

Basic Technical Mathematics with Calculus

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Preface

Purpose of This Book

This book is designed for students in technical and engineering technology programs. It provides a thorough coverage of the main precalculus topics from beginning algebra through trigonometry, and it continues with analytic geometry, differential and integral calculus, infinite series, differential equations, and double integrals.

Main Features

Just as the need for persons with a technical background is on the rise, so is the need for textbooks that help students prepare for technical and scientific careers. Traditional approaches are not enough: The student has to see how the mathematics is actually applied in technology. This book not only fills the needs of today's technology student; the presentation is exceptionally student-oriented.

- 1. The approach is nonrigorous and intuitive.
- 2. Algebraic drill exercises use notations commonly encountered in technical fields.
- 3. Most sections contain examples and exercises that illustrate how mathematics is applied to technical problems.
- 4. The most important concepts are boxed and labeled for easy reference; other concepts are identified by marginal labels.
- 5. Color ink is used for more than mere decoration. Its main function is to help explain difficult steps.

- 6. Important procedures are summarized; step-by-step procedures are provided whenever feasible.
- 7. Calculator operations are explained whenever appropriate. They are usually identified by the marginal label *Calculator Comment*.
- 8. Examples are worked out in great detail. Marginal notes are used to help explain the steps.
- 9. A large number of drill exercises is included to help reinforce basic concepts.
- 10. Common pitfalls are pointed out in special segments called *Common Errors*.

The use of realistic notations, even in drill exercises, is particularly important, since students tend to have great difficulty in transferring mathematical skills to technical problems that use a different notation. No prior knowledge of the different technical areas is assumed, however.

Most books on algebra and trigonometry encourage occasional use of calculators; some even go so far as to specify exercises to be solved with the aid of a calculator. Since this book is written for students in technology, it is assumed that they will be using scientific calculators throughout. Calculator procedures are explained whenever appropriate. Certain topics, such as approximation of irrational roots by linear interpolation, numerical integration, and solution of equations by Newton's method, are designed to be carried out with a calculator. In the discussion of limits, certain problems are done with a calculator to amplify the ideas presented.

Some other features are:

- 1. Throughout the book, the use of graphs has been given particular attention.
- 2. Both metric and English units are employed. For the metric units SI notations are generally used.
- 3. Every chapter begins with a set of clearly stated objectives and ends with a set of review exercises.
- 4. Cumulative Review Exercises are given at the end of every third chapter.
- 5. A background in algebra and geometry is assumed for most of the material in this book. However, the discussion begins at a sufficiently low level to help overcome possible deficiencies in some areas.

Flexibility

While this book allows considerable flexibility in the order of presentation of topics, careful attention has been paid to the fact that most technical curricula require certain topics at definite times. For example, although scientific notation is introduced in Chapter 1 and basic trigonometry in Chapter 4, a discussion of these topics may be postponed. On the other hand, trigonomet-

ric identities (Chapter 16) can be taken up after Chapter 10. Since determinants up to the third order are discussed in Chapter 3, higher-order determinants and matrices, which are first discussed in Chapter 15, can be introduced earlier or omitted altogether.

The basic plan of the second part of the book is to introduce the differential and integral calculus in Chapters 20–23, with two-year technology programs in mind. The later chapters treat various topics from more advanced areas and may be used according to the requirements of individual programs.

Coverage and Scope

Chapter 1 is a basic introduction to algebra. The topics covered range from signed numbers to operations with polynomials. Zero and negative exponents are introduced in case the teacher wishes to cover scientific notation early in the course (Section 1.11), but these topics may be postponed until Chapter 10 is covered. Because of the large number of applied problems in the book, rounding off numbers and significant figures are also discussed in Chapter 1.

Chapter 2 begins with the solution of linear equations, followed by a section on ratio and proportion and a section on variation. These topics are a natural continuation of the discussion of equations and formulas and provide a concrete basis for the definition of a function in Section 2.6.

Chapter 3 deals with systems of linear equations and their applications. The methods of solution discussed are graphing, addition or subtraction, and substitution. Determinants up to the third order are also introduced. (Higher-order determinants are covered in Chapter 15.)

Chapter 4 is an introduction to right-triangle trigonometry. (As already noted, many technical curricula require basic trigonometry early.)

Chapter 5 on factoring and fractions is organized to allow a gradual mastery of factoring: Basic special products, introduced in Section 5.1, are followed by the corresponding factoring cases in Section 5.2. Section 5.3 introduces more special products, and Section 5.4 the corresponding factoring cases. Factoring by grouping is then discussed in Section 5.5. The rest of the chapter is devoted to operations with fractions, complex fractions, and fractional equations.

Chapter 6 covers quadratic equations and their applications. The methods of solution discussed are factoring, completing the square, and the use of the quadratic formula.

Chapter 7 expands the discussion of trigonometric functions begun in Chapter 4. The topics covered are the functions of angles in any quadrant, functions of special angles, radian measure, and applications of radian measure, including linear and angular velocity. The emphasis is on the use of calculators rather than of tables.

Chapter 8 covers the graphs of trigonometric functions, as well as graphing by addition of coordinates. Considerable space is given to applications of sinusoidal functions.

Vectors and applications of vectors are treated in Chapter 9. The sections on the sine and cosine laws contain additional vector applications.

Chapter 10 comprises a detailed treatment of exponents and radicals: zero, negative and fractional exponents, and fundamental operations with radicals.

Chapter 11 covers complex numbers: rectangular, polar, and exponential forms of complex numbers and powers and roots by De Moivre's theorem. The chapter ends with a brief discussion of phasors.

Chapter 12 contains a thorough treatment of logarithmic and exponential functions with special emphasis on applications. Also included are discussions of natural logarithms, properties of logarithms, exponential and logarithmic equations, and graphing on logarithmic paper. As in the case of trigonometric functions, the emphasis is on the use of calculators rather than of tables. (Computations with logarithms are introduced only briefly.)

Chapter 13 begins with a brief discussion of conic sections to provide a basis for solving systems of two quadratic equations by means of graphs. Algebraic methods are discussed next, followed by equations in quadratic form, and fractional equations.

Chapter 14 contains a detailed treatment of higher-order equations. The method of approximating irrational roots by linear interpolation, which has become highly practical with the advent of the calculator, is also discussed in detail.

Higher-order determinants and a detailed treatment of matrices can be found in Chapter 15.

Chapter 16 is devoted to trigonometric identities, trigonometric equations, and inverse trigonometric functions.

Chapter 17 on inequalities covers both graphical and algebraic methods of solution. The chapter ends with an optional section on linear programming.

Geometric series, the binomial theorem, and binomial series are treated in Chapter 18.

Chapter 19 covers the traditional topics of analytic geometry. Although intended mainly for use in calculus, applications of conic sections are also discussed.

The derivative is introduced in Chapter 20 and applications of the derivative in Chapter 21.

Chapters 22 and 23 cover basic integration, including the trapezoidal rule and Simpson's rule. Although the integral is defined as a Riemann sum, the emphasis is on a shortcut using a single typical element, referred to as a "sloppy Riemann sum." This is used extensively in Chapter 23 on applications of integration. The purpose is to enable the student to gain the necessary insight for setting up integrals in many different situations, both physical and geometric.

Chapter 24 covers the transcendental functions and their applications to various technical fields. Also included is a brief discussion of L'Hospital's rule.

Chapter 25 develops the different integration techniques, covered in considerable detail. Techniques may be selected according to individual needs. For example, if time constraints do not allow a detailed discussion of trigonometric substitution, integrals of this form can be obtained by use of tables, discussed in Section 25.8.

Chapter 26 emphasizes power-series expansions but contains an optional section on tests of convergence. A section of Fourier series is included, with electrical technology curricula in mind.

Chapters 27, 28, and 29 are devoted to differential equations, since these provide particularly interesting and powerful applications in numerous technical fields. Chapter 29 on Laplace transforms includes a section on partial fractions. Although time-consuming, partial fractions are essential to the Laplace transform technique.

A brief discussion of Newton's method is given in Appendix A. This method of solving equations is particularly useful for technology students.

Supplements

The answers to the odd-numbered exercises are given in the answer section; answers to even-numbered problems are published in a separate Instructor's Manual. A Student Solutions Manual with approximately every fourth problem worked in detail is also available.

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