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ELASTICITY

(SECOND EDITION)

ZHANG-JIAN WU　HAI-JUN WU　FENG HAN

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Abstract

The purpose of this book is to introduce fundamentals of the classical theory of elasticity and associated recent research developments by the authors. The whole book is constructed on the basis of the elasticity theory course syllabuses and contents used in the past few years at Beijing Institute of Technology, China, and the University of Manchester, UK. The content arrangement in this edition refers to some classic text books on elasticity, the readers' feedback on the first edition of the book and some newly-obtained teaching and learning outputs. In order to meet the requirement of bilingual pedagogic development in higher education and facilitate the numerical simulation and engineering calculation in elasticity, finite difference and finite element methods are added in comparison with the first edition of the book.

By reading this book and other relevant Chinese- and English-version textbooks, readers should be able to command the fundamental knowledge of elasticity, comprehend relevant standard technical terms and enhance their level of professional English. The book is intended for senior undergraduate and postgraduate students, especially for engineering mechanics students, of higher education engineering institutes. It can also be considered as an English reference for engineers, researchers and novices.

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PREFACE

This book is, to a large extent, an extension of the lecture notes used by the authors during the past few years in the modules of Elasticity at Beijing Institute of Technology, China and the University of Manchester, UK. It is assumed that the students and readers have already had some acquaintance with the theory of rods and beams, concepts of stress and strain, and so forth, which may be found in basic textbooks on the strength of materials or "Mechanics of Materials". It is intended to give postgraduate and senior undergraduate students sound foundations in preparation for advanced courses such as thermal elasticity, plasticity, plates and shells, mechanics of composite materials, wave propagation, the finite element method and those branches of mechanics which require the analysis of stress and strain. To this purpose, the book has tried to bridge the branches mentioned with either a whole chapter in detail or a section giving a brief overview of related material, from the fundamental theory to the practical application. For example, finite difference and finite element methods are added in this new edition in comparison with the first edition of the book. Such inclusions in the book should be more appropriate and convenient for readers to study and carry out numerical simulations and engineering calculations in elasticity. The whole content of the book is arranged as follows:

Chapter 1 includes the fundamental assumptions adopted in the course of elasticity theory and, for ready reference, certain mathematical preliminaries. The main content of the book begins with stresses and equilibrium in Chapter 2. The theory of deformation is presented in Chapter 3. These two chapters emphasise the independence of stress and strain and also show their mathematical similarity. They are independent of material behaviour. Stress and strain are linked in Chapter 4 by the introduction of three-dimensional stress-strain constitutive relations—generalised Hooke's law. This chapter also introduces the strain energy density function, St. Venant's principle and the assembled basic field equations from Chapters 2 and 3 in order to elucidate general solution techniques in boundary value problems of elasticity. From Chapters 5 to 7, two-dimensional boundary value problems including plane stress and plane strain, torsion and bending of bars, are selected for the application of these solution techniques. The methods of solution are classical and elementary, but they show clearly what concepts and methods are adopted. The more sophisticated methods for solving the boundary value problems of elasticity, such as Love's strain function, the Galerkin method, complex potentials and integral transforms, have been deliberately omitted. They are covered extensively in most traditional treatises on the mathematical theory of elasticity. Readers are encouraged to appreciate some of these specialised approaches after they are familiar with the contents of Chapters 5 to 7.

Chapter 8 involves an innovative hybrid technique to the exact solution of three-dimensional elasticity—the state space method. This method is considered to be one of the most powerful approaches for the strong solution in elasticity and has been used in the area of automatic control theory for many years. To the best knowledge of the authors, this kind of method and relevant content materials are, for the first time, introduced into a text book on elasticity. It is believed that the method will have wide application in anisotropic elasticity and laminated systems.

Chapter 9 provides the elementary material for classical Kirchhoff plate theory, mainly applicable to the analysis of isotropic thin plates.

Chapter 10 is concerned with the formulations of energy principles and their application to continuum solid mechanics. The emphasis is on showing how the relevant equations of equilibrium and compatibility can be deduced for this “approximate” theory of elasticity in a systematic and consistent manner. It belongs to the scope of what is known as a “weak solution” in elasticity. Due to the spatial limitation of the current book, Cartesian tensor notations have been adopted in this chapter whenever they are necessary in order to eliminate tedious and lengthy algebraic and integral computation.

Chapters 11 and 12 are new additions in comparison with the content of the first edition of this book. The revision is mainly motivated by the constructive feedback from readers of the first edition. As a classical numerical method for the solution of differential equations, the finite difference method is described extensively in Chapter 11, with emphasis on the difference expressions for the Laplace and bi-harmonic operators which are two typically differential formats for many governing equations in elasticity. The treatment for different boundary conditions, especially for the curved boundary, has been formulated in principle. Readers can easily extend these difference formations to practical application confidently.

One of the most powerful modern numerical and simulation tools for a lot of engineering subject areas including elasticity is the finite element method (FEM). Chapter 12 outlines the formulation of displacement-based FEM only on the basis of the principle of minimum potential energy specified in Chapter 10. Although most responses of the examples under given loading in this chapter can be calculated by hand, the method itself is actually a computer-aided mathematical technique for obtaining approximate numerical solutions to a number of simultaneous equations. Readers are suggested to pay attention to the formulation of the nodal force vector and the last section of the chapter where some key issues related to the application of FEM have been extracted and discussed. It will help readers to assess, conceptually, if the computed results by FEM are reasonable or not.

A few special topics of elasticity in engineering such as thermal elasticity and propagation of elastic waves are shown in Chapter 13. Various strength theories and fracture criteria are listed too for potential engineering applications. This chapter is intended to be an introduction to the material for the interests of some readers. Lecturers can decide if they are delivered or not.

Most of the material presented in the book is classical (except for the state space method of Chapter 8), and for this reason limited references are provided at the end of the book. All important citations have been clearly indicated in the book. Hopefully, we will be forgiven for any omission deemed important. S. I. units are consistently used throughout the book.

The development of this textbook in English language was strongly influenced by some factors, such as the importance of English language due to the globalisation of the world economy. The bilingual teaching provides an excellent tool for those who wish to learn about elasticity using the standard terminology of mechanics in both English and Chinese. We also believe that the inclusion of large numbers of examples and exercises or Problems/Tutorial Questions make this book suitable for self-study. Students, researchers or practitioners, novices and experts, should all profit much from reading the book and having it for reference in the years to come.

We wish to express our gratitude to Prof. Feng-lei Huang and Qing-ming Zhang from Beijing Institute of Technology (BIT), Mr. Mike Maidens, and Dr. Lee Cunningham from the University of Manchester who read the manuscript and made useful suggestions and to Prof. Jia-rang Fan from Hefei University of Technology with whom many sections were discussed. Sincere thanks are due to Prof. Zhuo-cheng Ou from BIT for his critical review and positive comments, editors from the Press of BIT for their patience, proofreading and other support during the preparation of the final

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May, 2017

CONTENTS

CHAPTER 1 BASIC ASSUMPTIONS AND MATHEMATICAL PRELIMINARIES

1.1 Introduction

Elasticity is a subject associated with the determination of the stresses and displacements in a body as a result of applied mechanical or thermal loads, for those cases in which the body reverts to its original state on the removal of the loads. In this text book, we shall further restrict attention to the case of linearly infinitesimal elasticity, in which the stresses and displacements are linearly proportional to the applied loads and the displacements are small in comparison with the characteristic length dimensions of the body. Detailed assumptions and restrictions on the subject can refer to Section 1.2.

Elastic theory was once a problem solver on its own with closed form solutions which are still admirable and challenging even today. However, extensive and successful use of numerical approaches in engineering, such as finite elements, have diminished to a great extent the need to solve problems analytically, which usually requires enormous, sometimes overwhelming, mathematical skills. The challenge to engineers today appears to be to make correct and effective use of available problem solvers and to assess results obtained from the solvers. This can be achieved only when the engineer understands the formulation of the problem before the solvers are employed.

The main intention of our study of Elasticity is to promote the understanding of the concepts and formulation of elasticity problems. Problem solving skills and techniques are relegated to an adjunct role, being used to help in understanding or in making sense of a theoretical development. Citation of some solvable problems in Elasticity is necessary in this book for us to overview its theoretical frame. It is assumed that the readers possess preliminary knowledge of Structural and Stress Analysis/Mechanics of Solids and Structures. Sometimes such knowledge comes from the prerequisite course called Strength of Materials which will be referred to in the present book.

Strength of Materials deals with bars subjected to loads such as axial tension, compression, torsion about the longitudinal axis and lateral bending. A common assumption is usual for all these cases: the plane section assumption, i.e. a cross-section perpendicular to the axis of the bar deforms into a plane section perpendicular to the axis of the deformed bar. Other restrictions are also needed in order to validate the theoretical analysis, e.g. the bar should be sufficiently long compared with its other dimensions.

Questions arise at this point. How to quantify these restrictions? What happens if a problem goes beyond these restrictions? How to deal with objects other than bars, e.g. those shaped like a two-dimensional plate or three-dimensional laminated structures? These questions set the scene and the main content of the present study of Elasticity.

1.2 Basic Assumptions

All the assumptions in Elasticity have appeared in Strength of Materials (not the wrong way around!) and they will be put in a more rigorous and more explicit manner below.

For the material:

(1) ***Homogeneity*** means that material properties are the same at any point in the

material. Microscopically, no material can be homogeneous when one realises that materials are all composed of molecules or atoms, in the case of metals, a larger unit is a grain. However, if an "infinitesimal" volume contains a large enough number of these units, homogeneity is a reasonable assumption on a statistical basis.

Exceptions: reinforced concrete, sandwich laminates, etc.

(2) ***Continuity*** means that every material point is connected and infinitesimally close to the one next to it. All materials carry discontinuous nature in one way or another microscopically. When engineers describe the behaviour of a material at a point, they do not mean a mathematical point (a mathematical point has zero area and zero volume). What they really mean is a small but finite area or volume. They are sufficiently small that they can be regarded as "infinitesimal" without losing any macroscopic feature of the material. If such small areas and volumes are much larger than the characteristic dimension of the discontinuities involved (e.g. the grain size or the distance between atoms), the material can be reasonably assumed to be continuous. Therefore, whenever a small segment dx, a small area dA, etc. are mentioned, they are in fact a continued segment and a continued area, etc.

Exceptions: porous material, granular materials, material with cracks which could not be treated as parts of total boundary of the material, etc.

(3) ***Isotropy*** means that material properties at a point are the same in any direction. It depends on the arrangement of the microscopic material units within an "infinitesimal" volume. If they are arranged at random, isotropy is statistically true.

Exceptions: fibre reinforced composites, wood, etc.

(4) ***Elasticity*** requires that the internal state, such as the strains in the material, reverts to the original state after loads are removed, and, also, that the unloading have to follow the same path as when it was loaded up. It is reasonable for many materials when only small deformation is involved.

Exceptions: problems involving large strain into plastic regime, some plastics of