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Singular Perturbation Analysis of Discrete Control Systems

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Dedicated to

Mother and Father (DSN)

and

Mother (AKR)

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PREFACE

The dynamics of many control systems is described by higher order differential equations. However, the behaviour is governed by a few dominant parameters, a relatively minor role being played by the remaining parameters such as small time constants, masses, moments of inertia, inductances and capacitances. The presence of these "parasitic" parameters is often the source for the increased order and the "stiffness" of the system. The "curse" of this dimensionality coupled with stiffness causes formidable computational difficulties for the analysis and control of such large systems. The singular perturbation method using the reduced order models and relieving the stiffness, is a "gift" to control engineers. As such it is very attractive to formulate many control problems to fit into the framework of the mathematical theory of singular perturbations which has a rich literature [1-5]. The singular perturbation theory in continuous control systems has reached a certain level of maturity and is well documented [6-11].

Discrete systems are very much prevalent in science and engineering. There are three important sources of discrete models described by high order difference equations containing several small parameters [12]. The first source is digital simulation, where the ordinary differential equations are approximated by the corresponding difference equations [13]. The study of sampled-data control systems and computer-based adaptive control systems leads in a natural way to another source of discrete-time models [14,15]. Finally, many economic, biological and sociological systems are represented by discrete models [16,17]. In spite of its paramount importance, the area of singular perturbations in difference equations and its applications to discrete control problems has not so far received sufficient attention [18-28]. It is in this context that the present investigation is taken up in the field of singular perturbation analysis of discrete control systems. The motivation for the present investigation comes mainly from the work of Comstock and Hsiao [18]. The Monograph starts with the analysis of singularly perturbed difference equations in classical form and the various state space models [29-32] and contributes towards the development of singular perturbation methods for open-loop control [33], and closed-loop optimal control [34]. Various implications in casting the equations in a form suitable for singular perturbation analysis and many distinguishing features of the analysis are examined with relevance to each problem. Typical numerical examples are provided to illustrate the proposed methods.

The Monograph is organized as follows:

In Chapter 1, the singularly perturbed difference equations in classical form are formulated as initial value problems and boundary value problems. Methods are developed to obtain approximate solutions in terms of an "outer series" based on the degenerate (reduced) problem and a "boundary layer correction series" obtained by using certain transformations on the original problem.

In Chapter 2, we consider the state space modelling and analysis of singularly perturbed difference equations in order to give a general framework suitable for control engineers. Depending on the position of the small parameter, three state space discrete models are formulated and techniques are developed to obtain approximate series solutions [29-32]. The computational requirements of singularly perturbed differential equations are tremendous due to their stiffness [35-40]. A method is therefore suggested to cast singularly perturbed differential equations into the corresponding discrete models. The case of sampled-data control systems is also examined.

In Chapter 3, the three-time scale property of difference equations is first examined. Then, the open-loop optimal control of singularly perturbed discrete system is investigated [41]. For the resulting two-point boundary value problem, a method is developed consisting of an outer series and two correction series corresponding to "initial" and "final" boundary layers [33].

In Chapter 4, we investigate the basic ideas of the singularly perturbed nonlinear difference equations. Then the closed-loop optimal control of a linear singularly perturbed discrete system is examined [41]. For solving the resulting matrix Riccati difference equation, a method is developed. The steady state solution of the Riccati equation is also examined [34].

There are several other contributions made in singular perturbation and time scale analysis of discrete systems with reference to general control problems [46-49, 51, 53-55, 57-60, 62, 68-72, 75] optimal control problems [52, 65-67, 73], adaptive control problems [56, 63, 64], and stochastic systems and Markov chains [50, 61, 74].

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CONTENTS

CHAPTER - 1

SINGULAR PERTURBATION ANALYSIS OF DIFFERENCE EQUATIONS IN CLASSICAL FORM

In this Chapter the analysis of singularly perturbed difference equations in classical form is carried out. In order to get a clear insight into the analysis, both initial and boundary value problems are first considered in their simplest form. The various distinguishing features of singular perturbations such as order reduction, loss of boundary conditions, boundary layer and boundary layer correction are clearly brought out. Methods are developed where the approximate solution is sought in the form of an outer series solution and a correction series solution. A special feature of this chapter is to give a complete analysis of a general higher order difference equation with several small parameters [29]. Examples are provided to illustrate the proposed methods.

1.1 Small Parameter at the Right End: R-type

As a first step leading to the analysis of higher order difference equations and in order to get a clear insight into the method, consider a second order linear homogeneous equation

$$y(k+2) + ay(k+1) + hy(k) = 0 \qquad \ldots \quad (1.1)$$

where h is a small parameter negligibly small in comparison with the other coefficients a and 1.

The solution of the above equation is

$$y(k) = c_1(m_1)^k + c_2\ (m_2)^k$$

where m_1 and m_2 are the characteristic roots of (1.1) given by

$$m_1 = -\frac{a}{2} + \frac{a}{2}\,(1 - 4h/a^2)^{0.5} \qquad \ldots \quad (1.2a)$$

$$m_2 = -\frac{a}{2} - \frac{a}{2}\,(1 - 4h/a^2)^{0.5} \qquad \ldots \quad (1.2b)$$

and c_1 and c_2 are constants that depend on the given boundary conditions.

Case (a): Initial Value Problem (IVP)

Given the initial conditions y(0) and y(1), the solution of (1.1) is

$$y(k) = \frac{(y(0)m_2 - y(1))(m_1)^k + (y(1) - y(0)m_1)\ (m_2{}^k)}{m_2 - m_1} \qquad \ldots \quad (1.3)$$

where m_1 and m_2 are as in (1.2).

Using the Binomial expansion

$$(1 - 4h/a^2)^{0.5} = 1 - 2h/a^2 - 2h^2/a^4 \ .\ .\ .\ .\ .\ .$$

$$\text{if } |4h/a^2| < 1,$$

for sufficiently small values of h, the solution (1.3) is obtained as a power series in h.

Ignoring terms with coefficients of h and higher powers of h, the zeroth order approximate solution is

$$y(k) = y(1)(-a)^{k-1} + h^k\ [(y(0 + y(1)/a)(-1/a)^k] \qquad \ldots \quad (1.4a)$$

Similarly, ignoring terms with coefficients of h^2 and higher powers of h, the first order approximate solution is

$$y(k) = y(1)(-a)^{k-1} + h(\text{N}\frac{y(0)}{a} + \frac{y(1)}{a^2} + \frac{(1-k)}{a^2}\ y(1)0(-a)^{k-1}|$$

$$+h^k((y(0)+y(1)/a)(-1/a)^k + h\text{Nm}(y(0)+y(1)/a)\frac{k}{a^2} + \frac{y(0)}{a^2}\ \frac{2y(1)}{a^3}\%\text{m} \ -1/a\%^k)|$$

$$\ldots \quad (1.4b)$$

Solutions of higher order approximation can be obtained on similar lines.

Note that in (1.4), h^k is included to take account of the fact that the contribution to the series associated by this term is different when h and k tend to zero in different sequences (ie. h T 0 first and k T 0 later or vice versa).

Suppressing the small parameter h in (1.1), the resulting degenerate equation is

$$y^{(0)}(k+2) + a\, y^{(0)}(k+1) = 0 \qquad \ldots \quad (1.5)$$

This equation is of order one and can satisfy only one of the given two initial conditions. Hence (1.1) is said to be in the singularly perturbed form [18].

If (1.5) is solved with $y^{(0)}(1) = y(1)$, the solution obtained is

$$y^{(0)}(k) = y(1)\ (-a)^{k-1} \qquad \ldots \quad (1.6)$$

which is the same as the first term in (1.4a).

The above solution yields

$y^{(0)}(0) = -y(1)/a$] $y(0)$.

A close examination of (1.4) reveals that

(i) Limit y(k)] Limit y(k)
h T 0   k T 0
k T 0   h T 0

In other words, as h tends to zero the uniform convergence of the solution (1.3) to the degenerate solution (1.6) fails at k = 0. The degenerate solution is therefore not valid at that point and a boundary layer occurs at k = 0.

(ii) The solution in (1.4) not involving h^k is called the "outer series" solution, as it is valid outside the boundary layer. This satisfies only one of the given initial conditions y(1). The solution involving h^k is called the "boundary layer correction series" solution which recovers the lost boundary condition y(0).

(iii) The presence of h^k in the series solution (1.4) suggests that the correction series solution has the transformation

$$w(k) = y(k)/h^k$$

Using the above transformation in (1.1) and dividing throughout with h^{k+1}, the equation of the correction series is obtained as

$$hw(k+2) + aw(k+1) + w(k) = 0 \qquad \ldots \quad (1.7)$$

Putting h = 0 in the above equation, its degenerate form is

$$w^{(0)}(k+1) + \frac{1}{a} w^{(0)}(k) = 0$$

The above equation is now solved with the initial condition

$$w^{(0)}(0) = y(0) - y^{(0)}(0)$$

Yielding

$$w^{(0)}(k) = (y(0) - y^{(0)}(0)) \ (-1/a)^k$$

Using the value of $y^{(0)}(0)$ from (1.6),

$$w^{(0)}(k) = (y(0) + y(1)/a) \ (-1/a)^k$$

Note that the above solution is the same as the term associated with h^k in (1.4a). This illustrates the recovery of the initial condition lost in the process of degeneration.

Thus the total zeroth order series solution composed of the outer series solution and the correction series solution is given by

$$y(k) = y^{(0)}(k) + h^k w^{(0)}(k)$$

The above solution, not only satisfies both the given initial conditions but also is nearer to the exact solution (1.3) upto zeroth order approximation.