ALM 3

Advanced Lectures in Mathematics

Computational Conformal Geometry

Xianfeng David Gu • Shing-Tung Yau

高等教育出版社
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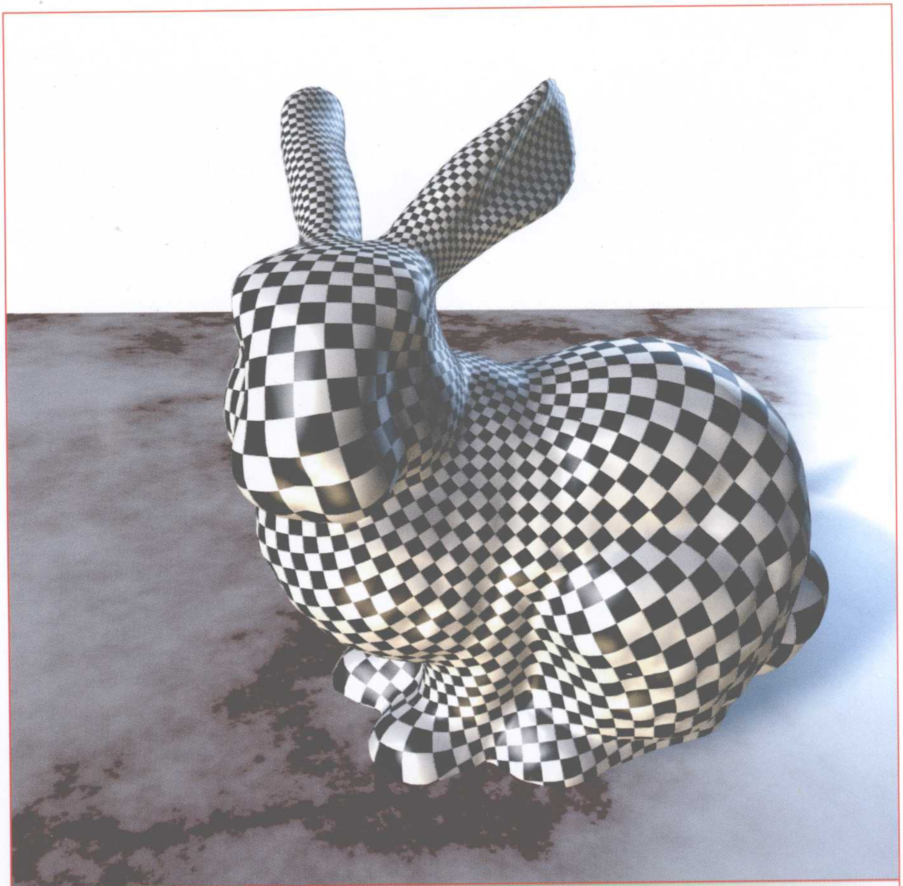
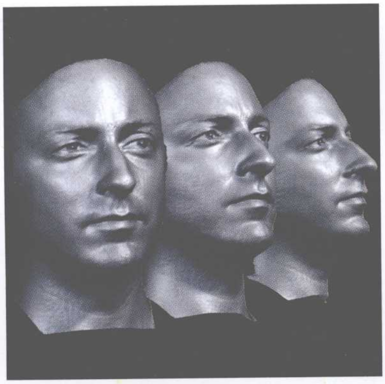
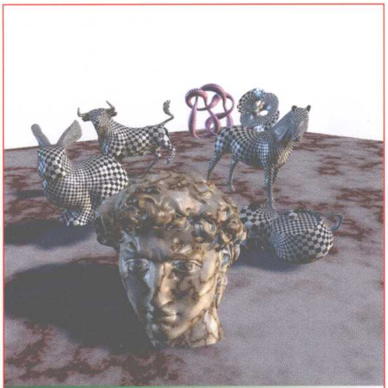


Fig. 1.1. Visualization of the conformal structure of the Stanford bunny surface.



A human face surface scanned by 3D scanner made by Geometric Informatics Inc.

Fig. 1.11. Human face surfaces with different expressions scanned by 3D scanner made by Geometric Informatics Inc.



Visualization of the conformal structures of the surfaces.

Fig. 1.2. Conformal Mappings.

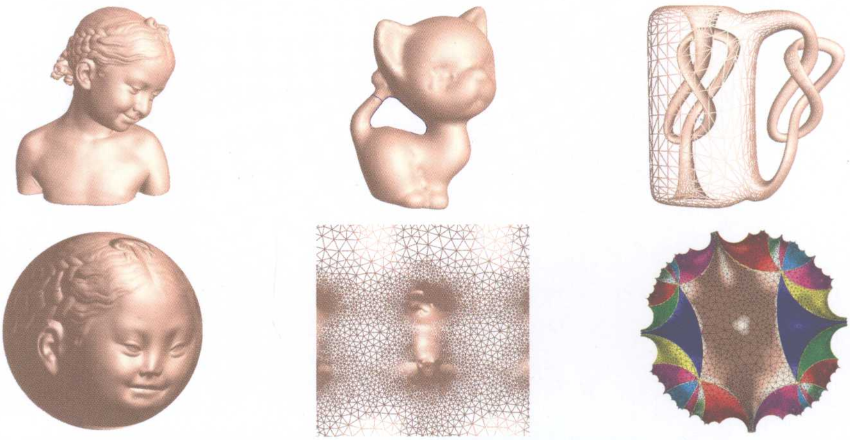
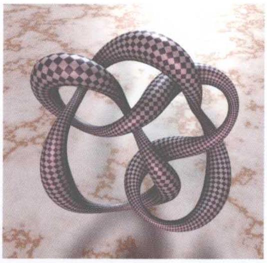


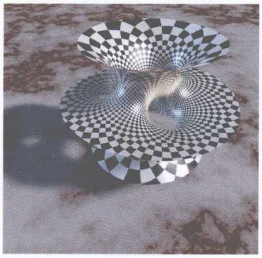
Fig. 1.8. Riemann uniformization theorem. Every closed surface has a Riemannian metric, which is conformal to the original metric and induces constant Gaussian curvature +1,0 or -1. Their universal covering spaces can be isometrically embedded into the sphere, the plane or the hyperbolic space.

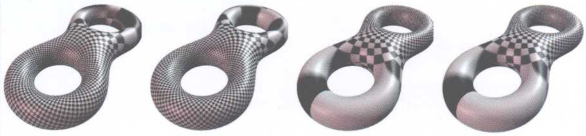
Fig. 1.38. Hyperbolic structures of high genus surfaces.



The conformal structure of a knot surface.

Harmonic 1-form and their conjugates on a genus two surface.

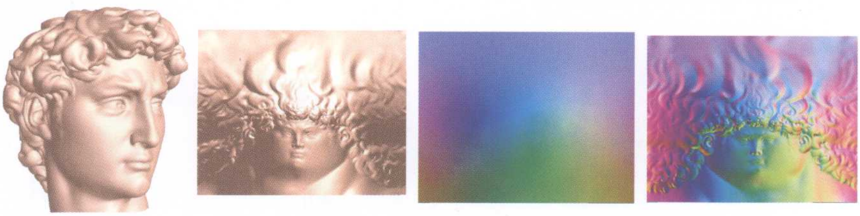




Holomorphic 1-form basis of a genus two surface.

Fig. 5.3. The conformal structure of the Costa minimal surface.

Fig. 10.2. The human face is double covered to form a topological sphere, then the double covering surface is conformally mapped to the unit sphere.



(a) Original Surface (b) Conformal Parameterization (c) Geometry Image (d) Normal map

Fig. 1.30. Geometry image of Michelangelo's David head model.

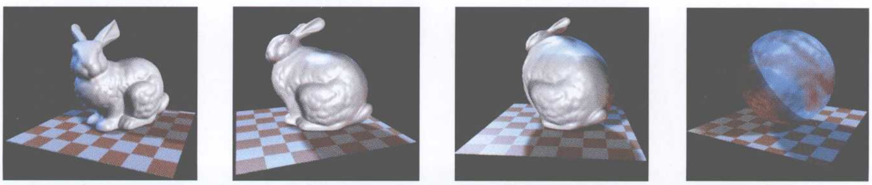


Fig. 1.28. Geometric morphing of the Standford bunny surface to the unit sphere.

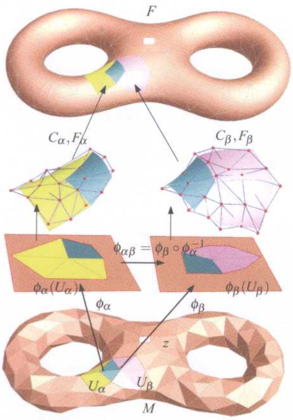


Fig. 1.40. Framework of manifold spline.

Fig. 1.42. Manifold Spline surface.

Spherical harmonic map.

Spherical harmonic map.

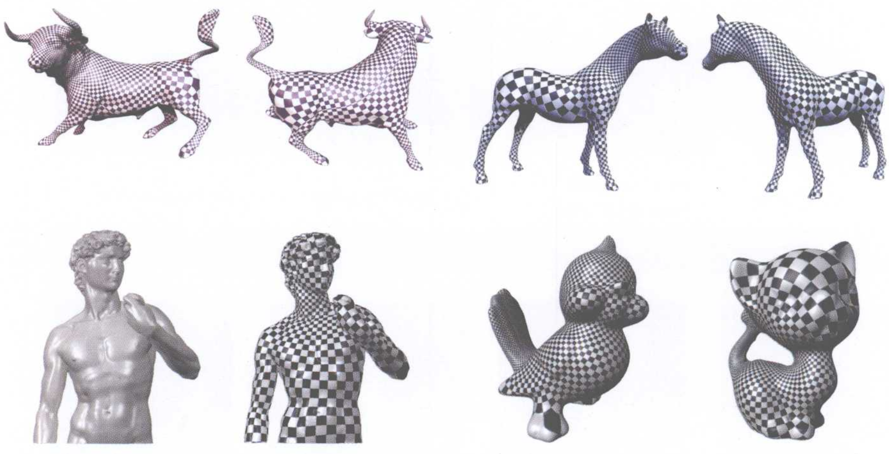


Fig. 11.5, 1.6, 1.7,. Conformal structures of surfaces.

ADVANCED LECTURES IN MATHEMATICS

Dedicated to all mathematicians and computer scientists.

Preface

Conformal geometry is in the intersection of many fields in pure mathematics, such as Riemann surface theory, differential geometry, algebraic curves, algebraic topology, partial differential geometry, complex analysis, and many other related fields. It has a long history in pure mathematics, and is an active field in both modern geometry and modern physics. For example, the conformal fields in super string theory and modular space in theoretic physics are research areas with very fast developments.

Recently, with the rapid development of three dimensional digital scanning technology, computer aided geometric design, bio-informatics, and medical imaging, more and more three dimensional digital models are available. The need for effective methods to represent, process, and utilize the huge amount 3D surfaces has become urgent. Digital geometric processing emerges as an inter-disciplinary field, combining computer graphics, computer vision, visualization, and geometry.

Computational conformal geometry plays an important role in digital geometry processing. It has been applied in many practical applications already, such as surface repairing, smoothing, de-noising, segmentation, feature extraction, registration, re-meshing, mesh spline conversion, animation, and texture synthesis. Especially, conformal geometry lays down the theoretic foundation and offers rigorous algorithms for surface parameterizations. Computational conformal geometry is also applied in computer vision for human face tracking, recognition, expression transfer; in medical imaging, for brain mapping, virtual colonoscopy, data fusion; in geometric modeling for constructing splines on manifolds with general topologies.

The fundamental reason why conformal geometry is so useful lies in the following facts:

- Conformal geometry studies conformal structure. All surfaces in daily life have a natural conformal structure. Therefore, the conformal geometric algorithms are very general.
- Conformal structure of a general surface is more flexible than Riemannian metric structure and more rigid than topological structure. It can handle large deformations, which Riemannian geometry cannot efficiently deal with; it preserves a lot of geometric information during the deformation, whereas topological methods lose too much information.
- Conformal maps are easy to control. For example, the conformal maps between two simply connected closed surfaces form a 6-dimensional space, therefore by fixing three points, the mapping is uniquely determined. This fact makes conformal geometric methods very valuable for surface matching and comparison.

- Conformal maps preserve local shapes, therefore it is convenient for visualization purposes.
- All surfaces can be classified according to their conformal structures, and all the conformal equivalent classes form a finite dimensional manifold. This manifold has rich geometric structures, and can be analyzed and studied. In comparison, the isometric classes of surfaces form an infinite dimensional space, which is really difficult to deal with.
- Computational conformal geometric algorithms are based on elliptic partial differential equations, which are easy to solve and the process is stable. Therefore, computational conformal geometry methods are very practical for real engineering applications.
- In conformal geometry, all surfaces in daily life can be deformed to three canonical spaces: the sphere, the plane, or the disk (the hyperbolic space). In other words, any surface admits one of the three canonical geometries: spherical geometry, Euclidean geometry, or hyperbolic geometry. Most digital geometric processing tasks in three dimensional space can be converted to the task in these two dimensional canonical spaces.

The major goals for writing this book are twofold. First, we want to introduce the beautiful theories of conformal geometry to general audiences, and make the elegant conformal structures better appreciated. The major concepts in conformal geometry are profound and abstract, which mainly existed in the imaginations of professional mathematicians. Our conformal geometric methods can compute those concepts explicitly on all kinds of surfaces in daily life, and display them using modern computer graphics and visualization technologies. Therefore, the students can see them, sense them, and accumulate intuition. Professional mathematicians can design experiments and use computers to help their exploration.

Furthermore, we would like to introduce the practical conformal geometric algorithms, and make them easily accessible for computer scientists and engineers. Therefore, the whole book is written to use less abstract mathematical reasoning, but more intuitive explanations and hands on experience. Major concepts and theorems are visualized by figures and computational algorithms are given. Students can implement the algorithms by themselves and see the abstract concepts represented as data structures on computers and create the images reflecting various geometric structures.

The book has two parts. The first part focuses on the theoretical foundations. It covers algebraic topology, differential exterior calculus, differential geometry, Riemann surface theory, surface Ricci flow, and general geometric structures. All of this knowledge is required for doing research in computational conformal geometry. Most of these topics are elementary, and some advanced topics are briefly touched with thorough references.

The second part focuses on computational algorithms, and is completely written in computer science language. It covers the computational algorithms for surfaces, which can be easily generalized to 3-manifolds. Then the algorithms on computing conformal structures for surfaces using various methods are explained in detail. Finally, algorithms for computing hyperbolic structure, and projective structure using Ricci flow method are examined. All algorithms are accompanied by pseudo-code, which is extremely easy to convert to programming language. We hope students can build the software system from scratch, and follow the book to implement various algorithms. The algorithms described in the book have already been applied in industrial applications.

The major content of the book is summarized from our research projects during the last several years. This textbook has been taught in graduate level courses in the Mathematics Department at Harvard University and the Computer Science Department at the State University of New York at Stony Brook. The theory part takes one semester, the computer science part takes one semester. The problem sets and programming exercises are valuable for students to improve their understanding and build their practical skill for developing geometric processing software. More teaching materials, coding samples and geometric surface data sets are available from the authors by requests. [1]

The first author is very grateful to all professors in the *Center for Visual Computing* at Stony Brook: Arie Kaufman, Hong Qin, Dimitris Samaras, Klaus Mueller and all faculty members in the Computer Science Department at Stony Brook University and the Computer Information Science and Engineering Department at the University of Florida. The first author wants to thank Steven Gortler, Hugues Hoppy, John Snyder, Julie Dorsey, Leonard McMillan, who led him to the graphics field; Tom Sederberg, Ralph Martin, Shi-Min Hu, Jörg Peters, who led him to the geometric modeling field; Tony Chan, Paul Thompson and Baba Vemuri, who led him to medical imaging field.

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Special thanks to Yalin Wang, who helped us to conduct the most advanced research in brain mapping, and adapted conformal geometric methods for real industrial applications. Special thanks to Song Zhang, who developed the 3D scanner with high accuracy and high speed, and used our conformal geometric technology for motion capture applications. Special thanks to Junho Kim, who developed an efficient random access mesh compression algorithm, Euclidean Ricci flow and volumetric harmonic forms. The first author also appreciates all the students in the Center for Visual Computing at Stony Brook.

Special thanks to Lance Cong, Alex Mohr, Luke Farrer, and Susan Frank, who contribute to the artwork of this beautiful textbook. Special thanks to Ralph Martin, Feng Luo and Joe Marino, who helped proof reading the manuscript.

Last but not least, we also want to thank both of our families, without their supports, this book could not be accomplished.

Stony Brook, New York, *David Gu*
Summer 2007 *Shing-Tung Yau*

[1] The color version of all of the figures, teaching materials, sample codes, and sample data sets can be found at http://www.cs.sunysb.edu/ gu/.

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