高等学校教材

大学物理简明教程(第二册)



A CONCISE COURSE IN UNIVERSITY PHYSICS

大学物理简明教程

VOLUME 2

(英文版)

电磁学

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主 编 王安安

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主　　编　王安安

编写人员　[illegible]　樊则宾　王安安

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主 编 王安安

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前　言

我们正处在一个高新技术飞速发展、科技信息量激增、知识更新加快、国际交流日益广泛的时代。我国的进一步改革开放，社会主义市场经济的建立都要求高校毕业生有更强的适应能力，在人才市场上，有较强的外语应用能力、交流型、综合型的毕业生供不应求。在这种形势下，我们的高等教育正向着重视素质教育的方向转变，而素质与能力是密切相关的，素质的培养要以一定的知识和能力为基础，其中包括独立获取知识的能力。勿庸置疑，直接用外语为工具获取知识、进行交流的能力是人才素质的一个重要方面。

然而，由于历史原因、文化背景、经济基础、外语教育模式和各类师资外语水平等诸多因素的影响，我们在应用外语进行教学方面的基础性工作十分薄弱。在普通高校本科生教育中，教材和教学过程基本上只使用中文这一单一语种，实际上已经制约了学生应用外语（主要是英语）获取知识能力的发展。为了改变这种现状，跟上时代的步伐，试用英文教材，使用英语进行教学的改革便应运而生了。

本教材的编写是编者主持的“试用英文物理教材”教改试点工作的继续，也是编者大学物理教学经验的总结。从国外引进的教材，虽有诸多优点，但在系统上与我国的大学物理教学基本要求不完全对应，为了满足师生对英文物理教材的需要，编写一套根据我国工科物理教学基本要求，顺应工科物理教学改革的形势，反映编者在多年物理教学实践中总结出来的教学方法与经验，与我们的学生在一、二年级的英文水平相适应的英文“简明物理学教程”的计划就提到日程上来了，这就是我组织编写这套教材的初衷。这套英文物理教材是1996年经国家教委批准列入正式出版计划的。本教材可供普通高等工科院校本科生物理课130～140学时使用。编者力求突出以下特点：

① 基本理论讲述方法简明、准确、易学易教。

② 为使学生读后达到基本训练的要求，配备了适量的例题、思考题和习题。除自编的题目及我校物理教研室常用的题目外还吸收了一部分英文教材中的优秀题目。

③ 每册最后编入一个阅读材料，选自优秀的英文原版教材供学生自学，内容力求反映物理学的新发展。不另编写绪论，用第一册的阅读材料“物理学是什么”代之。

④ 除物理学名词外，绝大多数英文词汇包含在大学英语四级词表内，全部词汇几乎都在英语六级词表内。通过三册书的教学，不但能达到教学基本要求，而且学生直接使用英文基础课教材及专业课教材学习的能力将大大提高。

⑤在第三册的部分插图中使用了编者在教学、科研中积累的素材。

全书21章，具体编写分工如下：

第一册：力学部分（第1～5章）由王安安编写，分子运动论和热力学基础（第6、7章）由陶亚琴和王安安编写。

第二册：电学及稳恒电流的磁场（第8～10章）由吴光敏编写，磁介质和电磁感应（第11、12章）由樊则宾编写，麦克斯威方程组（第13章）由王安安编写。

第三册：机械振动（第14章）由伏云昌编写，机械波（第15章）由王安安和陈劲波编写。电

磁振荡与电磁波(第 16 章)由吴光敏编写,波动光学部分(第 17 章)中干涉与衍射由陈劲波编写,光的偏振由樊则宾编写。近代物理部分(第 18 ~ 21 章)由伏云昌编写。

第一册绝大部分插图由刘富华用计算机绘制,其余插图由李俊昌教授绘制,封面也由李俊昌设计,谨此致以诚挚谢意。

编写大学英文物理教材是一种大胆的尝试,由于编者水平有限,错误疏漏之处在所难免,希望同行和读者批评指正。我们相信这本教材的出版将对物理教学的现代化和物理教学与国际接轨作出有益的贡献。

编者

1997 年 5 月于昆明

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Contents

Chapter 8
Electrostatic Field in Vacuum

From this chapter we will discuss the electromagnetic interaction that is one of four essential interaction in nature. The constructions of all, from the giant star to the mountains, rivers, forests and living things, are associated with electromagnetic interaction. The application achievement of electromagnetism can be found everywhere, from the modern science and technique to the people's daily live. In this chapter we want to discuss mainly to the property of the electric field set up by the static charge relative to a observer in vacuum. She contents involve the interaction between the charges—Cloulomb's law, for description the property of the electric field to introduce two physical quantity—electric field and electric potential, and the property of the electrostatic field—Gauss' law, and the relationship between electric field and electric potential.

§ 8-1 Coulomb's Law

1. Electric charges

(1) Two kinds of electric charges

The electric phenomenon was known to the ancient Greeks as long ago as 600 B.C. It was found that amber, rubbed by wool or cat's fur, would attract small objects, like bits of lint or dust. Nowadays, rubber rod and fur are commonly used in demonstration. The rubber rod, after rubbing with fur, will acquire the property of attracting light objects. In order to describe this property, we say that rubber rod is *electrified*, or possesses an electric charge. If we hold an electrified rubber rod near two small and very light pith balls that are suspended near each other by fine silk threads, at first they will be attracted to the electrified rubber rob and will cling to it. But after the pith balls touch the rubber rod, they will be repelled by the rubber rod and will also repel each other. If we do similar experiment with a glass rod that has been rubbed with silk, it will give rise to the same result. The pith balls electrified by contact with such a rod are repelled not only by the glass rod but by each other. On the other hand, when a pith ball that has been in contact with electrified rubber is placed near one that has been in contact with electrified glass, the pith balls attract each other. We explain these facts by saying that rubbing a rod give it an electric charge and that the charges on the rubber rod and on the glass rod must be different. The pith balls become charged by virtue of their contact with two kind of rod. We are therefore led to the conclusion that there are *two kinds* of electric charges. One is called a *negative charge*, which is possessed by rubber rod after being rubbed with fur, and the other is called *positive charge*, which is possessed by glass rod after being rubber with silk. We can sum up these experiments by saying that *like charges repel each other and unlike charges attract each other*.

Electric effects are not limited to glass rod rubbed with silk or to rubber rod rubbed with fur. Any substance rubbed with any other under suitable conditions will become charged to some extent. By comparing the unknown charge with glass rod which had been rubbed with silk or rubber rod which had been rubbed with fur, it can be labeled as either positive or negative.

Matter as we ordinarily experience it can be regarded as composed of atoms in which there are three kinds of subatomic particles, the negatively charged electron, the positively charged proton, and the neutral neutron. The negative charge of the electron is of the same magnitude as the positive charge of the proton and no charges of smaller magnitude have ever been observed. The protons and neutrons form a nucleus which has a net positive charge due to the protons. The diameter of the nucleus, as roughly spherical, is of the order of magnitude 10^{-14} m. Outside the nucleus, at relatively large distances from it, are the electrons, whose number is equal to the number of protons within the nucleus. The SI unit of charge is the Coulomb (abbr. C). The charge carried by an electron or proton is called fundamental charge. This charge, to which we give the symbol e, has the magnitude 1.60×10^{19} C. It is one of the important constant of nature.

The modern view of bulk matter is that, in its normal or neutral state, it contains equal amounts of positive and negative charges (the total number of protons equals the total number of electrons). If two bodies like glass and silk are rubbed together, some of the electrons is transferred from one to the other, upsetting the electric neutrality of each, so that one body has an excess and the other a deficiency of that electrons. The experiment, a rubber rod is rubbed with fur and a glass rod is rubbed with silk, indicates that rubber and silk obtain some electrons and fur and glass rod loss some of their electrons. The "charge" of a body refers to its excess charge only. The excess charge is always a very small fraction of the total positive or negative charge in the body.

(2) Quantization of charge

In former times electric charge was thought of as a continuous fluid. The atomic theory of matter, however, has shown that fluids themselves, such as water and air, are not continuous but are made up of atoms and molecules. Experiment shows that the "electric fluid" is not continuous either but that it is made up of multiples of the fundamental e. To give a body an excess negative charge, we may add a number of electrons to neutral body, while a number of electrons from neutral body will result in an excess positive charge. Therefore, any physically existing charge q, no matter what its origin, can be written as

$$q = Ne; N = \pm 1, \pm 2, \pm 3 \cdots \qquad (8\text{-}1)$$

where N is a positive or a negative integer. This conclusion is called *quantization of charge*. The quantum of charge e is so small that the "graininess" of electricity does not show up in large-scale experiments, just as we do not realize that the air we breathe is make up of atoms. The charge of a body is generally considered as continuously, for it contains practically a tremendous number of charged particles.

(3) The conservation of charge

In the case of the glass rod rubbed with silk, electrons are transferred from the glass to the silk, giving the silk a net negative charge and leaving the glass rod with an equal positive charge. This suggests that rubbing does not create charge but only transfers it from one object to another, disturbing slightly the electrical neutrality of each during the process. We say that charge is conserved in the rubbing process. But in addition to friction there are some process in which the charged particles are created or vanished.

An interesting example comes about when an electron, whose charge is $-e$, and a positron, charge $+e$, are brought close to each other. The two particles may simply disappear, converting themselves into gamma rays, without any inner electric structure. The net charge is zero both before and after the event so that charge is conserved. Another example of charge conservation is found in β-decay. A neutron becomes a proton and an electron and releases, meanwhile, a neutron whose charge is zero. The resultant charge is zero both before and after this decay process. The principle of conservation of electric charge can be stated as follows:

In any interaction the net algebraic amount of electric charge remains constant.

Conservation of charge has been tested repeatedly in the realm of high-energy physics and has been found to hold in all circumstances, no exceptions have ever been found.

2. Coulomb's law

We have known that there is an interacting force between two charged bodies and the force being repel or connect with the kinds of the charge, positive or negative, carried by two bodies. Experiment showed that magnitude of the force is related to with not only the distance between two charged bodies but also the shapes and sizes of them. If the sizes of charged bodies are much smaller than their distance, the effect caused by the charge distribution on the bodies can be ignored. Then the magnitude of the force between two charged bodies depends only on their distance apart and the net charges. Coulomb's law holds for charged bodies whose sizes or spatial dimensions are much smaller than the distance between the bodies. These charged bodies are called *point charges*. This means that the electric charge resides on a geometric point.

Charles Augustin de Coulomb (1736—1806) measured electrical attractions and repulsion quantitatively and deduced the law that governs them. He found, utilizing a torsion balance, that the force of attraction or repulsion between two point charges is inversely proportional to the square of their distance apart. The force between charges depends also on the magnitude of the charges. Specifically, it is proportional to their product. The complete expression for the magnitude of the force between two point charges is

$$F = k\frac{q_1 q_2}{r^2} \tag{8-2}$$

where F is the magnitude of the force exerted on charge q_1 by charge q_2 separated by a distance r. By Newton's third law, this quantity equals the magnitude of the force exerted by charge q_1 on charge q_2. The k is a proportionality constant whose magnitude depends on the units in which F, q_1, q_2 and r are expressed. k is usually written in a more complex way as

$$k = \frac{1}{4\pi\varepsilon_0} \tag{8-3}$$

ε_0 is called the *permittivity of vacuum*. In SI unit its value turns out to be

$$\varepsilon_0 = 8.854187818 \times 10^{-12} \quad C^2/N.m^2 \tag{8-4}$$

We may write the Coulomb's law in vector form as

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}^3}\boldsymbol{r}_{12} \tag{8-5}$$

where $\boldsymbol{F}_{12}$ is the force exerted on q_2 by q_1, $\boldsymbol{r}_{12}$ is a displacement vector from q_1 to q_2, $\boldsymbol{F}_{21}$ which is the force exerted by q_2 on q_1 is the equal and opposite force of $\boldsymbol{F}_{12}$. Figure 8-1 shows the direction of the force of two like or unlike point charges interaction. It indicate that two like point charge repel and two unlike point charge attraction, and the force direction along the line connecting two charges.

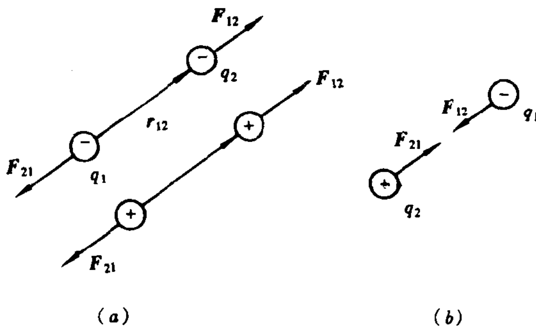

Fig.8-1

(a) Two like point charges repel each other;

(b) Two unlike point charges attract each other.

In SI unit system the dimensions of the length, mass, time and current are expressed by L, M, T, I. We can write the dimensions of the charge q and the permittivity ε_0 as IT and $I^2 L^{-3} T^4 M^{-1}$ by $q = It$ and Eq.8-4.

§ 8-2 The Electric Field

1. Concept of the electric field

If we place two point charges in the space, Coulomb's law precisely describes the force between the electric charges. But Coulomb's law offers no explanation for the force. To many scientists this situation was unacceptable. The forces with which they were familiar required contact between interacting objects, such as the contact of two colliding billiard balls. If a charge experienced a force, they reasoned, then there should be something acting directly at the location of the charge. We didn't feel the presence of any material connection between two point charges. We want to know "why" this is possible—it is an experimental fact that charged bodies behave in this way. The law explains nothing, however, about how one charge "senses" the distant presence of the other. If the charges are stationary we do not need to raise this question because we can solve all problems that arise if we know the magnitudes and the positions of charges. Suppose, however, that one charge suddenly moves a little closer to the second charge. According to Coulomb's law, the force on the second charge must increase. Speaking loosely, how does second charge "know" that the first charge has moved?

It had ever two view to answer above question. One, is called *action-at-a-distance* point of view, considered that the force acting between charged bodies was direct and instantaneous interaction. We can represent this view as

$$\text{Charge} \rightleftharpoons \text{Charge} \tag{8-6}$$

The other, is called *field* point of view, considered that there is an electric field as an intermediary between charges, that is, one charge sets up an electric field in space around itself and that field acts on another charge, producing a force on it. This situation is completely symmetrical, each charge being immersed in a field associated with the other charge. This view can be represented as

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$$\text{Charge} \rightleftharpoons \text{Field} \rightleftharpoons \text{Charge} \qquad (8\text{-}7)$$

If the problem was only that of the forces between stationary charges, the field and the action-at-a-distance point of view would be perfectly equivalent. Suppose, however, that the one point charge suddenly accelerates to the other. How quickly does the still charge learn that other charge has moved and that the force which it experiences must increase? Field theory predicts that the still charge learns about the other's motion by field disturbance that emanates from the moving charge, traveling with the speed of light. The action-at-a-distance point of view requires that information about moving charge's acceleration be communicated instantaneously to the still charge: this is not in accord with experiment.

The electric field, as a kind of matter will bring about an interaction with the charges, and have behavior on two hand. One is that electric field exerts a force on the charges in space, and the other is that electric field do a work on charges during a displacement in the electric field. About the force we will define the electric field in this segment, and the work done on a charge by electric field will introduce the conception of the electric potential in section § 8-5.

2. Electric field

(1) Definition of the electric field

To define the electric field, we place a points charge q_0, called the test charge and assumed positive for convenience, at a point in space that is to be examined. If the electric field exists everywhere in space, this test charge would be exerted by the field on a force $\boldsymbol{F}$ which can be measured. Experiment shows that the magnitude and direction of the force depends on the position and magnitude of the q_0, that indicates the electric field is a vector. Figure 8-2 shows the situation that a force acts on the test charges which are at the point around a charge called field source charge, which sets up an electric field in space. If the test charge is fixed at a point, then the rate of the $\boldsymbol{F}/q_0$ is a invariable vector which is not relative to the magnitude of the test charge. Therefore we can use this rate to represent quantitatively the property of the electric field. Thus the electric field, with a symbol $\boldsymbol{E}$, at the point is defined as

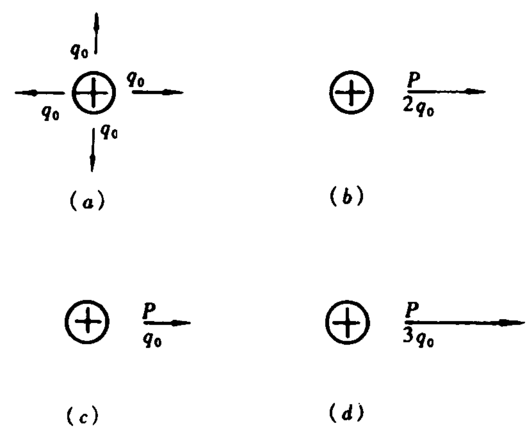

Fig. 8-2

(a) The test charges are around the field source charge

(b)、(c)、(d) The test charges q_0, $2q_0$, $3q_0$ are fixed at P point. The $\boldsymbol{F}/q_0$, $2\boldsymbol{F}/2q_0$, $3\boldsymbol{F}/3q_0$ are the invariable vector.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q_0} \qquad (8\text{-}8)$$

Here $\boldsymbol{E}$ is a vector because $\boldsymbol{F}$ is one, q_0 being a scalar. If the q_0 equals one unity, then $\boldsymbol{E} = \boldsymbol{F}$. We can say that the magnitude of the electric field in a point equals the force exerting on the unity charge at this point, and the direction of the electric field is the direction of the force when this charge is positive. The

SI unit for the electric field is the Newton/Coulomb (N/C). Its dimension is $I^{-1}LMT^{-3}$.

If the field source charges that set up the electric field that we are examining were fixed in position, we could use a test charge of any magnitude. If the field source charges are not fixed, however, the test charge should be as small as possible. Otherwise, it might cause the field source charges to change their positions. Because charge is quantized, we cannot, of course, use a test charge smaller than the elementary charge e.

If a point charge q is placed in space in which the electric field $\boldsymbol{E}$ has been known, the force $\boldsymbol{F}$ that electric field exerts on the point charge is represented by

$$\boldsymbol{F} = q\boldsymbol{E} \tag{8-9}$$

The force on a negative charge, such as an electron, is opposite to the direction of the electric field.

(2) The electric field of a point charge

Given a charged object, what electric field does it set up at nearby points? We look first at the case of a single point charge q, imagine a test charge q_0 to be place at P. Let $\boldsymbol{r}$ be the vector from the charge q to the P. The force on the test charge q_0, by Coulomb's law, is

$$\boldsymbol{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2} \cdot \frac{\boldsymbol{r}}{r} \tag{8-10}$$

and hence the electric field at P is, from the Eq. 8-8.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q_0} = \frac{q}{4\pi\varepsilon_0 r^2} \cdot \frac{\boldsymbol{r}}{r} \tag{8-11}$$

Since $\boldsymbol{r}/r$ is a unit vector, the magnitude of $\boldsymbol{E}$ is

$$E = \frac{q}{4\pi\varepsilon_0 r^2} \tag{8-12}$$

If q is positive, the direction of $\boldsymbol{E}$ is the same as that of the vector $\boldsymbol{r}$ (away from q), and if q is negative, it is opposite to $\boldsymbol{r}$ (toward q).

3. The Superposition principle of the electric field

If there is a group of charges in space, the resultant electric field $\boldsymbol{E}$ at a given point in space is determined by all these point charges. Placing a test charge q_0 at a given point, we can find the resultant electric field force exerted on the test charge. Since the force obeys a superposition principle, the electric field does also. We have

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \cdots = \sum_i \boldsymbol{F}_i$$

From the definition of the electric field, Eq. 8-8, we obtain

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q_0}; \boldsymbol{E}_1 = \frac{\boldsymbol{F}_1}{q_0}; \boldsymbol{E}_2 = \frac{\boldsymbol{E}_2}{q_0}; \boldsymbol{E}_3 = \frac{\boldsymbol{F}_3}{q_0}; \cdots$$

then yielding

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 + \cdots = \sum_i \boldsymbol{E}_i \tag{8-13}$$

The sum is a vector sum, taken over all charges. The $\boldsymbol{E}_i$ is the field set up by the ith charge. Eq. (8-13) is called superposition principle of the electric field.

If a number of point charges $q_1, q_2, \cdots, q_i \cdots$, are at distances $r_1, r_2, \cdots, r_i \cdots$, from the given point

P. The resultant field $\boldsymbol{E}$ is, from the Eq. (8-13) and Eq. (8-11), the vector sum of the individual electric fields, that is

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\cdot\frac{\boldsymbol{r}_i}{r_i} \tag{8-14}$$

Example 8-1 Fig. 8-3 shows two charges of magnitude q but of opposite sign, separated by a distance l. We call this configuration an *electric dipole*. The vector $\boldsymbol{l}$, whose magnitude is l and direction points from $-q$ to $+q$, is called the *dipole arm*. The product $q\boldsymbol{l}$: which involves intrinsic properties of the electric dipole, is called the *electric dipole moment*, and is expressed with $\boldsymbol{p}$, that is

$$\boldsymbol{p} = q\boldsymbol{l} \tag{8-15}$$

Find the field $\boldsymbol{E}$ due to the dipole of Fig. 8-3 at point P_1, a distance y from the midpoint of the dipole on its central axis, and at point P_2, a distance x along the perpendicular bisector of the line joining the charges.

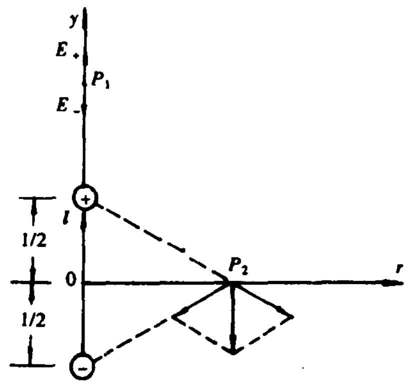

Fig. 8-3 The field of an electric dipole

Solution We first calculate the field at point P_1. Because both the point charges lie on the y axis, the electric field $\boldsymbol{E}$ at point P_1, and also the field E_+ and E_- due the separate charges that make up the dipole, must lie along the dipole axis, which we take to be the y axis. The superposition principle applies to electric field so that we can write for the magnitude of the field at P_1

$$E_{P_1} = E_+ - E_- = \frac{q}{4\pi\varepsilon_0 r_+^2} - \frac{q}{4\pi\varepsilon_0 r_-^2}$$

$$= \frac{q}{4\pi\varepsilon_0(y - l/2)^2} - \frac{q}{4\pi\varepsilon_0(y + l/2)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 y^2}\left[\left(1 - \frac{l}{2y}\right)^{-2} - \left(1 + \frac{l}{2y}\right)^{-2}\right] \tag{8-16}$$

Physically, we are usually interested in the electrical effect of a dipole only at distance which is large compared with the dimension of the dipole, that is, at distance such that $y \gg l$. At such a large distance, we have $\frac{l}{2y} \ll 1$ in Eq. 8-16. We can then expend the two quantities in the parentheses in that equation by the binomial theorem, obtaining

$$E_{P_1} = \frac{q}{4\pi\varepsilon_0 y^2}\left[\left(1 + \frac{l}{y} - \cdots\right) - \left(1 - \frac{l}{y} + \cdots\right)\right]$$

As an approximate result that holds at large distances, we then have

$$E_{P_1} \approx \frac{q}{4\pi\varepsilon_0 y^2}\cdot\frac{2l}{y} = \frac{ql}{2\pi\varepsilon_0 y^3} + \cdots$$

From the Eq. 8-15, we can write the field E_{P_1} in vector form

$$\boldsymbol{E}_{P_1} = \frac{\boldsymbol{p}}{2\pi\varepsilon_0 y^3} \tag{8-17}$$

Using the superposition principle, we can write for the vector of the field at P_2

$$\boldsymbol{E}_{P_2} = \boldsymbol{E}_1 + \boldsymbol{E}_2$$

where, from Eq.8-11,

$$E_1 = E_2 = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(\frac{l}{2})^2 + x^2}$$

The vector sum of $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ point vertically downward and has the magnitude

$$E_{P_2} = 2E_1 \text{ of } \cos\theta = 2E_2\cos\theta$$

where, from Eq.8-3 we have

$$\cos\theta = \frac{l/2}{[(l/2)^2 + x^2]^{1/2}}$$

Substituting the expressions $\cos\theta$ and E_1 into E_{P_2} yields

$$E_{P_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{ql}{[(\frac{l}{2})^2 + x^2]^{3/2}}$$

If $x \gg l$, we can neglect $l/2$ in the denominator, and from the Eq.8-15 we write this equation into vector form

$$\boldsymbol{E}_{P_2} = -\frac{\boldsymbol{p}}{2\pi\varepsilon_0 x^3} \tag{8-18}$$

Eq.8-17 and Eq.8-18 show that, if we measure the electric field of a dipole only at distant points, we can never find q and l separately, only their product. The field at distant points would be unchanged if, for example, q were doubled and l simultaneously cut in half.

4. The electric field set up by a continuously distributed charge

An electric charge is actually a collection of discrete charges such as electrons and protons. However, when we consider a large number of charges from a large distance away, the distribution of charges appears to be continuous and we can treat the discrete charges as a continuously distributed change.

Suppose a charged object on which the charge distributes continuously. The field set up at an point P, shown in Fig.8-4, can be computed by the superposition principle as before. The continuously distributed charge is regarded as being composed of many infinitesimal elements $\mathrm{d}q$. The field $\mathrm{d}\boldsymbol{E}$ due to each element at the point in question is then calculated, treating the elements as point charges. The field $d\boldsymbol{E}$ is given by

$$\mathrm{d}\boldsymbol{E} = \frac{\mathrm{d}q}{4\pi\varepsilon_0 r^2} \cdot \frac{\boldsymbol{r}}{r} \tag{8-19}$$

Where $\boldsymbol{r}$ is the vector from the charge element $\mathrm{d}q$ to the point P. The resultant field at P is then found from the superposition principle by integrating the field contributions due to all the charge elements

$$\boldsymbol{E} = \int \mathrm{d}\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\mathrm{d}q}{r^2} \cdot \frac{\boldsymbol{r}}{r} \tag{8-20}$$

The integration, like the sum in Eq.8-13, is a vector operation. We can resolve $\boldsymbol{E}$ into three components

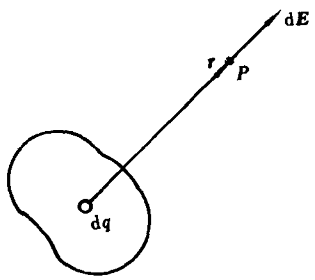

Fig.8-4 The field due to an element charge dq

and the Eq.8-20 becomes three scalar integrations. To evaluate the vector integral, we evaluate each of the three scalar integrals.

If we express the *volume charge density* with ρ (C/m^3), the dq being the charge distributing in the *volume element* dV equals the product ρdV. Then Eq.8-20 becomes

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho dV}{r^2}\cdot\frac{\boldsymbol{r}}{r} \qquad (8\text{-}21)$$

where the integral limit V is the region of all charge distribution. If the charge on the object is spread out on a surface or a line, the charge element can be written by $dq = \sigma ds$ or $dq = \lambda dl$, where σ is the *surface charge density* (C/m^2) and λ is the *linear charge density*, and ds is the *surface element* and dl is the *length element*. The Eq.8-20 can be written as

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\sigma ds}{r^2}\cdot\frac{\boldsymbol{r}}{r} \quad \text{or} \quad \boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int_l \frac{\sigma dl}{r^2}\cdot\frac{\boldsymbol{r}}{r} \qquad (8\text{-}22)$$

Example 8-2 The field of a ring of charge

Fig.8-5 shows a thin ring of radius R: charged to a constant linear charge density λ around its circumference. Find the electric field at point P, a distance z from the plane of the ring along its central axis.

Solution We break up the ring into charge elements that are small enough so that we can treat them as point charges. We will then find the electric field due to the ring by adding up the field contributions of all these charge elements.

Consider a differential element of the ring of length dl located at an arbitrary position on the ring in Fig.8-5. It contains an element of charge give by

$$dq = \lambda dl \qquad (8\text{-}23)$$

This element sets up a differential field $d\boldsymbol{E}$ at point P. We write the magnitude of $d\boldsymbol{E}$ from Eq.8-19 and Eq.8-23

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dl}{r^2} = \frac{\lambda dl}{4\pi\varepsilon_0(z^2+R^2)} \qquad (8\text{-}24)$$

Note that all charge elements that make up the ring are the same distance r from point P.

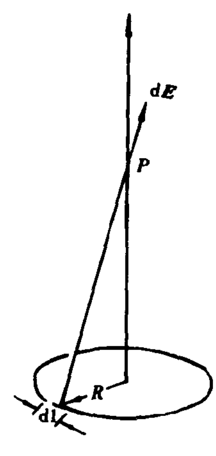

Fig.8-5 A uniform ring of charge

To find the resultant field at P we must add up, vectorially, all the field contributions $d\boldsymbol{E}$ made by the differential elements of the ring. From symmetry, we know that the resultant field $\boldsymbol{E}$ must lie along the axis of the ring. Thus, only the components of $d\boldsymbol{E}$ parallel to this axis need to be counted. Components of $d\boldsymbol{E}$ at right angle to the axis will cancel in pairs, the contribution from a charge element at any ring location being canceled by the contributiom from the diametrically opposite charge element. Thus: our vector integral becomes a scalar integral of parallel axial components.

The axial component of $d\boldsymbol{E}$ is $dE\cos\theta$. From Fig.8-5 we see that

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}} \tag{8-25}$$

Combing Eq.8-24 and Eq.8-25, we find

$$dE\cos\theta = \frac{\lambda z \mathrm{d}l}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \tag{8-26}$$

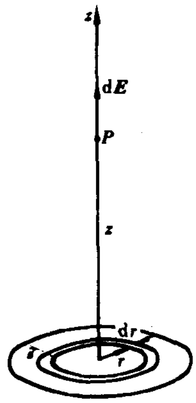

Fig. 8-6 The field of a charged disk

Except l is a variable, all other quantities in Eq. 8-26 have the same value for all charge elements. Thus, the integral need only for the dl elements

$$E = \int \mathrm{d}E\cos\theta = \frac{\lambda z}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int \mathrm{d}l = \frac{\lambda z \cdot (2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \tag{8-27}$$

in which the integral is simply $2\pi R$, the circumference of the ring. But $\lambda \cdot (2\pi R)$ is q, the total charge on the ring, so that we obtain

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \tag{8-28}$$

For points far enough away from the ring so that $z \gg R$, we can put $R = 0$ in Eq. 8-28. Doing so yields

$$E = \frac{q}{4\pi\varepsilon_0 z^2}$$

which (with z replaced by r) is Eq. 8-12. We are not surprised because, at large enough distance, the ring behaves electrically like a point charge. We note also from Eq. 8-28 that $E = 0$ for $z = 0$. This is also not surprising because a test charge at the center of the ring would be pushed or pulled equally in all directions in the plane of the ring and would experience no net force.

Example 8-3 **The field of a charged disk**

Fig. 8-6 shows a circular disk of radius R, carrying a uniform surface charge of density σ on its upper surface. Find the electric field at point P, a distance z from the disk along its axis.

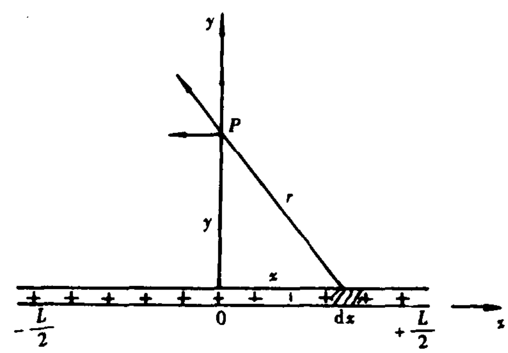

Fig. 8-7 The field of a line of charge

Solution We divide the disk up into concentric rings and calculate the electric field by integrating the contributions of the various rings. Fig. 8-6 shows a flat ring with radius r and of width dr, its total charge being

$$\mathrm{d}q = \sigma(2\pi r)\mathrm{d}r$$

where $(2\pi r)\mathrm{d}r$ is the differential area of the ring. We have already solved the problem of the electric field due to a ring of charge. Substituting dq for q in Eq. 8-28, and replacing R in Eq. 8-28 by r, we obtain

$$\mathrm{d}E = \frac{z\mathrm{d}q}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} = \frac{\sigma z r \mathrm{d}r}{2\varepsilon_0(z^2 + r^2)^{3/2}}$$

Now we can find E by integrating over the surface of the disk, that is, by integrating with respect to the variable r. Note that z remains constant during this process. Thus

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