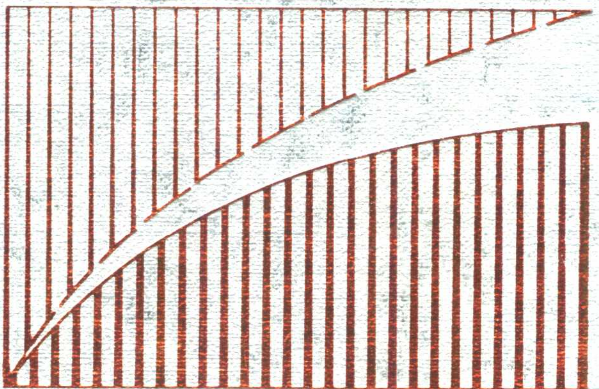
INTRODUCTION TO SYSTEM ANALYSIS

T. H. GLISSON



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PREFACE

This is a textbook for a first course in system analysis in electrical engineering and computer engineering curricula. Prerequisites assumed by the book are integral calculus, a first course in electric circuits, and a working knowledge of complex algebra. Neither prior nor concurrent study of differential equations and operational calculus is required. Chapters 1 to 5 provide all prerequisites for successful study of virtually all popular undergraduate textbooks on communication, control, power systems, instrumentation, and signal processing. These chapters also provide a meaningful introduction to system analysis for those who choose to pursue other areas of study.

The book began in 1975 as a set of notes for a required junior course in system analysis at North Carolina State University. The course bridges the gap between a sophomore course in circuits and senior electives in communication, control, instrumentation, power systems, and signal processing. This book has been used in various stages of revision in that course by more than 1500 students and by five different instructors. Explanations, examples, and problems given in the book have been tested thoroughly in the classroom.

The aim is to present concepts in a straightforward manner and in logical order and to illustrate definitions, principles, and procedures with an ample number of examples. The book contains almost 300 examples, more than 600 figures, and more than 450 problems. Approximately equal emphasis is placed on drill problems, multistep problems that require use of two or more principles, and substantive problems that require application of principles in realistic settings. A few problems require slight extensions of concepts developed in the body of the text.

The book stresses what I believe to be the three central ideas in system analysis: (1) methodology of system analysis is based on representing large systems as interconnections of simpler subsystems and on representing complicated signals as combinations of simpler signals; (2) performance of systems usually is described with reference to outputs for certain test inputs and in terms of a few fundamental properties, (e.g., realizability, stability, fidelity, and sensitivity to parameter variations); (3) the objective of system analysis is to determine whether a proposed system will perform as intended and, if not, to determine why, so that appropriate modifications can be made. These ideas are introduced in Chap. 1 and emphasized throughout the remainder of the book.

The first five chapters treat analog (continuous-time) systems. Chapter 1 describes elementary signals, elementary systems, block diagrams, and fundamental properties of systems. Chapter 2 treats time-domain methods (impulse response and convolution) for linear stationary systems, Chap. 3 treats sinusoidally excited systems, Chap. 4 treats frequency-domain methods (Fourier series and the Fourier integral), and Chap. 5 treats the Laplace transformation (both two-sided and one-sided).

Chapters 6 to 8 treat digital (discrete-time) systems. Chapter 6 describes analog-to-digital and digital-to-analog conversion, elementary digital systems, and fundamental properties of digital systems. Chapter 7 describes time- and frequency-domain methods for analysis of linear shift-invariant digital systems, and Chap. 8 treats the z transformation. Chapters 7 and 8 also introduce digital-filter design as a practical application of principles and techniques of digital system analysis.

Chapter 9, which provides an introduction to simulation and computer-aided analysis, can be studied in parallel with Chaps. 1 to 8. Section 9-1 provides an introduction to simulation using CSMP. Subsequent sections describe more advanced applications of simulation and also treat several other topics in computer-aided analysis, including numerical convolution, finding characteristic roots, finding partial-fraction coefficients, using fast Fourier transforms, and designing digital filters. Emphasis is on using existing software rather than developing algorithms.

I chose to treat analog and digital systems separately for two reasons: (1) In many curricula, as in ours, there is only one required course in systems analysis. I have found it impossible to carry a parallel treatment of analog and digital systems far enough in one semester to provide much of value to students who do not take a second course in system analysis. (2) I have found from my experience that pedagogical advantages of a parallel treatment are more apparent than real. Indeed, a parallel treatment fosters more confusion than insight; students become so concerned with parallels between analog and digital systems that they lose sight of more important relations between time-domain, frequency-domain, and complex-plane descriptions of signals and systems.

I chose to treat analog systems first for three reasons: (1) Most engineering applications of digital systems are found in analog systems (e.g., control systems, telephone networks, sonars, radars, and instrumentation systems). Intelligent design of digital systems for those applications requires some knowledge of analog systems. (2) Treating analog systems first makes use of and reinforces material just learned (in circuits and calculus)—thus reinforcing it before it is forgotten. (3) Students who take only one course in systems and subsequently specialize in other areas benefit more from studying differential equations, convolution integrals, the Fourier integral, and the Laplace transformation than by exposure to analogous discrete-time topics.

A few other comments on content and organization of the book are in order. State-space methods are omitted entirely because most juniors in electrical engineering have neither the background, the inclination, nor the need for a meaningful treatment of those methods. Simple nonlinear systems are treated at an appropriate level. Some exposure to nonlinear systems appears essential because virtually every practical system contains at least one nonlinear element and because study of nonlinear systems promotes deeper understanding of linearity and of limitations on linear models. Finally, signals and system parameters are treated throughout as dimensioned quantities. This makes the subject less abstract, simplifies practical application of principles, promotes insight, and gives students a powerful method for checking correctness of relations and reasonableness of numerical results.

I am truly grateful for the quantity and quality of help and encouragement given by students, colleagues, staff, reviewers, friends, and family. My wife, Robin, and my son, Jack, have been understanding beyond belief. My students have given encouragement and much constructive criticism. The present and two past heads of our department, Nino Masnari, Larry Monteith, and George Hoadley, have provided the best environment imaginable for me and my project. Larry Monteith, Russell Pimmell, and Kenneth Williams taught from my notes and made many valuable suggestions. Kenneth Williams wrote many of the computer programs used to produce figures and verify examples. He also read the manuscript carefully and eliminated many errors. Sande Maxim and Nancy Tyson typed many revisions of the manuscript with perfection and cheerfulness. I appreciate the helpful comments of several competent and conscientious reviewers, including Don Childers, Steve Director, David Fisher, Syed Nasar, Ronald Rohrer, Lee Rosenthal, Andy Sage, Ron Schaffer, and Michael Silevitch. I took their suggestions to heart, and the book is better for it. I also wish to express my thanks to L. E. Schoonmaker, who was a friend indeed; to Charley Black and Bud Flood, who taught me a lot; and to Andy Sage, who was an inspiration when inspiration was scarce. Finally, I owe special thanks to Sy Matthews, whose careful reading and thoughtful criticism are responsible for much of what is good about the style and pedagogy of the text.

T. H. Glisson

CONTENTS

CHAPTER

ONE

ELEMENTS OF SYSTEM ANALYSIS

The principal ideas introduced in this chapter are definitions of signal and system, definitions of five important signal models, the concept of a transfer characteristic, definitions of several important elementary systems, use of block diagrams for describing systems, and five important properties of systems.

In studying this chapter the reader may find it helpful to think of system analysis as a generalization (or analog) of circuit analysis. Circuit analysis deals with relations between voltages and currents. System analysis deals with relations between signals, which may be voltages, currents, temperatures, pressures, or other physical quantities that vary with time. Circuits are described by circuit diagrams, which are interconnections of idealized circuit elements (resistance, capacitance, inductance, and sources). Systems are described by block diagrams, which are interconnections of idealized elementary systems. Objectives of circuit analysis are to obtain and interpret relations between voltages and currents in an electric circuit. Objectives of system analysis are to obtain and interpret relations between signals in a system.

1-1 INTRODUCTION

We define signal, system, and system analysis; we discuss objectives of system analysis; and we describe some conventions regarding dimensions, units, and notation used in this book.

1-1A Signals and Systems

A *signal* is a physical (measurable) quantity that varies with time.† Examples are voltage across terminals of an electric circuit and temperature at a point in space. A

†In general, a signal may be a function of time and position, e.g., voltage on a long transmission line. Also, a signal may be a vector, e.g., electric field strength near an antenna. In this book a signal is a scalar function of time.

signal is represented as a function of time t, for example, $x(t)$. A *system* is a cause-and-effect relation between two or more signals. Signals identified as causes are called *inputs* or *excitations*. Signals identified as effects are called *outputs* or *responses*. For example, an audio amplifier can be regarded as a system whose input is voltage across the phono terminals and whose output is voltage across the speaker terminals.

Systems are represented by *block diagrams* (Fig. 1-1). The box represents a system, arrows entering the box represent inputs (excitations), and arrows leaving the box represent outputs (responses). A mathematical description of the system (of the relations between inputs and outputs) often is given in the box. For example, an audio amplifier can be represented as shown in Fig. 1-2. An input (a voltage across the phono terminals) is denoted by $x(t)$, the corresponding output (the voltage across the speaker terminals) is denoted by $y(t)$, and the relation between input and output is given in the box as $y(t) = Kx(t)$, where K denotes gain of the amplifier.

1-1B System Analysis

System analysis is the separation of systems into components for further study, which usually consists of examining the influence of one or more components on system performance. For example, an audio system might be separated into three components, as shown in Fig. 1-3, where $v(t)$ is motion of the phonograph stylus, $w(t)$ is voltage at the phono terminals, $x(t)$ is voltage at the speaker terminals, and $y(t)$ is pressure at a point in front of the speaker. Further study might show that the speaker has the poorest performance of the three components, suggesting that the speaker must be improved or replaced if performance of the audio system is to be improved.

System analysis plays an essential role in designing systems for communication, process control, data acquisition and processing, power generation and distribution, and other applications. Since construction of such systems is costly, it is economically necessary to have some assurance that a proposed system will perform as intended before construction is begun. *The central problem of system analysis is to determine whether a proposed system will perform as intended, and if not, why not, so the design can be corrected.*

Performance of a system usually is specified in terms of the output of the system for one or more test inputs. Consequently, the problem of system analysis as described above has two parts: (1) calculating the output of a system for one or more test inputs and (2), more important, *interpreting* the result of that calculation in terms of performance of the system. Test inputs used for specifying performance of systems are described in Sec. 1-2. Properties referred to in describing performance of systems are described in Sec. 1-5.

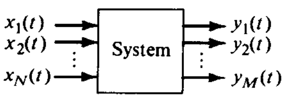

Figure 1-1 Block diagram for a system having inputs $x_1(t), x_2(t), \ldots, x_N(t)$ and outputs $y_1(t), y_2(t), \ldots, y_M(t)$.

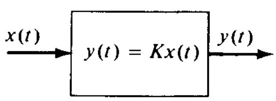

Figure 1-2 Block diagram for an audio amplifier.

Figure 1-3 Block diagram for an audio system.

1-1C Dimensions, Units, and Notation

A signal is a measurable quantity. A signal has a dimension and is specified at any instant by a number *and a unit*. Otherwise, it may be impossible to use the signal in a meaningful calculation. For example, suppose we are told that the signal at the terminals of a loudspeaker is $5 \cos \omega t$. What, exactly, have we been told? Not much; we do not know whether the signal is voltage, current, or gravitational field strength and we cannot, for example, calculate power delivered to the speaker even if we know the impedance of the speaker.

System parameters also are usually dimensioned quantities specified by a number *and a unit*. For example, the parameter K of a system whose output $y(t)$ for input $x(t)$ is given by $y(t) = Kx(t)$ must be expressed in the unit of $y(t)/x(t)$.

Dimensions and units are more than just extra baggage that must be carried along in order to get meaningful results. They provide powerful checks on correctness of relations and reasonableness of numerical values. A relation that is dimensionally incorrect is incorrect, period. A calculated current of 10^5 amperes through a loudspeaker is unreasonable, indicating that an error has been made in the calculation.

The SI system† of units is used in this book. Dimensions, SI units, and prefixes used in this book are given in Tables 1-1 and 1-2. Consistent use of certain symbols is helpful to students and practicing engineers alike. In this book we abide by certain conventions insofar as possible and reasonable. These conventions are pointed out where the need for them arises.

†The abbreviation SI is for the French Système Internationale d'Unités. An excellent discussion of dimensions and units is given in Kraus and Carver, chap. 1 and app. A-1. (References are listed in Appendix G.)

Table 1-1 Dimensions and SI units

Dimension or quantity	Name of SI unit	Symbol for SI unit	Dimension or quantity	Name of SI unit	Symbol for SI unit
Acceleration	meter/second^2	m/s^2	Inductance	henry	H
Angle	radian	rad	Length†	meter	m
Angular frequency	radian/second	rad/s	Mass†	kilogram	kg
Angular acceleration	radian/second^2	rad/s^2	Moment (torque)	newton-meter	Nm
Capacitance	farad	F	Power	watt	W
Charge	coulomb	C	Pressure	newton/meter^2	N/m^2
Current†	ampere	A	Resistance	ohm	Ω
Energy (work)	joule	J	Temperature†	Kelvin	K
Frequency	hertz	Hz	Time†	second	s
Force	newton	N	Velocity	meter/second	m/s
Impedance	ohm	Ω	Voltage	volt	V

†Fundamental unit in SI system.

Table 1-2 Some SI prefixes

Prefix	Abbreviation	Magnitude
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

1-2 ELEMENTARY SIGNALS

In this section we define signal models for a step, a rectangular pulse, an impulse, a sinusoid, and an exponential pulse. These five elementary signals, particularly the step and the sinusoid, are used widely as test inputs for specifying system performance. They are also used in mathematical descriptions of more complicated signals.

1-2A Step

The *unit step function* is denoted by $u(\alpha)$ and defined by†

$$u(\alpha) = \begin{cases} 0 & \alpha \leq 0 \\ 1 & \alpha > 0 \end{cases} \tag{1-1}$$

Figure 1-4 shows a graph of $u(\alpha)$ versus α. The *step signal* of Fig. 1-5 is described by

†The unit step function $u(\alpha)$ often is defined to be unity for $\alpha = 0$ and sometimes is defined to be $\frac{1}{2}$ for $\alpha = 0$. We prefer $u(\alpha) = 0$ because this simplifies using step functions to describe signals that change abruptly (but continuously) in response to an event that occurs at a specified time, e.g., closing a switch at $t = 0$.

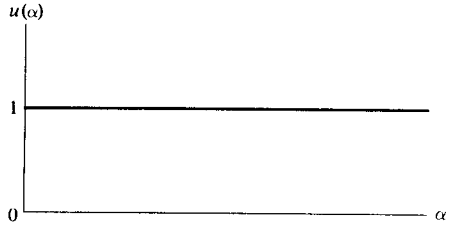

Figure 1-4 Unit step function.

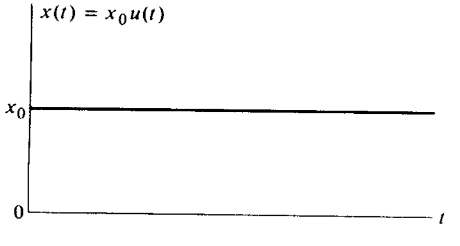

Figure 1-5 Step signal.

$$x(t) = x_0 u(t) \tag{1-2}$$

where t is time and x_0 is the *amplitude* of the step. Time t may be expressed in any convenient unit because the value of $u(t)$ depends only on whether t is positive or not. Amplitude x_0 is expressed in the unit of $x(t)$.

Example 1-1 In the circuit of Fig. 1-6 the switch is moved from contact A to contact B at $t = 0$. The voltage $v(t)$ is given by

$$v(t) = v_0 u(t)$$

where $v_0 = 5$ V.

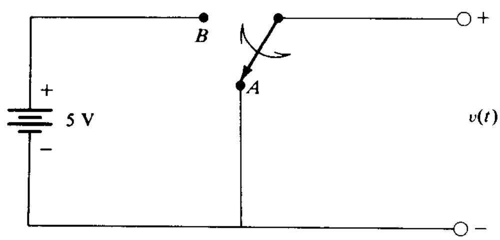

Figure 1-6 Circuit of Example 1-1.

1-2B Rectangular Pulse

The *rectangular function* is denoted by $r(\alpha)$ and defined by

$$r(\alpha) = \begin{cases} 1 & 0 < \alpha \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{1-3}$$

Figure 1-7 shows a graph of $r(\alpha)$ versus α. The *rectangular pulse* of Fig. 1-8 is described by

$$x(t) = x_0 r\left(\frac{t}{\tau}\right) \tag{1-4}$$

where x_0 is the *amplitude* of the pulse and τ is the *duration* of the pulse. The unit of x_0 is the unit of $x(t)$, and the unit of τ is the unit of t.

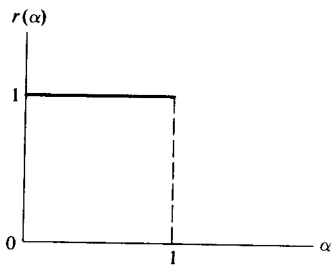

Figure 1-7 Rectangular function.

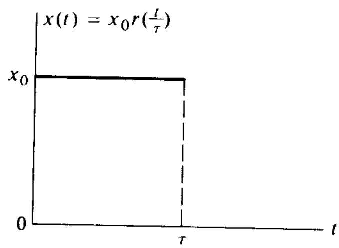

Figure 1-8 Rectangular pulse.

Example 1-2 In the circuit of Fig. 1-6 the switch is moved from contact A to contact B at $t = 0$ and from contact B back to contact A at $t = 10\ \mu s$. The voltage $v(t)$ is given by

$$v(t) = v_0 r\left(\frac{t}{\tau}\right)$$

where $v_0 = 5$ V and $\tau = 10\ \mu s$.

1-2C Impulse

The *delta function*† is denoted by $\delta(\alpha)$ and defined‡ by

$$\delta(\alpha) = \frac{du(\alpha)}{d\alpha} \tag{1-5}$$

where $u(\alpha)$ is the unit step function. The step function $u(\alpha)$ is dimensionless, and the operator $d/d\alpha$ has the dimension of α^{-1}; consequently the delta function $\delta(\alpha)$ has the dimension of α^{-1}.

The two most important properties of the delta function are expressed by the relations

$$\delta(\alpha) = 0, \qquad \alpha \neq 0 \tag{1-6}$$

and

$$\int_{-\infty}^{\infty} \delta(\alpha)\, d\alpha = 1 \tag{1-7}$$

Equation (1-6) follows from (1-5) because the derivative (the slope) of the unit step function $u(\alpha)$ is zero except for $\alpha = 0$. Equation (1-7) is derived from (1-5) as follows:

$$\int_{-\infty}^{\infty} \delta(\alpha)\, d\alpha = \int_{-\infty}^{\infty} d[u(\alpha)] = u(\infty) - u(-\infty) = 1 - 0 = 1$$

According to (1-6) and (1-7), the delta function is nonzero at only a single point ($\alpha = 0$), yet it has unit area. This peculiar property will become more understandable in applications. For now, it is sufficient to think of the delta function as a spike having nearly zero width, nearly infinite amplitude, and unit area. The delta function is represented graphically by an arrow, as shown in Fig. 1-9.

An *impulse* $x(t)$ is described by

$$x(t) = \mathbf{a}\delta(t) \tag{1-8}$$

and is represented graphically as shown in Fig. 1-10. The quantity $\mathbf{a}$ in (1-8) is called the *strength* of the impulse $x(t)$. The unit of $\delta(t)$ is that of t^{-1}. The unit of strength $\mathbf{a}$ is that of $tx(t)$. For example, if $x(t)$ is voltage in volts and t is time in milliseconds, the appropriate unit for $\mathbf{a}$ is volt-milliseconds (Vms). The strength $\mathbf{a}$ is *not* the amplitude (height) of the impulse $x(t)$; it is the *area* bounded by the impulse and the time axis because, from (1-7),

†Also called the dirac delta, after the English physicist P. A. M. Dirac (1902–), and the unit impulse. We reserve the term "impulse" for a *signal* whose amplitude is given by a delta function of time t [see (1-8)].

‡This definition has been known to cause apoplexy in mathematicians. Nonetheless, its consequences are consistent with those of a rigorous treatment, and it allows us to avoid a great deal of tedious mathematics.

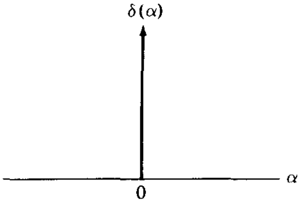

Figure 1-9 Graphical representation of a delta function.

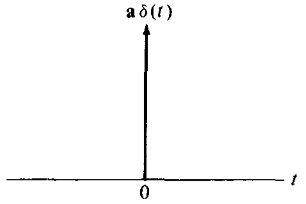

Figure 1-10 Graphical representation of an impulse.

$$\int_{-\infty}^{\infty} \mathbf{a}\delta(t)\,dt = \mathbf{a}\int_{-\infty}^{\infty} \delta(t)\,dt = \mathbf{a} \tag{1-9}$$

Example 1-3 A billiard ball at rest is struck by a cue ball whose momentum before the collision is p_0. After the collision the cue ball is at rest. Describe the force on the cue ball as a function of time.

SOLUTION We assume that the collision is instantaneous and that it occurs at $t = 0$. The momentum of the cue ball is given by

$$p_c(t) = p_0[1 - u(t)]$$

By Newton's law† the force on the cue ball is given by

$$f_c(t) = \frac{dp_c(t)}{dt} = -p_0\delta(t)$$

Note that this result is dimensionally correct because the dimension of momentum p_0 is force × time and the dimension of $\delta(t)$ is time^{-1}

1-2D Sinusoid

The *sinusoidal signal* of Fig. 1-11 is described by

$$x(t) = x_0 \cos \frac{2\pi t}{T} \tag{1-10}$$

†Force is rate of change of momentum; thus $f = dp/dt$, where f is force, p is momentum, and t is time. For $p = mv$, where m is a fixed mass and v is velocity, the relation $f = dp/dt$ becomes the familiar $f = ma$, where a is acceleration.

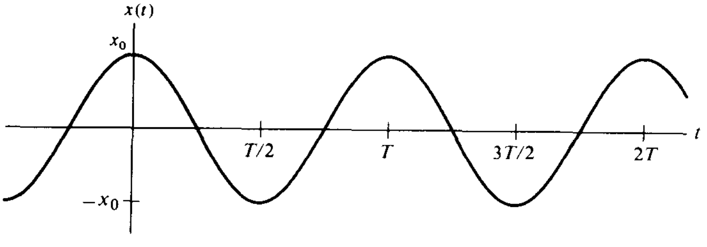

Figure 1-11 Sinusoidal signal described by (1-10).

where x_0 [unit of $x(t)$] is the *peak amplitude* of the sinusoid and T (unit of t) is the *period* of the sinusoid.

In engineering a sinusoid usually is described in terms of *frequency* f, defined by

$$f = \frac{1}{T} \tag{1-11}$$

or in terms of *angular frequency* ω, defined by

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{1-12}$$

Thus, the sinusoid of (1-10) is described by

$$x(t) = x_0 \cos 2\pi f t \tag{1-13}$$

or by

$$x(t) = x_0 \cos \omega t \tag{1-14}$$

The SI unit of frequency f is the hertz (Hz), with 1 Hz = 1 s^{-1}. The SI unit of angular frequency ω is the radian per second with 2π rad/s = 1 Hz.

Example 1-4 The sinusoid of Fig. 1-12 is described by

$$i(t) = i_0 \cos 2\pi f t$$

where $f = 1/0.002 = 500$ Hz and $i_0 = 50$ mA.

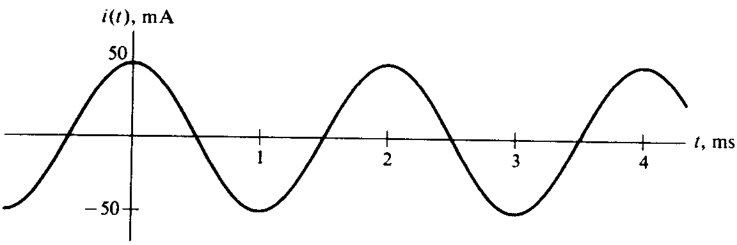

Figure 1-12 Sinusoidal signal of Example 1-4.

1-2E Exponential Pulse

The *exponential pulse* of Fig. 1-13 is described by

$$x(t) = x_0 e^{-t/\tau} u(t) \tag{1-15}$$

The quantity x_0 [unit of $x(t)$] is called the *initial amplitude* of the pulse. It is the amplitude of the exponential for $t = 0^+$.† The quantity τ (unit of t) is called the *time constant* of the exponential. In any interval of duration $n\tau$ the amplitude of the exponential pulse of (1-15) decreases by the factor e^{-n}; that is,

$$x(t + n\tau) = e^{-n} x(t) \tag{1-16}$$

†The symbol 0^+ (0^-) denotes a very small positive (negative) time; thus $x(0^+)$ is the value of $x(t)$ immediately after $t = 0$.