Advanced Strength and Applied Elasticity

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Preface

This text is a development of classroom notes prepared in connection with advanced undergraduate and first year graduate courses in elasticity and the mechanics of solids. It is designed to satisfy the requirements of courses subsequent to an elementary treatment of the strength of materials. In addition to its applicability to aeronautical, civil, and mechanical engineering and to engineering mechanics curricula, the authors have endeavored to make the text useful to practicing engineers. Emphasis is given to *numerical techniques* (which lend themselves to computerization) in the solution of problems resisting *analytical treatment*. The stress placed upon numerical solutions is not intended to deny the value of classical analysis, which is given a rather full treatment. It instead attempts to fill what the authors believe to be a void in the world of textbooks.

An effort has been made to present a balance between the theory necessary to gain insight into the mechanics, but which can often offer no more than crude approximations to real problems because of simplifications related to geometry and conditions of loading, and numerical solutions, which are so useful in presenting stress analysis in a more realistic setting. The authors have thus attempted to emphasize those aspects of theory and application which prepare a student for more advanced study or for professional practice in design and analysis.

The theory of elasticity plays three important roles in the text: it provides exact solutions where the configurations of loading and boundary are relatively simple; it provides a check upon the limitations of the strength of materials approach; it serves as the basis of approximate solutions employing numerical analysis.

To make the text as clear as possible, attention is given to the presentation of the fundamentals of the strength of materials. The physical significance of the solutions and practical applications are given emphasis. The authors have made a special effort to illustrate important principles and applications with numerical examples. Consistent with announced national policy, included in the text are problems in which the physical quantities are expressed in the International System of Units (SI).

It is a particular pleasure, upon the completion of a project of this

nature, to acknowledge the contributions of those who assisted the authors in the evolution of the text. Thanks are, of course, due to the many students who have made constructive suggestions throughout the several years when drafts of this work were used as a text. To Professor F. Freudenstein of Columbia University and Professor R. A. Scott of the University of Michigan, we express our appreciation for their helpful recommendations and valuable perspectives in connection with their review of the manuscript. And, as has always been the case, Mrs. Helen Stanek has provided intelligent editorial and typing assistance throughout the several drafts of this work; to her, the authors express their special thanks and appreciation.

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Teaneck, New Jersey
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Chapter 1

Analysis of Stress

1.1 Introduction

The basic structure of matter is characterized by nonuniformity and discontinuity attributable to its various subdivisions: molecules, atoms, and subatomic particles. Our concern in this text is not with the particulate level of matter, however, and it is to our advantage to replace the actual system of particles with a *continuous* distribution of matter. There is the clear implication in such an approach that any small volumes with which we may deal are large enough to contain a great many particles. Random fluctuations in the properties of the material are thus of no consequence. Of the states of matter, we are here concerned only with the solid, with its ability to maintain its shape without the need of a container, and to resist continuous shear and tension.

In contrast with rigid body statics and dynamics, which treat the external behavior of bodies (i.e., the equilibrium and motion of bodies without regard to small deformations associated with the application of load), the mechanics of solids is concerned with the relationships of external effects (forces and moments) to internal stresses and strains.

External forces acting on a body may be classified as *surface forces* and *body forces*. A surface force is of the *concentrated* type when it acts at a point; a surface force may also be distributed *uniformly* or *non-uniformly* over a finite area. Body forces act on volumetric elements rather than surfaces, and are attributable to fields such as gravity and magnetism.

The principal topics under the general heading *mechanics of solids* may be summarized as follows:

(a) Analysis of the stresses and deformations within a body subject to a prescribed system of forces. This is accomplished by solving the governing equations which describe the stress and strain fields (theoretical stress analysis). It is often advantageous, where the shape of the structure or conditions of loading preclude a theoretical solution or where verification is required, to apply the laboratory techniques of experimental stress analysis.

(b) Determination by theoretical analysis or by experiment of the limiting values of load which a structural element can sustain without suffering damage, failure, or compromise of function.

(c) Determination of the body shape and selection of those materials which are most efficient for resisting a prescribed system of forces under specified conditions of operation such as temperature, humidity, vibration, and ambient pressure. This is the *design* function and more particularly that of *optimum design*. Efficiency may be gaged by such criteria as minimum weight or volume, minimum cost, or any criterion deemed appropriate.

The design function, item (c) above, clearly relies upon the performance of the theoretical analyses cited under (a) and (b), and it is these to which this text is directed. The role of analysis in design is observed in examining the following steps comprising the systematic design of a load carrying member:

(1) Evaluation of the most likely modes of failure under anticipated conditions of service.

(2) Determination of expressions relating external influences such as force and torque to such effects as stress, strain, and deformation. Often, the member under consideration and the conditions of loading are so significant or so amenable to solution as to have been the subject of prior analysis. For these situations textbooks, handbooks, journal articles, and technical papers are good sources of information. Where the situation is unique, a mathematical derivation specific to the case at hand is required.

(3) Determination of the maximum or allowable value of a significant quantity such as stress, strain, or energy, either by reference to compilations of material properties or by experimental means such as a simple tension test. This value is used in connection with the relationship derived in (2).

(4) Selection of a *design factor of safety*, usually referred to simply as the factor of safety, to account for uncertainties in a number of aspects of the design, including those related to the actual service loads, material properties, or environmental factors. An important area of uncertainty is connected with the assumptions made in the analysis of stress and deformation. Also, one is not likely to have a secure knowledge of the stresses which may be introduced during machining, assembly, and shipment of the element. The design factor of safety also reflects the consequences of failure, e.g. the possibility that failure will result in loss of human life or injury, and the possibility that failure will result in costly repairs or danger to other components of the overall system. For the abovementioned

reasons the design factor of safety is also sometimes called the *factor of ignorance*. The uncertainties encountered during the design phase may be of such magnitude as to lead to a design carrying extreme weight, volume, or cost penalties. It may then be advantageous to perform thorough tests or more exacting analysis rather than to rely upon overly large design factors of safety. The so-called *true factor of safety* can only be determined after the member is constructed and tested. This factor is the ratio of the maximum load the member *can sustain* under severe testing without damage to the maximum load *actually* carried under normal service conditions.

The foregoing procedure is not always conducted in as formal a fashion as may be implied. In some design procedures, one or more steps may be regarded as unnecessary or obvious on the basis of previous experience.

We conclude this section with an appeal for the reader to exercise a degree of skepticism with regard to the application of formulas for which there is uncertainty as to the limitations of use or the areas of applicability. The relatively simple form of many formulas usually results from rather severe restrictions in their derivation. These relate to simplified boundary conditions and shapes, limitations upon stress and strain, and the neglect of certain complicating factors. The designer and stress analyst must be aware of such restrictions lest their work be of no value, or worse, lead to dangerous inadequacies.

In this chapter, we are concerned with the state of *stress at a point* and the *variation of stress* throughout an elastic body. The latter is dealt with in Secs. 1.4 and 1.12, and the former in the balance of the chapter.

1.2 Definition of Stress

Consider a body in equilibrium, subject to the system of forces shown in Fig. 1.1*a*. An element of area ΔA, located on an exterior or interior surface (the latter as in Fig. 1.1*b*), is acted on by force $\Delta \mathbf{F}$. Let n, s_1, s_2 constitute a set of orthogonal axes, origin placed at the point P, with n normal and s_1, s_2 tangent to ΔA. In general $\Delta \mathbf{F}$ does not lie along n, s_1, or s_2. Decomposition of $\Delta \mathbf{F}$ into components parallel to n, s_1, and s_2 (Fig. 1.1*c*) leads to the following definitions of the normal stress σ_n and the shear stresses τ_s:

$$\sigma_n = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$

$$\tau_{s_1} = \lim_{\Delta A \to 0} \frac{\Delta F_{s_1}}{\Delta A}, \qquad \tau_{s_2} = \lim_{\Delta A \to 0} \frac{\Delta F_{s_2}}{\Delta A} \tag{1.1}$$

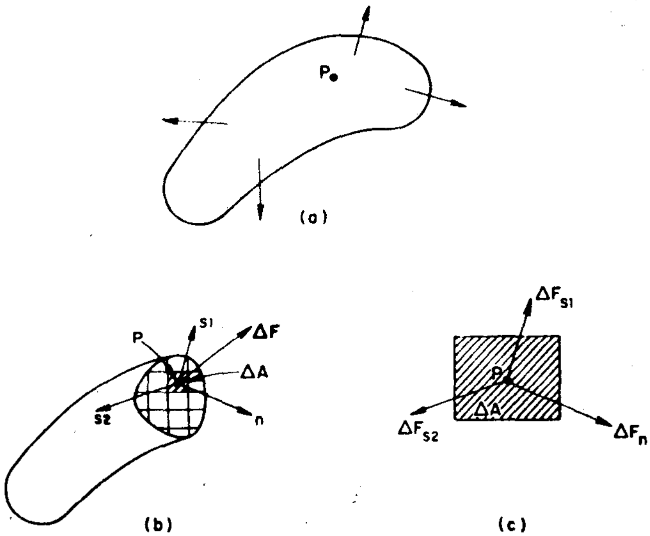

Fig. 1.1

These expressions provide the stress components at a point P to which the area ΔA is reduced in the limit. Clearly, the expression $\Delta A \rightarrow 0$ depends upon the idealization discussed in Sec. 1.1. In this country, stress is generally measured in pounds per square inch or kilopounds per square inch. In the International System of Units (SI), a system likely to gain widespread acceptance, stress is measured in newtons per square meter. Table 1.1 shows the equivalence between the two systems of units.

Table 1.1

	System of units	
	U.S.	SI
Length	inch	meter (m)
Force	pound force	newton (N)
Time	second	second (s)
Mass	pound mass, slug	kilogram (kg)

Some conversion factors

1 in. = 0.0254 m   1 lbm = 0.4536 kg

1 lbf = 4.448 N   1 psi = 6,895 N/m^2

The values obtained in the limiting processes of Eq. (1.1) differ from point to point on the surface as **ΔF** varies. The stress components depend not only upon **ΔF**, however, but also upon the orientation of the plane on which it acts at point *P*. Even at a given point, therefore, the stresses will differ as different planes are considered. The complete description of stress at a point thus requires the specification of the stress on all planes passing through the point.

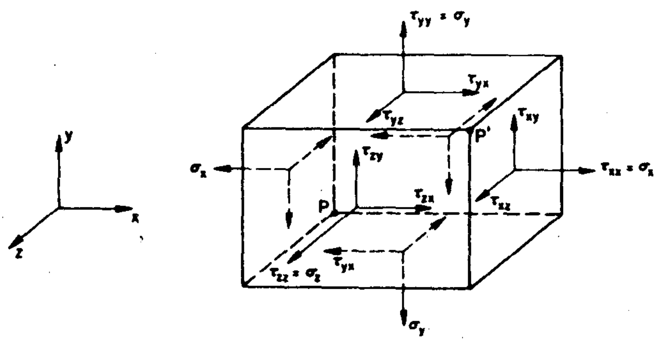

Fig. 1.2

1.3 The Stress Tensor

It is verified in Sec. 1.8 that in order to enable the determination of the stresses on an infinite number of planes passing through a point *P*, thus defining the stress at that point, one need only specify the stress components on three mutually perpendicular planes passing through the point. These three planes, perpendicular to the coordinate axes, contain three sides of an infinitesimal parallelepiped (Fig. 1.2). A three-dimensional state of stress is shown in the figure. Consider the stresses to be identical at points *P* and *P'*, and uniformly distributed on each face, represented by a single vector acting at the center of each face. In accordance with the foregoing, a total of nine scalar stress components define the state of stress at a point. The stress components can be assembled in the following *matrix form*, wherein each row represents the group of stresses acting on a plane passing through $P(x, y, z)$:

$$\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{1.2}$$

The above array represents a tensor of second rank (refer to Sec. 1.8), requiring two indices to identify its elements or components. A vector is a tensor of first rank; a scalar is of zero rank.

The double subscript notation is interpreted as follows: the first subscript indicates the direction of a normal to the plane or face on which the stress component acts; the second subscript relates to the direction of the stress itself. Repetitive subscripts will be avoided in this text, so that the normal stresses τ_{xx}, τ_{yy}, and τ_{zz} will be designated σ_x, σ_y, and σ_z, as indicated in Eq. (1.2). *A face or plane is usually identified by the axis normal to it*, e.g., the x faces are perpendicular to the x axis.

Referring again to Fig. 1.2, we observe that *both* stresses labeled τ_{yx} tend to twist the element in a clockwise direction. It would be convenient, therefore, if a sign convention were adopted under which these stresses carried the same sign. Applying a convention relying solely upon the coordinate direction of the stresses would clearly not produce the desired result, inasmuch as the τ_{yx} stress acting on the upper surface is directed in the positive x direction, while τ_{yx} acting on the lower surface is directed in the negative x direction. The following sign convention, which applies to both normal and shear stresses, is related to the deformational influence of a stress, and is based upon the relationship between the direction of an outward normal drawn to a particular surface, and the directions of the stress components on the same surface. When *both* the outer normal and the stress component face in a positive direction relative to the coordinate axes, the stress is positive. When *both* the outer normal and the stress component face in a negative direction relative to the coordinate axes, the stress is positive. When the normal points in a positive direction while the stress points in a negative direction (or vice versa), the stress is negative. In accordance with this sign convention, tensile stresses are always positive and compressive stresses always negative. Figure 1.2 depicts a system of positive normal and shear stresses.

Many of the equations of elasticity become quite unwieldy when written in full, unabbreviated from; see, for example, Eq. (1.17). As the complexity of the situation described increases, so does that of the formulations, tending to obscure the fundamentals in a mass of symbols. For this reason the more compact *indicial or tensor notation* described in Appendix A is sometimes found in technical publications. A stress tensor is written in indicial notation as τ_{ij}, where i and j each assume the values x, y, and z as required by Eq. (1.2). Generally, such notation is not employed in this text.

1.4 Variation of Stress Within a Body

As pointed out in Sec. 1.2, the components of stress generally vary from point to point in a stressed body. These variations are governed by the