A Guide to Introductory Physics Teaching

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Preface

Starting approximately twenty years ago, members of the physics teaching community began conducting systematic observations and research on student learning and understanding of physical concepts, models, and lines of reasoning. Some of these investigations began with, or subsequently spilled over into, research on more general aspects of the development of the capacity for abstract logical reasoning. In this book, I have tried to bring together as many as possible of the relevant insights into the teaching of the most basic aspects of introductory physics—covering high school through first year college level, including basic aspects of the course aimed at physics and engineering majors, without penetrating the full depth of the latter.

Very little that I present is based on conjecture. I have invoked and referred to most of the systematic research of which I am aware, and I have drawn on my own observations, which have been under way for more than forty years and have been extensively replicated over that time. One of my sources has been the direct interview in which one asks questions and listens to the individual student response; the other has been the analysis of students' written response to questions on tests and examinations. It is impossible to give all of the protocols of student interviews and all of the detailed supporting evidence without producing a book of impossible length. Although I give specific examples of student response from time to time, some of the insights are asserted without the full support they deserve. I can only ask the careful and critical reader to bear with these gaps, test them as opportunity arises, or turn to the more detailed literature for deeper penetration.

It is also impossible to include, in a book of reasonable length, all of the insights emerging from research on teaching, learning, and cognitive development. The literature is rich, varied, and rapidly increasing. I have been selective and have tried to include observations having the most direct bearing on classroom practice at the most basic levels of subject matter; the list of references will open the door to those wishing to pursue greater detail and explore primary evidence. Where a significant reference at this level is missing, the fault is in my judgment or in my not having fully encompassed the extensive literature.

Both the *American Journal of Physics* and *The Physics Teacher* are rich in articles discussing the logic and epistemology of various laws and concepts, outlining improved modes of presentation, suggesting demonstrations and other ways of making abstractions clearer and more concrete, describing ways of engaging students in direct activities, criticizing loose and faulty approaches, introducing new derivations, new laboratory experiments, and so forth. Every one of these functions is valuable and important to our community, and I wish someone, more competent than I, would undertake to bring together the heritage that has accumulated over the years in these areas into another book on physics teaching.

It is necessary for me to make clear, however, that my own purpose is different. I have undertaken to discuss some of the elements that I believe *underlie* and *precede* a great many of the ideas and presentations appearing in the journals. In fact, many of the excellent suggestions appearing in the journals turn out to be ineffective with large numbers of students, not because of anything wrong with the suggestions, but because the students have not had a chance to master the necessary *prior* concepts and lines of abstract logical reasoning. It is to this end that I have elected to concentrate on some of these prior aspects of cognitive development and on underlying problems of learning and understanding that have been commanding increasing attention in recent years. In doing this, I in no way disparage the valuable materials and modes of presentation that are described in the journals and that enter in full force at the points where I leave off.

It must further be emphasized that I am *not* formulating prescriptions as to how items of subject matter should be presented to the students or how they should be taught, nor am I suggesting that there is one single way of getting any particular item "across to the student." There is tremendous diversity in style and method of approach among teachers, and such diversity should flourish. My objective is to bring out as clearly and explicitly as possible the conceptual and reasoning difficulties many students encounter and to point up aspects of logical structure and development that may not be handled clearly or well in substantial segments of textbook literature. With respect to modes of attack on these instructional problems (avenues of explanation and presentation, balance of laboratory versus classroom experience, use of computers and of audiovisual aids), I defer to the style and predilections of the individual teacher.

I have endeavored to cover the range from high school physics through college and university calculus-based courses. Some of the material, therefore, goes well beyond high school level, and high school teachers should draw appropriate lines, limiting the more sophisticated material to their front running students if invoking it at all. At the other end of the spectrum, teachers in elite colleges, dealing with highly selected students, or teachers with a highly selected student body in calculus-based engineering-physics courses will find less relevance in the discussions of some of the more mundane underpinnings. However, it is necessary to issue a warning: there is much more overlap between the disparate populations than most teachers realize, and it is frequently startling to find how many students, at a presumably fairly high level, have the same difficulties, preconceptions, and misconceptions as do much less sophisticated students. It is only the *percentage* of students having a certain difficulty that changes as one goes up or down the scale; there is not an abrupt drop to zero at some intermediate level. Also, students at higher levels of scholastic ability, especially verbal skills, can usually remediate or overcome such initial difficulties at a more rapid pace than do other students, and a teacher needs to calibrate each of the classes with which he or she must deal.

Some of the chapters contain appendixes giving illustrations of possible test questions or homework problems. To keep down discursive length, I have not included detailed discussions of these questions, but they are all designed to implement some of the knowledge gained in the research protocols. They illustrate the kinds of questions that might be *added* to the normal regimen of quantitative end-of-chapter problems to confront the mind of the learner with aspects otherwise not being made explicit. The examples being given are an invitation more than an end point. The pool of such questions must be greatly expanded to enhance variety and flexibility. Such expansion will take place not through the output of one individual, whose imagination gives out at some finite point, but through the superposition of effort on the part of numerous interested individuals, each of whom brings a new imagination to the effort. I long to see my limited set of examples greatly expanded.

Finally I point to the following unwelcome truth: much as we might dislike the implications, research is showing that didactic exposition of abstract ideas and lines of reasoning (however engaging and lucid we might try to make them) to passive listeners yields pathetically thin results in learning and understanding—except in the very small percentage of students who are specially gifted in the field. Even in the calculus-based course, many students have the difficulties, and need all of the help, outlined in these pages. In expressing this caveat, I am, of course, *not* advocating *un*clear exposition. I am pointing to the necessity of supplementing lucid exposition with exercises that engage the mind of the learner and extract explanation and interpretation in his or her own words.

It is obvious that ideas and information such as I have summarized here cannot be developed in seclusion. I am deeply indebted to the hundreds of students who have submitted to my questioning, accepting the tension that goes with my shutting up and waiting for their answers. I am indebted also to the many colleagues and associates with whom I have discussed physics, prepared test questions, and worried about the meaning of learning and understanding. Among these are my former colleagues at Amherst College: Bruce Benson, Colby Dempesy, Joel Gordon, Robert Romer, Theodore Soller, and Dudley Towne; at the University of Washington: David Bodansky, Kenneth Clark, Ronald Geballe, James Gerhart, Patricia Heller, Lillian McDermott, James Minstrell, and Philip Peters. Robert Romer, Kenneth Clark, and Phillip Peters have read extensive sections of this book and have supplied me with valued criticism, corrections, and suggestions.

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CHAPTER 1

Underpinnings

1.1 INTRODUCTION

Several fundamental gaps in the background of students may seriously impede their grasp of the concepts and lines of reasoning that we seek to cultivate from the beginning of an introductory physics course. These gaps, having to do with understanding the concepts of "area" and "volume" and with reasoning involving ratios and division, are often encountered, even among students at the engineering-physics level.

In principle, these gaps should not exist because the ideas are dealt with, and should have been mastered, at earlier levels in the schools. It is an empirical fact, however, that such mastery has not been achieved, and ignoring the impediment is counterproductive.

Unfortunately, it is illusory to expect to remediate these difficulties with a few quick exercises, in artificial context, at the start of a course. Most students can be helped to close the gaps, but this requires *repeated* exercises that are spread out over time and are integrated with the subject matter of the course itself. This statement is *not* a matter of conjecture; it reflects empirical experience our physics education research group at the University of Washington has encountered repeatedly [Arons (1976), (1983b), (1984c)].

This chapter describes some of the learning difficulties that are involved in the development of a number of underpinnings, including arithmetical reasoning, and suggests exercises that can be made part of the course work.

1.2 AREA

The concept of area is the foundation of many of the other basic physical concepts, such as pressure, stress, energy flux, and coefficients of diffusion and heat conduction. It underpins all the ratio reasoning associated with geometrical scaling. Furthermore, it is essential to the interpretation of velocity change as area under the graph of acceleration versus clock reading, to the interpretation of position change as area under the graph of velocity versus clock reading, to the definitions of work and impulse, and to the interpretation of integrals in general.

If you ask students how one arrives at numerical values for "area" or "extent of surface," many—if they have any response at all—will say "length times width." If you then sketch some very irregular figure without definable length or width and ask about assigning a numerical value to the area of the figure, very little response of any kind is forthcoming. Students who respond in this way have not formed a clear operational definition of "area."

The reason for this is fairly simple: Although the grade school arithmetic books, when they introduce the area concept, do have a paragraph about selecting a unit square, imposing a grid on the figure in question, and counting the squares within the figure, virtually none of the students have ever gone through such a procedure themselves in homework exercises. They have never been asked to define "area." All they have ever done is calculate areas of regular figures such as squares, rectangles, parallelograms, or triangles, using memorized formulas that they no longer connect with the operation of counting the unit squares, even though this connection may have been originally asserted.

Furthermore, virtually none of the students have had any significant exposure to the notion of operational definition. They have had little or no practice in defining a term by reference to shared experience or by describing, in simple words of prior definition, the actions through which one goes to develop the numerical value being referred to in the name of a technical concept.

1.3 EXERCISES WITH "AREA"

In introductory physics teaching, it is desirable to invoke the area concept at the earliest possible opportunity. Students should be led to articulate the operational definition in their own words—and to do so on tests. (This is an excellent opportunity to introduce the concept of operational definition in a context that is familiar and relatively unthreatening.) The fact that they had been using the technical term "area" without adequate mastery of the concept behind it makes a salutory impression on many students.

Homework and test problems should give students opportunity to execute the operations they describe in the definition, right through the selection of the unit square, superposition of the grid on the figure in question, and actually counting the squares. The operation of counting must involve the estimation of squares contained around the periphery of the figure. To many students the necessity of estimating the fractions appears in some sense "sinful," since it involves "error" and is not "exact," as seems to be the value obtained from a formula. The actual experience of counting and estimating should begin with "pure" areas, that is, surface extent of arbitrarily and irregularly shaped geometrical figures. Then, as soon as it becomes appropriate, the exercises should be extended to measurement and interpretation of areas under v versus t and under a versus t graphs. (This, of course, adds the arithmetical reasoning associated with the dimensionality of the coordinates.)

In calculus-physics courses, the latter exercises should be explicitly linked with the mathematical concept of "integral." Although this might seem so obvious as to be not worthy of mention, many students have not actually established this connection even though they may be taking, or may have completed, a calculus course. Although they have been *told*, perhaps many times, that the integral can be interpreted as an area, the idea has not registered because it has not been made part of the individual student's concrete experience; and they have never had the opportunity to articulate the idea in their own words.

Such exercises should be repeated still later when the context begins to involve "work" and "impulse." It is only such recycling of ideas over fairly extended periods of time, reencountered in increasingly rich context, that leads to a firm assimilation in many students.

In noncalculus-physics courses, the concept of "integral" is not at hand and is not necessary. Dealing with the areas, however, breaks the shackles to eternally constant quantities and shows the students how physics can easily and legitimately deal with continuous change. "Capturing the fleeting instant" was one of the great intellectual triumphs of the seventeenth century, and students can be given some sense of this part of their intellectual heritage through calculations that they can easily make without the necessity of a formal course in the calculus.

1.4 VOLUME

Initially, most students have the same difficulty with "volume" as with "area." They grasp for formulas without having registered an operational definition of the concept. As a result, quite a few students do not, in fact, discriminate between area and volume; they use the words carelessly and interchangeably as metaphors for size.

Once the operational definition of "area" has been carefully developed and anchored in the concrete experience of counting squares, however, the operational definition of "volume" can be elicited relatively easily. The analogy to "area" is readily perceived, and the counting of unit cubes is quickly accepted.

1.5 MASTERY OF CONCEPTS

It should be emphasized at this point that mastery of the operational definitions of "area" and "volume" up to the point of recognizing the counting of unit squares or cubes is only a beginning; it is still far short of the ability to use the concepts in more extended context. At this stage, for example, some students (particularly those who have had little or no prior work in science) do not discriminate between mass and volume.[1] Many students, including those in engineering-physics courses, are, at this stage, still unable to compare final with initial areas or volumes when the linear dimensions of an object have been scaled up or down.

The problem of scaling is a particularly important one. It involves ratio reasoning and will be discussed in more detail in Section 1.12.

1.6 RATIOS AND DIVISION

One of the most severe and widely prevalent gaps in cognitive development of students at secondary and early college levels is the failure to have mastered reasoning involving ratios. The poor performance reproducibly observed on Piagetian tasks of ratio reasoning has become well known during the past 15 years [McKinnon and Renner (1971); Karplus, et al. (1979); Arons and Karplus (1976); Chiappetta (1976)]. This disability, among the very large number of students who suffer from it, is one of the most serious impediments to their study of science.

For convenience, I separate reasoning with ratios and division into two levels or stages: (1) verbally interpreting the result obtained when one number is divided by another; (2) using the preceding interpretation to calculate some other quantity.

[1]For evidence concerning this assertion and for strategies that help students achieve such discrimination see McDermott, Piternick, and Rosenquist (1980); McDermott (1980); McDermott, Rosenquist, and van Zee (1983).

1.7 VERBAL INTERPRETATION OF RATIOS

Reasoning with ratios and division requires, as a first step, the capacity to interpret verbally the meaning of a number obtained from a particular ratio. The verbal interpretations are somewhat different in different contexts. Many students are deficient in this capacity and need practice in interpreting ratios in their own words.

In the primitive case in which the numbers have not been given specific physical meaning, we interpret the result of, say 465/23, as the number of times 23 is contained in 465. This may sound like a trivial statement, but it is not. Most students have memorized (successfully or unsuccessfully, as the case may be) the algorithm of division but have never been given the opportunity to recognize it as a shorthand procedure for counting successive subtractions of 23 from 465. Thus they do not see the operation of division in perspective or translate it into simpler prior experience. The phrase "goes into" is memorized without relation to other contexts. Those who have not developed this perspective should be given the opportunity to count the successive subtractions and to begin to see what they are doing in the memorized algorithm. They should finally have to tell the whole story in their own words.

At a next higher level of sophistication, we may be dealing with a ratio of dimensionally identical quantities, for example, L_2/L_1, the ratio, say, of the heights of two buildings, or of distances from a fulcrum in balancing, or the linear scaling of a geometrical figure. Here the numerical value of the ratio serves as a *comparison*: it tells us how many times larger (or smaller) one length is compared to the other.

Next we encounter division of dimensionally *inhomogeneous* quantities: mass in grams divided by volume in cubic centimeters; position change in meters divided by a time interval in seconds; dollars paid divided by number of pounds purchased. Here the result of division tells us how much of the numerator is associated with *one* unit of whatever is represented in the denominator.

Finally, if we have 500 g of a material that has 3.0 g in each cubic centimeter, the numerical value of 500/3.0 tells us how many "packages" of size 3.0 g are contained in the 500 g sample. Since each such "package" corresponds to one cubic centimeter, we have obtained the number of cubic centimeters in the sample.

1.8 EXERCISES IN VERBAL INTERPRETATION

Many students have great difficulty giving verbal interpretations like those illustrated in the preceding section, since they have almost never been asked to do so. Without such practice in at least several different contexts, students do not think about the meaning of the calculations they are expected to carry out, and they take refuge in memorizing patterns and procedures of calculation, manipulating formulas, rather than penetrating to an understanding of the reasoning. As a consequence, when they find themselves outside the memorized situations, they are unable to solve problems that involve successive steps of arithmetical reasoning.

Explaining or telling students who are in such difficulty the meaning of particular ratios, however frequently or lucidly this may be done, has very little effect. It is necessary to ask questions that lead the students to articulate the interpretations and explanations in their own words. Here are some typical excerpts from such conversations:

Suppose students having difficulty with a problem involving the use of the density concept are asked: "We took the measured mass (340 g) of an object and divided it by the volume (120 cm^3). How do you interpret the number 340/120? Tell what it means, using the simplest possible words." Some will answer "That is the density." These students have not separated the technical term, the *name* of the resulting number, from the verbal interpretation of its meaning. (This involves an important cognitive process that will be discussed in another chapter.)

When it is pointed out that the name is not an interpretation, some students will say "mass per volume"; others might say "the number of grams in 120 cubic centimeters." (Exactly parallel statements are likely to be given if the ratio is position change divided by time interval.) Very few students having trouble with the original problem will give a simple statement to the effect that we have obtained the number of grams in *one* cubic centimeter of the material.

One can now adopt the strategy of going back to some more familiar context: "Suppose we go to a store and find a box costing $5.00 and containing 3 kg of material. What is the meaning of the number 5.00/3?" Some students will still say "That is how much you pay for 3 kg" but, in this more familiar context, many will recognize that we have calculated how many dollars we pay for *one* kilogram. (The former group is in need of further dialog, using more concrete examples, before a correct response is found.) One can now try to get the students to the generalization that in such situations the resulting number tells us "how many of these (in the numerator) are associated with *one* of those (in the denominator)."

If one then asks: "In the case of the box costing $5.00 and containing 3 kg, suppose we now consider the number 3/5.00. In light of what we concluded in the previous example, does this number have an interpretation?" Many students, including some who gave the correct interpretation of 5.00/3, now encounter difficulty. Some revert to earlier locutions such as "how many kilograms you get for 5.00"; many consider the number meaningless or uninterpretable.

In such instances there seem to be two difficulties superposed: (1) although the students may have previously been given some opportunity to think about or calculate "unit cost" (how much we pay for one kilogram), they rarely, if ever, have been asked about the inverse (how much one gets for one dollar). (2) 5.00/3 involved the division of a larger number by a smaller one. To many students this is more intelligible and less frightening than the fraction 3/5.00.

After students have been led through the parallel interpretation of *both* ratios, one can usually go back to a case such as mass divided by volume or change of velocity divided by time interval and elicit a correct interpretation of the new ratio and its inverse. Then one can elicit the generalization being sought, namely, that such a ratio tells us how much of the numerator is associated with *one* unit of whatever is represented in the denominator. It is essential, however, to elicit the word "one"; use of the word "per" by the student is no assurance that he or she understands the concept (see the discussion in the next section).

1.9 COMMENT ON THE VERBAL EXERCISES

Note the strategy being employed in the dialogs suggested in the preceding section: although some students have responded previously to problems such as "calculate the cost of one kilogram if 3 kg cost $5.00," very few students have ever been confronted with the ratio and asked to interpret it in words, that is, they have never reversed the line of thought, traversing it in the direction opposite to that previously experienced.

In Piagetian terminology, the term "operations" denotes reasoning processes that can be reversed by the user. Thus students who can calculate the unit cost