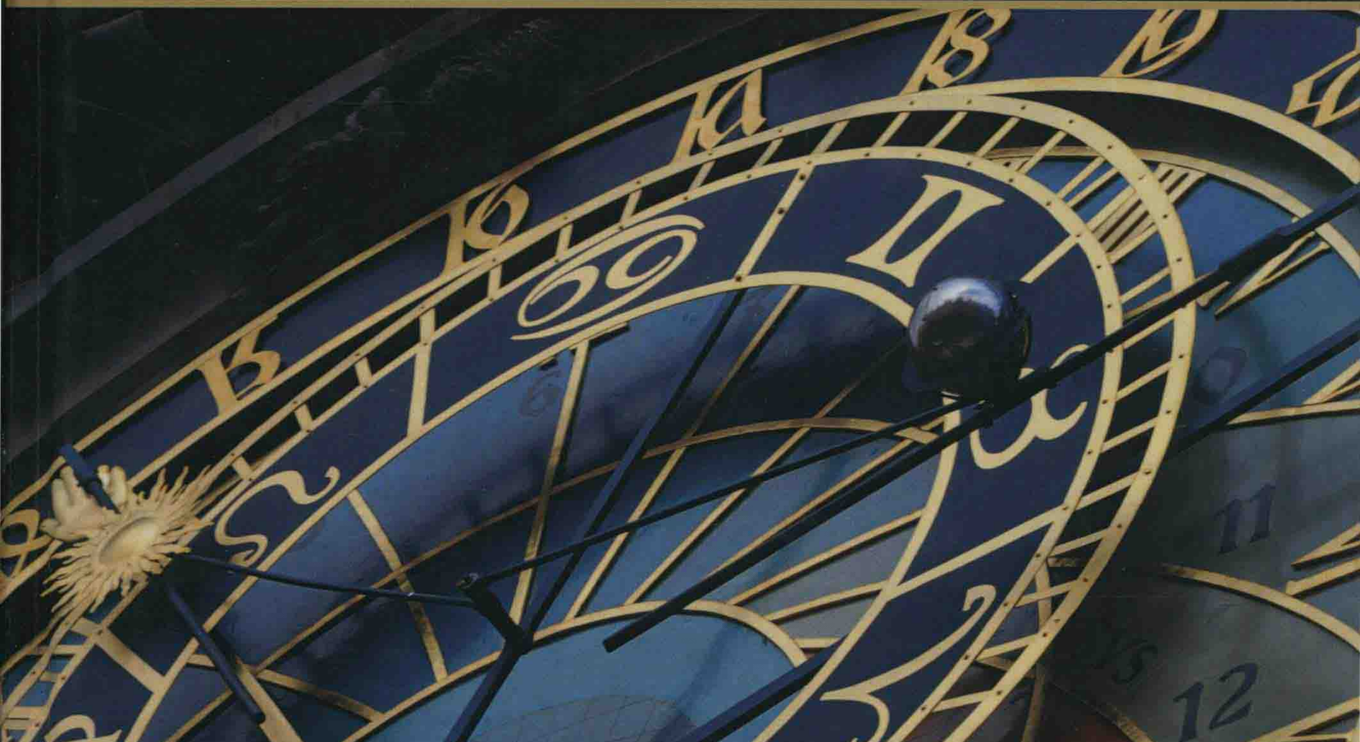
SIGNALS, SYSTEMS & INFERENCE



Alan V. Oppenheim & George C. Verghese

Prentice Hall Signal Processing Series | Alan V. Oppenheim, Series Editor

SIGNALS, SYSTEMS & INFERENCE

ALAN V. OPPENHEIM & GEORGE C. VERGHESE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

PEARSON

Boston • Columbus • Indianapolis • New York • San Francisco
Hoboken • Amsterdam • Cape Town • Dubai • London • Madrid • Milan • Munich
Paris • Montreal • Toronto • Delhi • Mexico City • São Paulo
Sydney • Hong Kong • Seoul • Singapore • Taipei • Tokyo

Vice President and Editorial Director, ECS: *Marcia J. Horton*
Acquisitions Editor: *Julie Bai*
Executive Marketing Manager: *Tim Galligan*
Marketing Assistant: *Jon Bryant*
Senior Managing Editor: *Scott Disanno*
Program Manager: *Joanne Manning*
Global HE Director of Vendor Sourcing and Procurement: *Diane Hynes*
Director of Operations: *Nick Sklitsis*
Operations Specialist: *Maura Zaldivar-Garcia*
Cover Designer: *Krista Van Guilder*
Manager, Rights and Permissions: *Rachel Youdelman*
Associate Project Manager, Rights and Permissions: *Timothy Nicholls*
Printer/Binder: *Edwards Brothers, Malloy*
Cover Printer: *Phoenix Color, Hagerstown, MD*
Composition/Full Service Project Management: *Pavithra Jayapaul, Jouve India*

10 9 8 7 6 5 4 3 2

Library of Congress Cataloging-in-Publication Data

Oppenheim, Alan V., 1937-
Signals, systems and inference / Alan V. Oppenheim and George C. Verghese
Massachusetts Institute of Technology.
pages cm
ISBN 978-0-13-394328-3 -- ISBN 0-13-394328-3
1. Signal processing. I. Verghese, George C. II. Title.
TK5102.9.O673 2016
621.382'2--dc23

2014043376

PEARSON

ISBN-13: 978-0-13-394328-3
ISBN-10: 0-13-394328-3

We dedicate this book to

Amar Bose, Bernard Gold, and Thomas Stockham

&

George Sr. and Mary Verghese, and Thomas Kailath

These extraordinary people have had a profound impact on our lives and our careers

PREFACE

This book has grown out of an undergraduate course developed and taught by us in MIT's Department of Electrical Engineering and Computer Science. Our course is typically taken by third- and fourth-year undergraduate students from many engineering branches, as well as undergraduate and graduate students from applied science. There are two formal prerequisites for the course, and for this book: an introductory subject in time- and frequency-domain analysis of signals and systems, and an introductory subject in probability. These two subjects are typically taken by most engineering students early in their degree programs. The signals and systems subject almost invariably builds on an earlier course in differential equations, ideally with some basic linear algebra folded into it.

In many engineering departments, students with a strong interest in applied mathematics have then traditionally gone on to a more specialized undergraduate subject in control, signal processing, or communication. In addition to being specialized, such subjects often focus on deterministic signals and systems. Our aim instead was to build broadly on the prerequisite material, folding together signals, systems, and probability in ways that could make our course relevant and interesting to a wider range of students. The course could then serve both as a terminal undergraduate subject and as a sufficiently rigorous basis for more advanced undergraduate subjects or introductory graduate subjects in many engineering and applied science departments.

The course that gave rise to this book teaches students about signals and signal descriptions that are typically new to them, for example, random signals and their characterization through correlation functions and power spectral densities. It introduces them to new kinds of systems and system properties, such as state-space models, reachability and observability, optimum filters, and group delay. And it highlights model-based approaches to inference, particularly in the context of state estimation, signal estimation, and signal detection.

Although some parts of our course are well covered by existing textbooks, we did not find one that fit our needs across the range of topics. This led to lecture notes, which was the easier part, and then eventually this book. In the process, we continually experimented with and refined the content and order of presentation. Along the way we also at times included other material or excluded some that is now back in the book. Among the conclusions of these experiments was that we did not have time in a one-semester class to fold in even basic notions of information theory, despite its central importance to communication systems and, more generally, to inference.

As suggested in the Prologue to this book, signals, systems and probability have been and will continue to be usefully combined in studying fields such as signal processing, control, communication, financial engineering, biomedicine, and many others that involve dynamically varying processes operating in continuous or discrete time, and affected by disturbances, noise, or uncertainty. This premise forms the basis for the overall organization and content of our course and this text.

The book can be thought of as comprising four parts, outlined below. A more detailed overview of the individual chapters is captured in the table of contents. Chapters 1 and 2 present a brief review of the assumed prerequisites in signals and linear time-invariant (LTI) systems, though some portions of the material may be less familiar. A key intent in these chapters is to establish uniform notation and concepts on which to build in the chapters that follow. Chapter 3 discusses the application of some of this prerequisite material in the setting of digital communication by pulse amplitude modulation.

Chapters 4–6 are devoted to state-space models, concentrating on the single-input single-output LTI case. The development is largely built around the eigenmodes of such systems, under the simplifying assumption of distinct natural frequencies. This part of the book introduces the idea of model-based inference in the context of state observers for LTI systems, and examines associated feedback control strategies.

Chapters 7–9 provide a brief review of the assumed probability prerequisites, including estimation and hypothesis testing for static random variables. As with Chapters 1 and 2, we felt it important to set out our notation and perspectives on the concepts while making contact with what students might have encountered in their earlier probability subject. Again, some parts of this material, particularly on hypothesis testing, may be previously unfamiliar to some students.

In Chapters 10–13, we characterize wide-sense stationary random signals, and the outputs that result from LTI filtering of such signals. The associated properties and interpretations of correlation functions and power spectral densities are then used to study canonical signal estimation and signal detection problems. The focus in Chapter 12 is on linear minimum mean square error signal estimation, i.e., Wiener filtering. In Chapter 13, the emphasis is on signal detection for which optimum solutions involve matched filtering.

As is often said, the purpose of a course is to uncover rather than to cover a subject. In this spirit, each chapter includes a final section with some suggestions for further reading. Our intent in these brief sections is not to be exhaustive but rather to suggest the wealth of learning opened up by the material in this text. We have pointed exclusively to books rather than to papers in the research literature, and have in each case listed only a fraction of the books that could have been listed.

Each chapter contains a rich set of problems, which have been divided into Basic, Advanced, and Extension. Basic problems are likely to be easy for most students, while the Advanced problems may be more demanding. The Extension problems often involve material somewhat beyond what is developed in the chapter. Certain problems require simulation or computation using some appropriate computational package. Given the variety and ubiquity of such packages, we have intentionally not attempted to structure the computational exercises around any specific platform.

There is more material in this book than can be taught comfortably in a one-semester course. This allows the instructor or self-learner to choose different routes through the text, and over the years we have experimented with various paths. For a course that is more oriented towards communication or signal processing, Chapters 4, 5 and 6 (state-space models) can be omitted, or addressed only briefly. For a course with more of a control orientation, Chapter 3 (pulse amplitude modulation), Chapter 9 (hypothesis testing) and Chapter 13 (signal detection) can perhaps be considered optional.

A third version of the course, and the one that we currently teach, is outlined in a little more detail below. This version involves two weekly lectures over a semester of approximately thirteen weeks. The lectures are interleaved with an equal number of small-group recitation sections, devoted to more interactive discussion of specific problems that illustrate the lectures and help address the weekly homework. In addition, we staff optional small-group tutorials. Finally an optional evening "common room" that we run several times each week allows students in the class to congregate and interact with each other and with a member of the teaching staff while they work on their homework.

In our teaching in general, we like to emphasize that the homework is intended to provide an occasion for learning and engaging with the concepts and mechanics, rather than being an exam. We recommend that the end-of-chapter problems in this book be approached in the same spirit. In particular, we encourage students to work constructively together, sharing insights and approaches. Our grading of the problems is primarily for feedback to the students and to provide some accountability and motivation. The course does typically have a midterm quiz and a final exam, and many of the end-of-chapter problems in this text were first created as quiz or exam problems. There are also many possibilities for term projects that can grow out of the material in the class, if desired.

An introductory lecture in the same spirit as the Prologue to this text is followed by a brief review of the signals and systems material in Chapter 1. The focus in class is on what might be less familiar from the prerequisite subject, and students are tasked with reviewing the rest on their own, guided by appropriate homework problems. We then move directly to the state-space material in Chapters 4, 5 and 6. Even if students have had some prior exposure to state-space models, there is much that is likely to be new to them here, though they generally relate easily to the material. We have not held students responsible for the more detailed proofs, such as those on eigenvalue placement for LTI observers or state feedback, but do expect them to develop an understanding of the relevant results and how to apply them to small examples. An important lesson from the state-space observer framework is the role of a system model in going from measured signals to inferences about the system.

Our course then turns to probabilistic models and random signals. The probability review in Chapter 7 is mostly woven into lectures covering minimum mean square error (MMSE) and linear MMSE (LMMSE) estimation, which are dealt with in Chapter 8. In order to move more quickly to random signals rather than linger on review of material from the prerequisite probability course, we defer the study of hypothesis testing in Chapter 9 to the end of the course, using it as a lead-in to the signal detection material in Chapter 13. Part of the rationale is also that Chapters 9 and 13 are devoted to making inferences about discrete random quantities, namely the hypotheses, whereas Chapters 8 and 12 on (L)MMSE estimation deal with inferences about continuous random variables. We therefore move directly from Chapter 8 to Chapter 10, studying random signals, i.e., stochastic processes, focusing on the time-domain analysis of wide-sense stationary (WSS) processes, and LTI filtering of such processes.

The topic of power spectral density in Chapter 11 connects back to the development of transforms and energy spectral density in Chapter 1, and also provides the opportunity to refer to relevant sections of Chapter 2 on all-pass filters and spectral factorization. These topics are again important in Chapter 12, on LMMSE (or Wiener) filtering for WSS processes. In most offerings of the course, we omit the full causal Wiener filter development, instead only treating the case of prediction of future values of a process from past values of the same process.

The last part of the course refers strongly back to Chapter 3, using the context of digital communication via pulse amplitude modulation to motivate the hypothesis testing problem. The return to Chapter 3 can also involve reference to the material in Chapter 2 on channel distortions and group delay. The hypothesis testing paradigm is then treated as in Chapter 9. This serves as the foundation for the study of signal detection in the last chapter, Chapter 13.

The breadth of this book, and the different backgrounds we brought to the project, meant that we had much to learn from each other. We also learn each term from the very engaged students, teaching assistants and faculty colleagues who are involved in the course, as well as from the literature on the subjects treated here. This book will have amply met its objectives if it sparks and supports a similar voyage of discovery in its readers, as they construct their own individual re-synthesis of the themes of signals, systems and inference.

Alan V. Oppenheim & George C. Verghese
Cambridge, Massachusetts

The Cover

In planning the cover, we were fortunate to have the opportunity to work closely with Krista Van Guilder, a talented graphic artist who is also manager of media and design in MIT's Research Laboratory of Electronics. The cover design originated in our desire to suggest the book's themes in a visually pleasing and striking way. Our explorations for the cover began with images of sundials, clocks, and astrolabes. The astrolabe (www.astrolabes.org), invented over two thousand years ago and used well into the 17th century, was an important instrument for numerous astronomical calculations, such as determining time from latitude and the positions of heavenly bodies. Inspired by these images, Krista's search led her to photographs of the beautiful astronomical clock that has kept time over the old town square in Prague since 1410. Time, both discrete and continuous, is of course central to our book. This clock, the *Prague Orloj*, with astrolabe markings on its face, and showing the position of the sun and moon along with many other astronomical details, was chosen as the primary image for the front cover.

The *Prague Orloj* is much photographed, but the wonderful photograph that we selected for the cover was particularly striking to us. It was taken in 2009 by Matteo Foiadelli, a computer engineer from Bergamo, Italy, whose hobby is photography. He was on holiday in Prague with friends, and becoming familiar with a new digital SLR camera. As he explained to us, "*I just wanted to get an image of the astronomical clock different than the usual frontal view. So I tried to close up a little bit and to frame it from the side.*" We feel fortunate to have found his photograph, and are grateful for his permission to use it on our cover and as part of the opening art for each chapter.

The image that adorns the back cover is from a very different type of camera: the Hubble telescope, deployed in orbit in 1990. This image from 2001, a striking counterpoint to the astronomical clock, is the result of capturing photons that left those galaxies more than half a billion years ago. As described on the Hubble website (www.hubblesite.org):

> *"This Hubble image of ESO 77-14 is a stunning snapshot of a celestial dance performed by a pair of similar sized galaxies. Two clear signatures of the gravitational tug of war between the galaxies are the bridge of material that connects them and the disruption of their main bodies. ... This interacting pair is in the constellation of Indus, the Indian, some 550 million light-years away from Earth. The dust lanes between the two galaxy centers show the extent of the distortion to the originally flat disks that have been pulled into three-dimensional shapes."*

The launch of the Hubble telescope into precise orbital position, and the subsequent recording, retrieval, and processing of data from it to yield such revealing and awe-inspiring images, reflect the mastery of signals, systems and inference that humankind has attained in the four centuries after the astrolabe faded from use. But the image also evokes the boundless opportunities for new advances and horizons.

ACKNOWLEDGMENTS

This text has its origins in an MIT subject that was first planned, designed and taught by us over twenty years ago. It has subsequently evolved to its current form through continual experimentation and with many variations of the material and presentation. The subject was conceived as part of the curriculum for a five-year Master of Engineering degree program that was being launched at that time in our Department of Electrical Engineering and Computer Science (EECS). We are grateful to Paul Penfield (as then department head), Jeffrey Shapiro (as associate head) and William Siebert for their part in defining the curricular structure that provided the opening for such a subject. Jeff Shapiro also worked with us on the initial definition of the content. Continued support of the curriculum, and of revisions to it, from subsequent department heads – John Guttag, Rafael Reif, Eric Grimson, Anantha Chandrakasan – and their administrations has been important, and we thank them for their support. More generally, we consider ourselves very fortunate to have had our academic careers develop in this highly collegial and vibrant department. MIT's culture of dedication to teaching and learning informed by research, and the Institute's recognition and celebration of excellence in teaching, have had a significant influence on us.

The staffing of the course, as taught in our department, includes a faculty member who gives two weekly lectures and has overall responsibility for running the course, as well as recitation instructors and teaching assistants who meet regularly with smaller groups of students. Numerous faculty colleagues in our department have collaborated with us over the years, as recitation instructors or as lecturers for the subject. Many students have served as able and enthusiastic teaching assistants. We have also benefited from the help of excellent administrative assistants. We take the opportunity in what follows to thank all these people for their multifaceted contributions to the development and running of the course, to the student experience in the course, and to this text.

In addition to each of us individually and jointly lecturing and overseeing the administration of the course many times, other colleagues who have served in that role are Bernard Lesieutre, Charles Rohrs, Jeffrey Shapiro, Gregory Wornell, and John Wyatt. In the process they have provided valuable feedback on the course content and course notes, as well as bringing new insights and developing new exam and homework problems.

Over the years, we have been privileged to work with a superbly talented and committed roster of faculty and senior graduate students serving as recitation instructors. The recitation instructors who have participated in the teaching of the subject are Jinane Abounadi, Elfar Adalsteinsson, Babak Ayazifar, Duane Boning, Petros Boufounos, John Buck, Mujdat Cetin, Jorge Goncalves, Julie Greenberg, Christoforos Hadjicostis, Peter Hagelstein, Thomas Heldt, Steven Isabelle, Franz Kaertner, James Kirtley, Amos Lapidoth, Bernard Lesieutre, Steve Massaquoi, Shay Maymon, Alexandre Megretski, Jose Moura, Asuman Ozdaglar, Michael Perrott, Rajeev Ram, Charles Rohrs, Melanie Rudoy, Jeffrey Shapiro, Ali Shoeb, William Siebert, Vladimir Stojanovic, Collin Stultz, Russell Tedrake, Mitchell Trott, Thomas Weiss, Alan Willsky, Gregory Wornell, John Wyatt, Laura Zager, and Lizhong Zheng. These colleagues have helped provide a rich experience for the students, and have made many contributions to the content of the course and this text.

Both we and the students in the class have been the beneficiaries of the dedication and energy of the stellar teaching assistants during this period: Irina Abarinov, Abubakar Abid, Anthony Accardi, Chalee Asavathiratham, Thomas Baran, Leighton Barnes, Soosan Beheshti, Ballard Blair, Petros Boufounos, Venkat Chandrasekaran, Jon Chu, Aaron Cohen, Roshni Cooper, Ujjaval Desai, Vijay Divi, Shihab Elborai, Baris Erkmen, Siddhartan Govindasamy, Hanhong Gao, James Geraci, Michael Girone, Carlos Gomez-Uribe, Christoforos Hadjicostis, Andrew Halberstadt, Nicholas Hardy, Everest Huang, Irena Hwang, Zahi Karam, Asif Khan, Alaa Kharbouch, Ashish Khisti, Lohith Kini, Alison Laferriere, Ryan Lang, Danial Lashkari, Adrian Lee, Karen Lee, Durodami Lisk, Karen Livescu, Lorenzo Lorilla, Zhipeng Li, Peter Mayer, Rebecca Mieloszyk, Jose Oscar Mur Miranda, Kirimania Murithi, Akshay Naheta, Kenny Ng, Tri Ngo, Paul Njoroge, Ehimwenma Nosakhare, Uzoma Orji, Tushar Parlikar, Pedro Pinto, Victor Preciado, Andrew Russell, Navid Sabbaghi, Maya Said, Peter Sallaway, Sridevi Sarma, Matthew Secor, Mariam Shanechi, Xiaomeng Shi, Andrew Singer, Lakshminarayan Srinivasan, Brian Stube, Eduardo Sverdlin-Lisker, Kazutaka Takahashi, Afsin Ustundag, Kathleen Wage, Tianyu Wang, Keyuan Xu, HoKei Yee, and Laura Zager. Their inputs are reflected in myriad ways throughout this text.

Over the many years of offering this subject, we have been guided by the wisdom of our colleague Frederick Hennie in matters of instructional staffing. Agnes Chow's strategic yet detailed oversight of the EECS department's administrative and financial operations has allowed us and other faculty to focus on our teaching. Lisa Bella, as assistant to the department's education officers, attends almost single-handedly and with incredible

responsiveness and good humor to the practical administrative aspects of supporting a hundred professors and over a hundred teaching assistants across the department's teaching enterprise each semester. For administrative assistance with our course in its many offerings, we would like to thank Alecia Batson, Margaret Beucler, Dimonika Bray, Susan Davco, Angela Glass, Vivian Mizuno, Sally Santiago, Darla Secor, Eric Strattman, and Diane Wheeler.

As the class subject has continued to evolve over the two-decade period, the accompanying course notes that ultimately led to this text have also grown and changed. The students in the class have been key participants in that process, through their questions, requests, challenges, suggestions, critiques, and encouragement. It is a continuing privilege to work with the gifted, engaged, thoughtful, and vocal students whom we have in our classrooms at MIT.

We have sometimes said, either ruefully or in jest, that the current text is the fourth edition of a book whose first three editions we never formally published. As any textbook author knows, however, the final phase of producing a polished text from what initially seem to be very good course notes is still a formidable task. Some of our teaching assistants and other students have more recently provided substantial help and feedback in advancing our lecture notes closer to a text. We would like to specifically acknowledge the efforts of Leighton Barnes and Ballard Blair, as well as Manishika Agaskar, Ganesh Ajjanagadde, Michael Mekonnen, and Guolong Su. For cheerfully, efficiently, and discerningly pulling together and keeping track of all the fragments and versions and edits as we advanced towards a text, we are enormously indebted to Laura von Bosau.

Our department leadership has consistently encouraged us to take the course notes beyond their role as a supplement to the classroom subject and into a published book, so that the material would be more widely and independently accessible. Anantha Chandrakasan's urging in recent years was a key catalyst in making this text happen. Also significant, and greatly appreciated, was the interest from several publishers. Tom Robbins saw the potential early on, and regularly offered helpful advice through the first decade of the course, during his time at Prentice Hall. Phil Meyler generously arranged for detailed feedback at a later stage. Our respect for the vision and integrity of vice president and editorial director Marcia Horton and executive editor Andrew Gilfillan at Pearson were major factors in our choice of publisher; their patience, commitment and confidence in the project meant a lot to us. Special thanks are due to the strong and accommodating editorial and production staff, particularly senior managing editor Scott Disanno at Pearson for his personal attention, and senior project manager Pavithra Jayapaul at Jouve for her outstanding and steady marshaling of the production through its countless details over the past year.

As described more fully on our page explaining the cover design, it was a pleasure to work closely with Krista Van Guilder, manager of media and design in MIT's interdisciplinary Research Laboratory of Electronics (RLE). RLE is the research home for both of us; the creative environment that it provides for research also impacts our teaching, including the development of this

text. The forthright leadership of Yoel Fink, and Jeffrey Shapiro before him, and the exemplary competence and friendliness of the RLE headquarters staff, set the tone for RLE.

Getting to a bound book has naturally included weathering various challenges along the way. Not the least of these was reconciling our sometimes differing opinions, instincts, approaches, or styles on many minor and sometimes major issues. It helped that we started as friends, and as respectful colleagues. And the experience of working so closely and extensively together in coauthoring this text has, happily, deepened that respect and friendship.

In concluding, we express some individual and more personal thoughts and acknowledgments.

Al Oppenheim

Much of the DNA in my contributions to this text derives, both literally and metaphorically, from my mother, as an extraordinary mentor and role model for me. It still astonishes me that as one of ten children in a poor immigrant family, whose parents arrived from Eastern Europe through Ellis Island, she managed to make her way through college and then medical school in the late 1920's. And then how, as a single parent, she very successfully raised three children while working full time in public health. An incredible and inspiring woman.

I landed at MIT, somewhat by accident, as a freshman in 1955, and shortly after wrote a letter home indicating that at the end of the first year I likely would leave for somewhere that was more fun. Clearly, before long MIT became fun and gratifying for me, and has been a wonderful place at which to have spent my entire academic life, first as a student and then as a faculty member. A tremendous expression of gratitude is due to MIT and more specifically to all of my teachers and mentors throughout this entire period at MIT. And, as indicated on the dedication page of this book, in particular and in very special ways to three mentors: Amar Bose, Ben Gold, and Tom Stockham, whose support and encouragement had a truly profound impact on me.

One of the most fortunate days of my life was the day I walked into the office of a then young assistant professor, Amar Bose, and subsequently signed on as his first teaching assistant. And he eventually signed on as my doctoral thesis advisor. What I learned from him about teaching, research, and life over the many decades of our relationship affected me in ways too numerous to describe. He set the highest standards in everything that he did, and his accomplishments as a teacher, an inventor, and an entrepreneur are legendary. Tom Stockham was another young assistant professor whom I met during my doctoral program. His excitement about and enthusiasm for my ideas gave me the courage to pursue them. During his years at MIT as a faculty member and then as research staff at MIT's Lincoln Laboratory, Tom was one of the pioneers of the then unknown field of digital signal processing. Through that and his later research at the University of Utah, Tom became widely acknowledged as the father of digital audio. Tom was an extraordinary teacher, researcher, practical engineer, and friend. I first met Bernard (Ben) Gold during my early days on the MIT faculty while he was a visiting faculty member in EECS. His work on speech compression was the context for his many pioneering contributions to digital signal processing. Ben's brilliance, creativity, and unassuming style were inspirational to me. He was as eager to learn from those around him as they were from him. Amar, Tom and Ben taught me so many things by example, including the importance of passion and extraordinary standards in every pursuit. Their influence on me is woven into the fabric of my life, my career, and this text. I miss them all, and their spirit remains deeply within me.

As any author knows, textbook writing is a long, difficult, but ultimately rewarding process. Throughout my career I've had the opportunity to write and edit a number of books, and in some cases through two or three editions. In that process, I've had the good fortune of collaborating with other wonderful co-authors in addition to George Verghese, specifically Ron Schafer and Alan Willsky. Such major collaborative projects can often strain relationships, but I'm delighted to say that in all cases, strong bonds and friendships have been the result.

I have often been asked whether I enjoy writing. My response typically has been that "writing is difficult and sometimes painful, but I enjoy *having* written." Projects of this magnitude inevitably require tolerance, patience, support and understanding from family and close friends. I've been incredibly fortunate to have had all of that throughout my career from my wife Phyllis, and from our children Justine and Jason, who have always been the source of tremendous joy. And I'm deeply appreciative of Nora Moran for her special friendship and encouragement (and chicken soup) during the completion of this book.

George Verghese

My parents, George Sr. and Mary, grew up in small towns a mere fifteen miles apart in Kerala, India, but first met each other 2500 miles away in Addis Ababa, Ethiopia, where – young, confident, and adventurous – they had traveled in the early 1950's as teachers. Two further continents later, they continue to set a model for me, of lives lived gracefully. I have everything to thank them for, including the brothers they gave me.

Growing up with physics books to chew on at home surely played a part in landing me at the Indian Institute of Technology, Madras, for undergraduate studies. My favorite professors there, V.G.K. Murti (for network theory) and K. Radhakrishna Rao (for electronic circuits), treated their students with respect, and earned it back many times over with the clarity and integrity of their thinking and teaching, and with their friendly approachability. They are probably why becoming a professor began to seem an attractive proposition to me.

I was fortunate to be introduced to linear system theory by Chi-Tsong Chen at the State University of New York at Stony Brook, and still recall the excitement of my first course taught – and so elegantly – by the author of a

textbook. A few months later I drove cross-country for a life-changing period at Stanford, to work under Thomas Kailath. It was an exceptional time to be there, particularly for the opportunity to learn from him as he completed his own text on linear systems, but also for the interactions with his other students, an amazing group. He undoubtedly thought forty years ago that he was only signing on to be my doctoral thesis advisor, but fifteen years later found himself a part of my family. I continue to learn from him on other fronts, and am still in awe of his acuity, bandwidth, energy, and generosity.

When I joined the faculty at MIT, I thought I would try it out for two years to see how I liked it. I've stayed for over 35. It has been a privilege to be affiliated with such an extraordinary institution, and with the people – students, faculty, and staff – who make it so. Working with Al Oppenheim has been a highlight.

My friends and extended family have helped me keep my labors on this text in necessary perspective, and I'm grateful to them for that. They will no doubt be relieved, the next time they ask, to hear that I'm not still working on the same book as the last time they checked. Throughout this, my dear wife Ann has been much more patient and understanding than I had any right to expect. And whenever she hit her limits, she hauled us off for a vacation that I invariably discovered I needed as much as she did. I could not have completed this project without her cheerful forbearance. Our daughters Deia and Amaya, now launched on trajectories of their own devising, keep us and each other smiling; they are our greatest blessings.