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**THEORETICAL
FOUNDATIONS
of
DIGITAL IMAGING
Using MATLAB[®]**

Leonid P. Yaroslavsky



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THEORETICAL FOUNDATIONS of DIGITAL IMAGING Using MATLAB®

Leonid P. Yaroslavsky

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**THEORETICAL
FOUNDATIONS
of
DIGITAL IMAGING
Using MATLAB[®]**

CHAPMAN & HALL/CRC MATHEMATICAL AND COMPUTATIONAL IMAGING SCIENCES

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This series aims to capture new developments and summarize what is known over the whole spectrum of mathematical and computational imaging sciences. It seeks to encourage the integration of mathematical, statistical and computational methods in image acquisition and processing by publishing a broad range of textbooks, reference works and handbooks. The titles included in the series are meant to appeal to students, researchers and professionals in the mathematical, statistical and computational sciences, application areas, as well as interdisciplinary researchers involved in the field. The inclusion of concrete examples and applications, and programming code and examples, is highly encouraged.

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To Dima and Yaro on their 25th and 5th birthdays

Preface

With the advent and ubiquitous spreading of digital imaging, a new profession has emerged: imaging engineering. This book is intended as a textbook for studying the theoretical foundations of digital imaging to master this profession. It is based on the experience accumulated by the present author for 50 years of working in the field and teaching various courses in digital image processing and digital holography in the Russian Academy of Sciences, the National Institutes of Health (Bethesda, Maryland, USA), the University of Erlangen-Nuremberg (Germany), the Tampere University of Technology (Tampere, Finland), Agilent Labs (Palo Alto, California, USA), the Autonomous University of Barcelona (Spain), the Institute of Henri Poincaré (Paris, France), the Kiryu University (Kiryu, Japan), and the University of Tel Aviv (Israel).

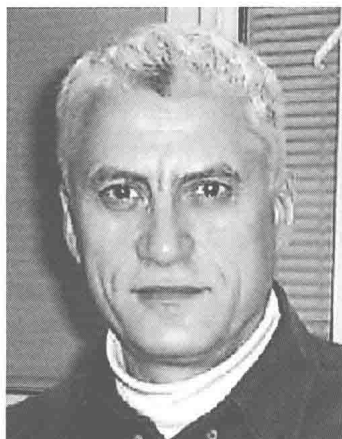
The book is addressed to young students who opt to pursue a scientific and research career in imaging science and engineering. The most outstanding minds of mankind, such as Galileo Galilei, René Descartes, Isaac Newton, James Clerk Maxwell, and many other scientists and engineers contributed to this branch of modern science and technology. At least 12 Nobel Prizes have been awarded for contributions directly associated with image science and imaging devices, and a majority of others would not be possible without one or the other imaging methods. You will be in good company, dear reader. Let this book help you to become a master of digital imaging.

A number of MATLAB programs are available at the Download section of this book's web page on the CRC Press website (<http://www.crcpress.com/product/isbn/9781439861400>).

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color images of the surface of Mars and the first panoramas of the surface of Venus. From 1983 to 1995, he headed the Laboratory of Digital Optics at the institute. From 1995 to 2008, he was a professor at the Faculty of Engineering, Tel Aviv University, where, at present, he is a professor emeritus. He was also a visiting professor at the University of Erlangen, Germany; National Institutes of Health, Bethesda, Maryland, USA; Institute of Optics, Orsay, France; Institute Henri Poincare, Paris, France; International Center for Signal Processing, Tampere University of Technology, Tampere, Finland; Agilent Laboratories, Palo Alto, California, USA; Gunma University, Kiryu, Japan; and the Autonomous University of Barcelona, Spain. He has supervised 20 PhD candidates and is an author and editor of several books and more than 100 papers on digital image processing and digital holography.

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