formula. This is the method usually used for evaluating the "anisotropy" energy.

The other approach is to use an atomic model, perhaps drastically simplified, to obtain an expression for a particular term in the internal energy U at $T = 0$, where thermal agitation does not complicate the calculation. The expression thus obtained may also be considered an expression for A at $T = 0$. It may be adapted to arbitrary T by replacing the constants in the formula by temperature-dependent parameters. This is the method that is convenient for evaluating the contribution of dipole-dipole forces to the "magnetic" and "anisotropy" energies.

In either case, the temperature-dependent parameters in the formula must be evaluated primarily by analysis of experimental data; atomic models, however, facilitate the estimation of orders of magnitude.

1.5 Scope of This Book

The primary aim of this book is to present, with reasonable soundness and completeness, the basic principles and methods of the theory just outlined. Illustrative applications will also be described, but this aspect of the treatment will not be exhaustive—partly because the details are too tedious for a book of this size, and partly because not all of the attempts to apply the theory have been equally fruitful. Attention will also be paid to the relations between this theory, "micromagnetics," and certain other branches of ferromagnetic theory: specifically, domain theory, the theory of ferromagnetic resonance, and spin-wave calculations. Here again the treatment will make no attempt at exhaustiveness; rather, the attempt will be to show to what extent a unified approach to these various fields of study is possible and desirable.

Chapter 2 traces the historical origins of the theory. Chapter 3 summarizes the principles and methods to be drawn upon, and Chapter 4 uses them to develop some of the basic equations of the theory. The following three chapters apply these to specific classes of problem: Chapters 5 and 6 to static problems (linear and nonlinear respectively), Chapter 7 to dynamic problems. In all these calculations, the material is assumed to be rigid; Chapter 8 summarizes and criticizes the usual methods of taking account of magnetostrictive phenomena.

Chapter 2

Historical Background

2.1 Early Concepts: Rotation and Inversion

The foundations of modern magnetization-curve theory were laid by Akulov (1928, 1929a, b, 1930a, 1931a, b) and Becker (1930). By laborious evaluation of lattice sums, Akulov derived the now familiar formula for the crystalline-anisotropy energy density in a cubic crystal,

$$F = K_1(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2), \tag{2-1}$$

where K_1 is a constant related to an assumed quadrupole moment. In the range of positive field strengths H in which HM_s is comparable with K_1, the experimental magnetization curves of crystals can be fitted quite well (with K_1 evaluated from the data) by assuming that the magnetic moment of the whole specimen rotates rigidly; its direction is then found by minimizing $F - \mathbf{H}\cdot\mathbf{M}$ at constant $\mathbf{H}$.

The rotation theory failed at small and negative H. When $\mathbf{H}$ is along [100] in iron, the rigid-rotation model predicts a magnetization M_s in the original direction until H reaches the negative value $-2K_1/M_s$. Experimentally, the curves are qualitatively similar to those of polycrystals, with a coercive force two or more orders of magnitude smaller than $2K_1/M_s$. At low fields, therefore, the "rotation" mechanism was assumed to be replaced by another mechanism, called "inversion." About the nature of this process there was considerable doubt and argument. Akulov (1930b) for a while maintained that in it the spontaneous magnetization ceased