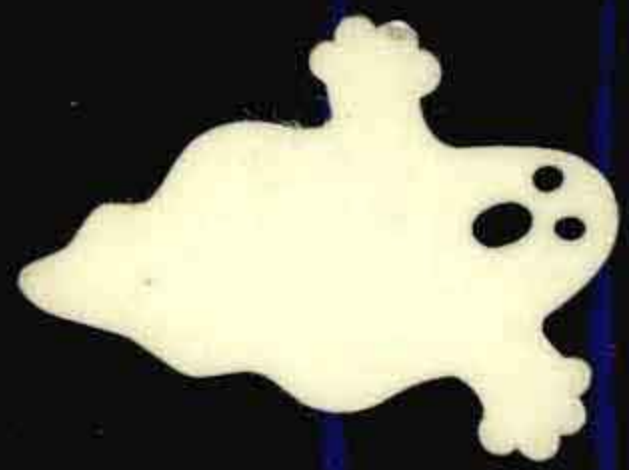


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7TH

Analytic Geometry

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A Brief Table of Integrals

1. $\int u \, dv = uv - \int v \, du$
2. $\int a^u \, du = \frac{a^u}{\ln a} + C, \quad a \neq 1, \quad a > 0$
3. $\int \cos u \, du = \sin u + C$
4. $\int \sin u \, du = -\cos u + C$
5. $\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$
6. $\int (ax + b)^{-1} \, dx = \frac{1}{a} \ln |ax + b| + C$
7. $\int x(ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a^2} \left[\frac{ax + b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$
8. $\int x(ax + b)^{-1} \, dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax + b| + C$
9. $\int x(ax + b)^{-2} \, dx = \frac{1}{a^2} \left[\ln |ax + b| + \frac{b}{ax + b} \right] + C$
10. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$
11. $\int (\sqrt{ax + b})^n \, dx = \frac{2}{a} \frac{(\sqrt{ax + b})^{n+2}}{n+2} + C, \quad n \neq -2$
12. $\int \frac{\sqrt{ax + b}}{x} \, dx = 2\sqrt{ax + b} + b \int \frac{dx}{x\sqrt{ax + b}}$
13. (a) $\int \frac{dx}{x\sqrt{ax + b}} = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax + b}{-b}} + C, \quad \text{if } b < 0$
 (b) $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right| + C, \quad \text{if } b > 0$
14. $\int \frac{\sqrt{ax + b}}{x^2} \, dx = -\frac{\sqrt{ax + b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax + b}} + C$
15. $\int \frac{dx}{x^2\sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}} + C$
16. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
17. $\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$
18. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x + a}{x - a} \right| + C$
19. $\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{2a^2} \int \frac{dx}{a^2 - x^2}$
20. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln |x + \sqrt{a^2 + x^2}| + C$

$$21. \int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + C$$

$$22. \int x^2 \sqrt{a^2 + x^2} dx = \frac{x(a^2 + 2x^2) \sqrt{a^2 + x^2}}{8} - \frac{a^4}{8} \sinh^{-1} \frac{x}{a} + C$$

$$23. \int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \sinh^{-1} \left| \frac{a}{x} \right| + C$$

$$24. \int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \sinh^{-1} \frac{x}{a} - \frac{\sqrt{a^2 + x^2}}{x} + C$$

$$25. \int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \sinh^{-1} \frac{x}{a} + \frac{x \sqrt{a^2 + x^2}}{2} + C$$

$$26. \int \frac{dx}{x \sqrt{a^2 + x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$$

$$27. \int \frac{dx}{x^2 \sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$$

$$28. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$29. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$30. \int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$$

$$31. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$32. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$$

$$33. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C$$

$$34. \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$35. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$36. \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C = \ln |x + \sqrt{x^2 - a^2}| + C$$

$$37. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + C$$

$$38. \int (\sqrt{x^2 - a^2})^n dx = \frac{x(\sqrt{x^2 - a^2})^n}{n+1} - \frac{na^2}{n+1} \int (\sqrt{x^2 - a^2})^{n-2} dx, \quad n \neq -1$$

$$39. \int \frac{dx}{(\sqrt{x^2 - a^2})^n} = \frac{x(\sqrt{x^2 - a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{x^2 - a^2})^{n-2}}, \quad n \neq 2$$

$$40. \int x(\sqrt{x^2 - a^2})^n dx = \frac{(\sqrt{x^2 - a^2})^{n+2}}{n+2} + C, \quad n \neq -2$$

$$41. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \cosh^{-1} \frac{x}{a} + C$$

$$42. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$$

$$43. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \cosh^{-1} \frac{x}{a} - \frac{\sqrt{x^2 - a^2}}{x} + C$$

Continued overleaf.

$$\begin{aligned}
44. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx &= \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + \frac{x}{2} \sqrt{x^2 - a^2} + C \\
45. \int \frac{dx}{x\sqrt{x^2 - a^2}} &= \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C = \frac{1}{a} \cos^{-1} \left| \frac{a}{x} \right| + C \\
46. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} &= \frac{\sqrt{x^2 - a^2}}{a^2 x} + C & 47. \int \frac{dx}{\sqrt{2ax - x^2}} &= \sin^{-1} \left(\frac{x - a}{a} \right) + C \\
48. \int \sqrt{2ax - x^2} dx &= \frac{x - a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + C \\
49. \int (\sqrt{2ax - x^2})^n dx &= \frac{(x - a)(\sqrt{2ax - x^2})^n}{n + 1} + \frac{na^2}{n + 1} \int (\sqrt{2ax - x^2})^{n-2} dx, \\
50. \int \frac{dx}{(\sqrt{2ax - x^2})^n} &= \frac{(x - a)(\sqrt{2ax - x^2})^{2-n}}{(n - 2)a^2} + \frac{(n - 3)}{(n - 2)a^2} \int \frac{dx}{(\sqrt{2ax - x^2})^{n-2}} \\
51. \int x\sqrt{2ax - x^2} dx &= \frac{(x + a)(2x - 3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2} \sin^{-1} \frac{x - a}{a} + C \\
52. \int \frac{\sqrt{2ax - x^2}}{x} dx &= \sqrt{2ax - x^2} + a \sin^{-1} \frac{x - a}{a} + C \\
53. \int \frac{\sqrt{2ax - x^2}}{x^2} dx &= -2 \sqrt{\frac{2a - x}{x}} - \sin^{-1} \left(\frac{x - a}{a} \right) + C \\
54. \int \frac{x dx}{\sqrt{2ax - x^2}} &= a \sin^{-1} \frac{x - a}{a} - \sqrt{2ax - x^2} + C \\
55. \int \frac{dx}{x\sqrt{2ax - x^2}} &= -\frac{1}{a} \sqrt{\frac{2a - x}{x}} + C \\
56. \int \sin ax dx &= -\frac{1}{a} \cos ax + C & 57. \int \cos ax dx &= \frac{1}{a} \sin ax + C \\
58. \int \sin^2 ax dx &= \frac{x}{2} - \frac{\sin 2ax}{4a} + C & 59. \int \cos^2 ax dx &= \frac{x}{2} + \frac{\sin 2ax}{4a} + C \\
60. \int \sin^n ax dx &= \frac{-\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax dx \\
61. \int \cos^n ax dx &= \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax dx \\
62. (a) \int \sin ax \cos bx dx &= -\frac{\cos (a+b)x}{2(a+b)} - \frac{\cos (a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2 \\
& (b) \int \sin ax \sin bx dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)}, \quad a^2 \neq b^2 \\
& (c) \int \cos ax \cos bx dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)}, \quad a^2 \neq b^2 \\
63. \int \sin ax \cos ax dx &= -\frac{\cos 2ax}{4a} + C \\
64. \int \sin^n ax \cos ax dx &= \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1
\end{aligned}$$

This table is continued on the endpapers at the back.

65. $\int \frac{\cos ax}{\sin ax} dx = \frac{1}{a} \ln |\sin ax| + C$
66. $\int \cos^n ax \sin ax dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$
67. $\int \frac{\sin ax}{\cos ax} dx = -\frac{1}{a} \ln |\cos ax| + C$
68. $\int \sin^n ax \cos^m ax dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax dx,$
 $n \neq -m \quad (\text{If } n = -m, \text{ use No. 86.})$
69. $\int \sin^n ax \cos^m ax dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax dx,$
 $m \neq -n \quad (\text{If } m = -n, \text{ use No. 87.})$
70. $\int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right] + C, \quad b^2 > c^2$
71. $\int \frac{dx}{b+c \sin ax} = \frac{-1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \sin ax + \sqrt{c^2-b^2} \cos ax}{b+c \sin ax} \right| + C, \quad b^2 < c^2$
72. $\int \frac{dx}{1+\sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$
73. $\int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$
74. $\int \frac{dx}{b+c \cos ax} = \frac{2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2} \right] + C, \quad b^2 > c^2$
75. $\int \frac{dx}{b+c \cos ax} = \frac{1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \cos ax + \sqrt{c^2-b^2} \sin ax}{b+c \cos ax} \right| + C, \quad b^2 < c^2$
76. $\int \frac{dx}{1+\cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$
77. $\int \frac{dx}{1-\cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$
78. $\int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$
79. $\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C$
80. $\int x^n \sin ax dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax dx$
81. $\int x^n \cos ax dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax dx$
82. $\int \tan ax dx = \frac{1}{a} \ln |\sec ax| + C$
83. $\int \cot ax dx = \frac{1}{a} \ln |\sin ax| + C$
84. $\int \tan^2 ax dx = \frac{1}{a} \tan ax - x + C$
85. $\int \cot^2 ax dx = -\frac{1}{a} \cot ax - x + C$
86. $\int \tan^n ax dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax dx, \quad n \neq 1$
87. $\int \cot^n ax dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax dx, \quad n \neq 1$
88. $\int \sec ax dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$
89. $\int \csc ax dx = -\frac{1}{a} \ln |\csc ax + \cot ax| + C$

Continued overleaf.

90. $\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$ 91. $\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$
92. $\int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx, \quad n \neq 1$
93. $\int \csc^n ax \, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx, \quad n \neq 1$
94. $\int \sec^n ax \tan ax \, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$
95. $\int \csc^n ax \cot ax \, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$
96. $\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1 - a^2 x^2} + C$
97. $\int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1 - a^2 x^2} + C$
98. $\int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \ln(1 + a^2 x^2) + C$
99. $\int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1 - a^2 x^2}}, \quad n \neq -1$
100. $\int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1 - a^2 x^2}}, \quad n \neq -1$
101. $\int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{1 + a^2 x^2}, \quad n \neq -1$
102. $\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$ 103. $\int b^{ax} \, dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0, \quad b \neq 1$
104. $\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) + C$ 105. $\int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$
106. $\int x^n b^{ax} \, dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} \, dx, \quad b > 0, \quad b \neq 1$
107. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$
108. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$
109. $\int \ln ax \, dx = x \ln ax - x + C$
110. $\int x^n \ln ax \, dx = \frac{x^{n+1}}{n+1} \ln ax - \frac{x^{n+1}}{(n+1)^2} + C, \quad n \neq -1$
111. $\int x^{-1} \ln ax \, dx = \frac{1}{2} (\ln ax)^2 + C$ 112. $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$
113. $\int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$ 114. $\int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$
115. $\int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$ 116. $\int \cosh^2 ax \, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$
117. $\int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$

$$118. \int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$$

$$119. \int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$$

$$120. \int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$$

$$121. \int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$$

$$122. \int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$$

$$123. \int \tanh ax \, dx = \frac{1}{a} \ln (\cosh ax) + C$$

$$124. \int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$$

$$125. \int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$$

$$126. \int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$$

$$127. \int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$$

$$128. \int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$$

$$129. \int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$$

$$130. \int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$$

$$131. \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$$

$$132. \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C$$

$$133. \int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx, \quad n \neq 1$$

$$134. \int \operatorname{csch}^n ax \, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx, \quad n \neq 1$$

$$135. \int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$$

$$136. \int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

$$137. \int e^{ax} \sinh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$138. \int e^{ax} \cosh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$139. \int_0^\infty x^{n-1} e^{-x} \, dx = \Gamma(n) = (n-1)!, \quad n > 0.$$

$$140. \int_0^\infty e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad a > 0$$

$$141. \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \cdot \frac{\pi}{2}, & \text{if } n \text{ is an even integer } \geq 2, \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n}, & \text{if } n \text{ is an odd integer } \geq 3 \end{cases}$$

7 TH EDITION

Calculus and Analytic Geometry

7 TH EDITION

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Preface

In this new edition of *Calculus and Analytic Geometry* we have tried to retain all the qualities and features that users of the previous editions have found most useful. Yet this is one of the most comprehensive revisions of both text and art in the more than thirty-year history of the book. The revision is based on dozens of reviews; numerous conversations with users of earlier editions; letters of advice from friends, students, and instructors from all over the world; and extensive classroom trials of revised lessons. Our overall revision plan has been to make the book more readable and the material more accessible to beginners without compromising the standards or coverage its users want to see.

Audience and Prerequisites This book provides everything necessary for the standard three-semester or four-quarter calculus sequence in the freshman and sophomore years of college. The prerequisites are the usual exposure to algebra and trigonometry, but to refresh everyone's memory Chapter 1 begins with brief reviews of coordinates, lines, functions, and graphs. There is also a review of trigonometry in Chapter 2. The book is available in a two-volume version: Volume I covers Chapters 1–12 and Volume II covers Chapters 11–20.

Features From Previous Editions

As always, our aim has been to teach the mathematics of calculus and to provide the training readers will need to use calculus effectively in their later academic and professional work. To accomplish this, we have preserved the book's mathematical level, its orientation toward applications, its concentration on worked examples, and the number and variety of the exercises, and continued to show the connections between calculus and some of the numerical methods used in other courses.

Mathematical Level Although many of the presentations in this new edition are noticeably easier than in earlier editions, the level of rigor is about the same. We try to explain things without belaboring the obvious and without answering questions that readers are not yet ready to ask. For example, we state the max-min theorem for continuous functions on closed intervals and use it to develop the Mean Value Theorem, but we do not prove the max-min theorem or explore the properties of the real number system on which the theorem

depends. We give a few simple ϵ - δ proofs about limits in Chapter 1 but put the proofs on the more complicated limit theorems in the appendixes.

Applications Calculus was invented to solve problems in physics and astronomy, and although it has since developed into a far-reaching mathematical discipline in its own right, most of its applications outside of mathematics still involve science and engineering. As in earlier editions, the applications of the present book are directed mainly along these lines. Typical applications include calculating extreme values, centers of mass, work, and hydrostatic force; predicting satellite orbits; and describing fluid flow. (See pp. 192, 345, 356, 363, 827, and 1018.) However, in recent years calculus has become important in many other fields, including economics, business, the life sciences, and even the physics of sports. We have therefore included a variety of applications from these fields as well, on such topics as average daily inventory, birth rates and population growth, and the work required to hit a golf ball or serve a tennis ball, to mention only a few. (See pp. 343, 428, and 361.) Whenever we feel we can do so without burdening the text, we make connections between calculus and real life. We also take more time in this edition with the problem-solving steps in applications that involve mathematical modeling, as in the introduction to related rates (pp. 203–204) and the solution of the hanging cable problem (pp. 577–578).

Worked Examples The exercises are where the readers do the work but the examples are where *we* do the work. We have kept the favorite examples and added a number of fresh ones, often showing solution steps in more detail than before. We have also replaced some of the harder examples with easier ones that make the same points. The topics range from insulating the Trans-Alaska Pipeline to draining and filling a swamp; from household electric power to a method for drawing parabolas; from analyzing the shape of the Gateway Arch to the West in St. Louis to the mysteries of computer calculations and the variation of the temperature below the earth's surface. (See pp. 139, 299, 342, 526, 581, 742, and 864.)

Problem Sets Each problem set contains a mixture that runs from routine mathematical exercises to more challenging problems. In many of the problem sets, the “ramp up” is more gradual than before. Nearly every set offers practice in applications and many offer calculator exercises. (See pp. 126–127, 199–203, 299–301.) Each chapter concludes with a section of miscellaneous problems that cover topics in the order in which the mathematics they depend on appears in the chapter. Many of these concluding sections present interesting but seldom-taught applications in which the mathematics is an extension of the material in the chapter. (See pp. 370 and 623.)

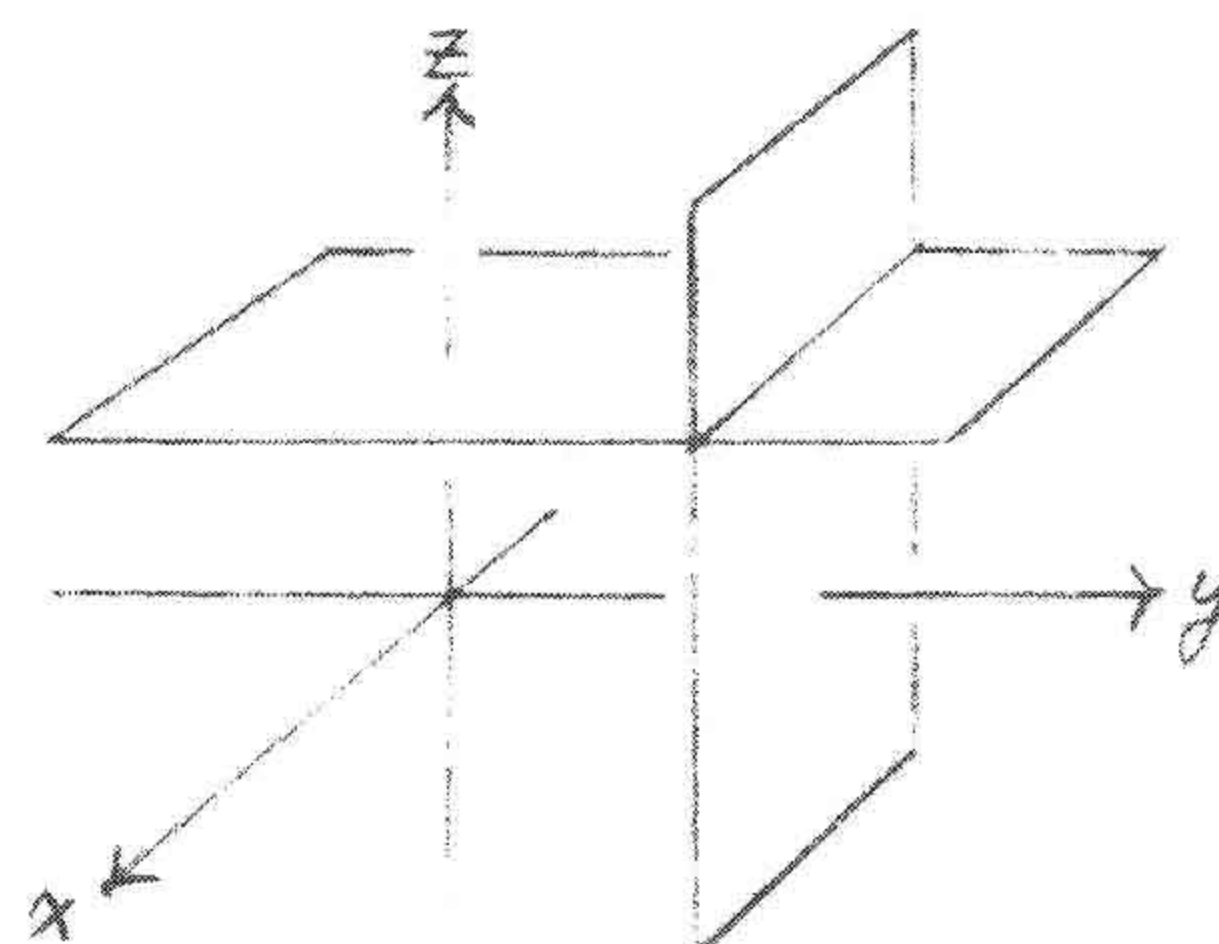
Numerical Methods We have retained the discussions of root finding, linear and quadratic approximations of functions, and the numerical approximations of integrals, topics that are becoming increasingly important both in mathemat-

ics and in other fields. As in earlier editions, there are occasional calculator exercises and there are references at the ends of many problem sets to micro-computer programs in *The Calculus Toolkit*. However, the text does not require anyone to have experience with or access to either a calculator or a computer.

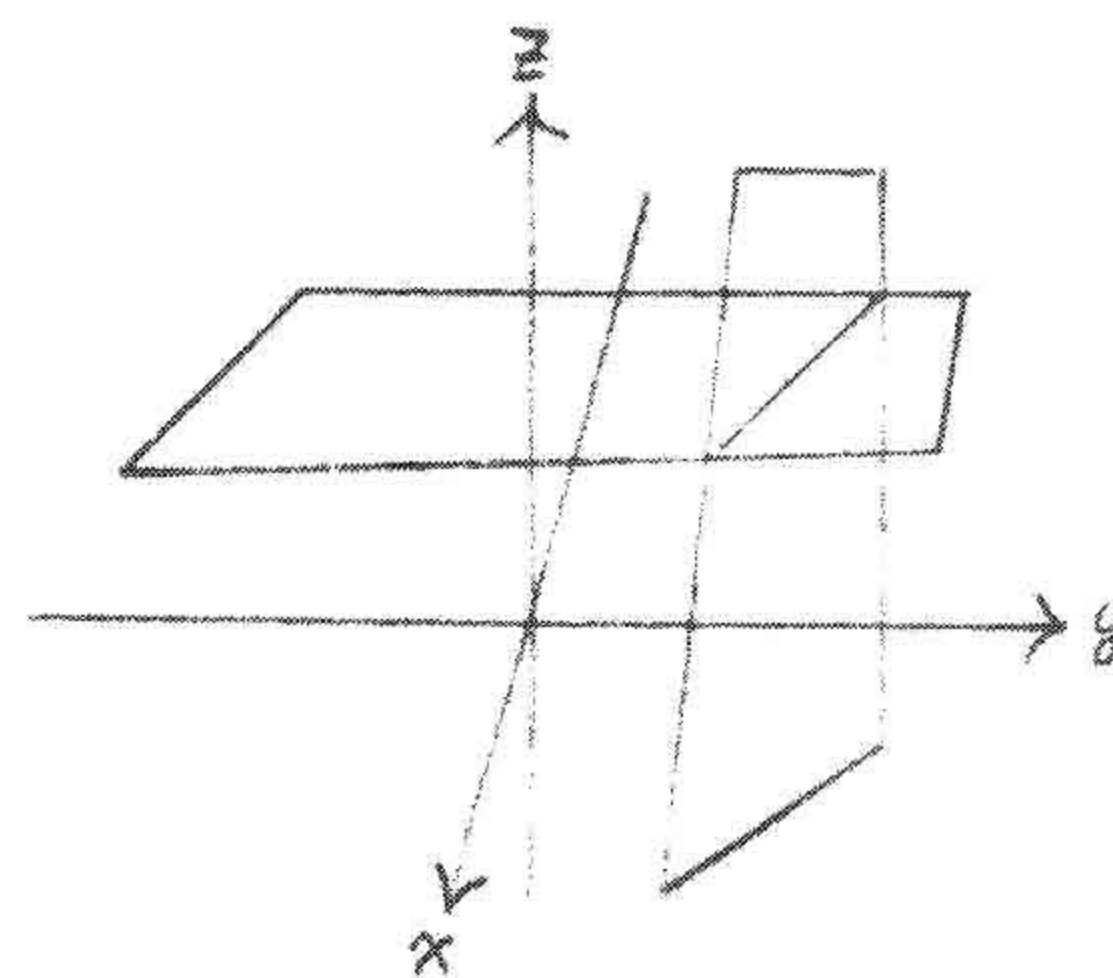
New Features

In addition to trying to preserve what users have indicated were the best features of earlier editions, we have added a number of new features to meet present classroom needs:

Drawing Lessons Drawing things in three dimensions is often difficult. We have therefore included step-by-step guidelines for drawing planes, cylinders, and quadric surfaces and for making three-dimensional objects look three-dimensional. The surfaces in the drawing lessons and practice exercises are the surfaces readers will work with later in the book when they study multivariable calculus. (See pages 770, 842, and 851.)

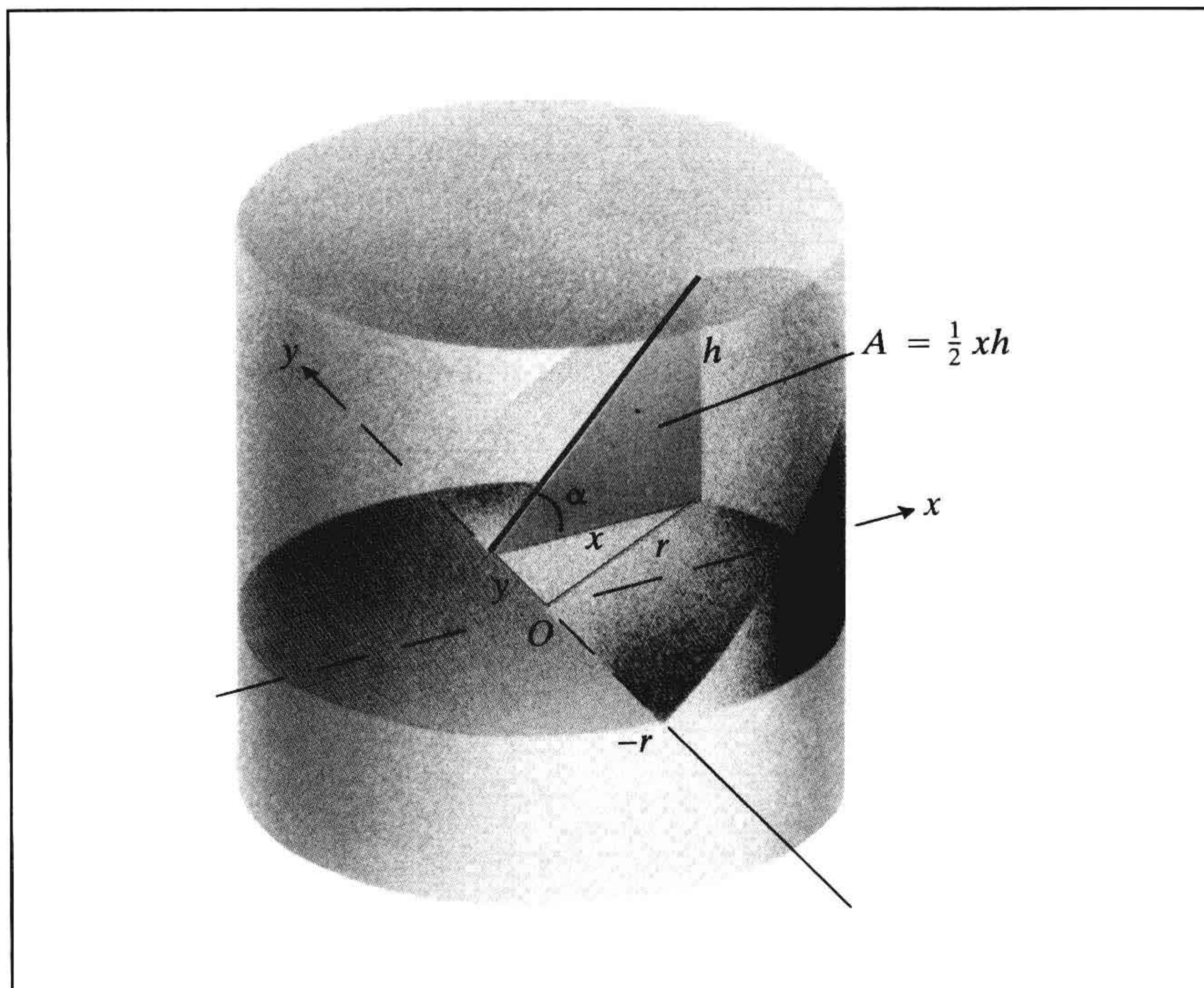


Advice followed



Advice ignored

Enhanced Artwork We have redrawn many of the older figures and added a number of new ones to make it easier to understand the mathematical arguments and to visualize the curves, surfaces, and solids that arise in the examples and exercises. There are more figures than in previous editions, and the problem sets have more art than before. We have used additional colors to highlight the important parts of many of the three-dimensional figures. (See the figure below, for example.) There are also full-page displays of related figures. (See pp. 261, 265, and 605.)



Chapter Overviews Each chapter begins with an overview that connects the forthcoming material with other topics in the book and describes its importance in theory and applications. (See pp. 236 and 859.) Most articles in the book also begin with brief introductions of their own to set the context for the topics they present. (See pp. 390 and 657.)

Historical Notes Calculus is the result of centuries of human endeavor, and to help to communicate this idea, we have included marginal notes about people who were involved with its development and about the contributions they made. (See the notes on Maria Agnesi's contributions, p. 556, and Taylor series, p. 704.)

Quick-Reference Charts We have added quick-reference charts that list groups of related formulas, emphasize procedures, and describe problem-solving strategies. (See pp. 327, 451, 469, and 691.)

Other Changes

There have been significant changes in this edition beyond the new features already mentioned:

Distinction between Core and Optional Topics Calculus books must meet the needs of many different courses simultaneously, with the result that they normally contain more than any individual instructor wants to cover. To help readers distinguish between core topics and topics that may be somewhat tangential, we have marked a number of articles and subsections with open squares (\square). The squares identify optional topics not needed for later parts of the book.

Reorganization Differentials are now introduced in Chapter 2 and inverse functions have been moved from Chapter 2 to where they are first needed in Chapter 6. The introduction of the Chain Rule has been separated from the introduction to parametric equations. Long articles have been shortened or divided in two (e.g., the presentation of functions and graphs in Chapter 1 and the presentation of gradients in Chapter 16) and some of the shorter articles have been combined to bring them closer to lesson-length (Rolle's theorem and the Mean Value Theorem now appear in a single lesson). Some of the less frequently covered topics—such as the theorems of Pappus, the substitution $z = \tan(x/2)$, direction cosines, and the division of power series—have been abbreviated, marked as optional, or moved into the problem sets.

Multivariable Calculus The presentation of multivariable calculus has been reorganized and much of it has been rewritten. Quadric surfaces have been removed from the chapter on vectors and placed in a new Chapter 15 with cylinders, cylindrical coordinates, spherical coordinates, and drawing lessons. Chapter 14 on vector functions and motion has been reordered and condensed to simplify the introduction of curvature, torsion, and the **TNB** tangent frame. The old chapter on partial derivatives has been divided into separate chapters, one on theory and the other on applications. The treatment of the vector integral theorems in Chapter 19 is entirely new. It begins with line integrals, vector fields, and Green's theorem in the plane, moves on to surface integrals, the divergence theorem, and Stokes's theorem, and closes with conservative fields and potential functions. We also discuss more of the real-world connections that originally motivated the subject's development.

New Applications We have added dozens of new applications, including the analysis of a truck's motion from its time-distance graph, the calculation of the radial rate of change of a soap bubble, the discussion of the decibel scale of loudness, and the determination of the path of a home run in Boston's Fenway Park. (See pp. 162, 203, 425, and 762.)

Revised Treatments of Standard Topics Among the many topics that now have revised treatments are

Differentials (Article 2.4)

The Chain Rule (Article 2.5)

The Fundamental Theorems of Calculus (Article 4.7)

Volumes of revolution (Articles 5.3–5.4)
 Moments and centers of mass (Articles 5.8 and 18.5)
 Parametric equations (Articles 2.8 and 8.9)
 Sequences, including a subsection on recursion and computer language (Article 11.1)
 Convergence tests (Articles 11.5–11.9)
 Jacobians, in two optional subsections (Articles 18.3 and 18.6)
 Line integrals in vector fields (Article 19.2)
 Surface area and surface integrals (Article 19.4)
 Path independence and conservative fields (Article 19.7)

Supplements for the Instructor

Solutions Manual This supplement contains the worked-out solutions for *all* the exercises in the text. Extra care has been taken to ensure the accuracy of this manual.

Complete Answer Book Contains the answers to all exercises in the text.

Computerized Test Generator (AWTest) Based on the learning objectives of the text, this easy-to-use algorithm-based system allows the instructor to generate tests or quizzes. Questions are available in open-ended, multiple choice, and true-false formats. AWTest is available for the IBM PC* and is free to adopters. (Questions prepared by Jeffery Cole.)

Printed Test Bank At least three alternate tests per chapter are included in this valuable supplement. Instructors can use this as a reference for creating tests with or without a computer.

Transparency Masters Includes a selection of key definitions, theorems, proofs, formulas, tables, and figures that appear in the text.

Computer Supplements for the Instructor and the Student

The Calculus Toolkit Consisting of twenty-seven programs ranging from functions to vector fields, this software enables the instructor and students to use the microcomputer as an “electronic chalkboard.” Three-dimensional graphics are incorporated where appropriate. The Calculus Toolkit is available for both the Apple* and the IBM PC.

Student Edition of MathCAD This software package is a very powerful free-form scratchpad. When you input equations, MathCAD automatically calculates and displays your results as numbers or graphs. It also allows you to plot the results, annotate your work with text, and print your entire document.

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