

# Lial/Miller Intermediate Algebra •

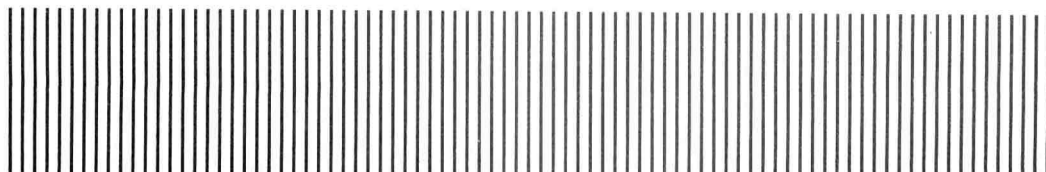


# Intermediate Algebra • 5th

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### To the Student

If you need further help with algebra, you may want to get copies of both the *Study Guide* and the *Student's Solutions Manual* that accompany this textbook. The additional worked-out examples and practice problems that these books provide can help you study and understand the course material. Your college bookstore either has these books or can order them for you.

Cover: A detail from *Spirals I* by Pauline Burbidge, Nottingham, England. A quilt, pieced and quilted by machine in cottons, which first appeared in *The Art Quilt*, published by The Quilt Digest Press, San Francisco.

Some exercises in Sections 4.3 and 4.7 are based on "Composing 'Interesting' Exercises Involving Rational Expressions" by John Hornsby and Jeffrey Cole, *Mathematics Teacher*, March 1984.

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# Preface

The fifth edition of *Intermediate Algebra* continues to address the needs of both the student and the instructor. Clear explanations keyed to objectives, numerous examples detailing the structure of each problem, carefully graded exercise sets keyed to examples, interactive tutorial software, a study guide, and a solutions manual are designed for the student. For the instructor, a complete instructional package is provided, including alternate forms of tests, additional test items keyed to objectives, computer-assisted testing, a complete answer key, and complete solutions to all exercises, audiotapes, and videotapes.

## Key Features

**Keyed Objectives** Each section begins with a list of skills that the student should learn in that section. The objectives are keyed to appropriate discussions in the text by numbered symbols such as **1**.

**Examples** Nearly 800 worked-out examples clearly illustrate concepts and techniques. Second color is used to identify pertinent steps within examples and to highlight explanatory side comments. For clarity, the end of each example is indicated with the symbol  $\boxplus$ .

**Word Problems** A problem-solving approach gives students early and repeated experience in solving applied problems. A list of steps for solving word problems is first presented in Chapter 2. Throughout the text students are given practice translating words into algebraic symbols. In this way, students see word problems early, work with them continuously, and thus, gradually improve their problem-solving skills. To enhance this feature, new word problems using roots and radicals, the equation of a line, and the conic sections have been added. Other new word problems require the use of geometric formulas.

**Pedagogical Use of Second Color** Key definitions, formulas, and procedures are printed inside color boxes, helping students review easily. Color side comments within examples explain the structure of the problem. Cautionary comments and reminders are printed in color type where appropriate.

## Exercises

**Graded Exercises** The range of difficulty in the exercise sets affords ample practice with drill exercises. Students are eased gradually through problems of increasing difficulty to those that will challenge outstanding students. More than 4500 drill exercises and 350 word problems, keyed to examples, are provided at the ends of sections. Including end-of-chapter exercises, the book gives a total of more than 6200 exercises, nearly 450 of them word problems. Answers to odd-numbered exercises are given at the back of the book.

**Challenging Exercises** A group of challenging exercises is given at the end of most exercise sets. These are intended to give students an opportunity to go somewhat beyond the discussion in the text. Because of this intent, these exercises have no corresponding examples. These challenging exercises are not labeled in the text but are listed in the *Instructor's Guide* and the *Instructor's Answer Manual*.

**Calculator Exercises** Calculator exercises have been included throughout the book. These optional exercises are identified by colored exercise numbers.

**Review Exercises** Beginning in Chapter 2, most exercise sets end with a few problems reviewing earlier concepts that help students prepare for the next section.

**Supplementary Exercises** A few sets of supplementary exercises, designed to integrate and clarify difficult or confusing topics, have been included. For example, a set in Chapter 4 requires students to distinguish between performing operations on rational expressions and solving equations involving rational expressions.

**Chapter Review Exercises** Extensive review exercises at the end of each chapter, more than 1000 in all, provide further opportunity for mastery of the material before taking an examination. These exercises are divided into two groups in most chapters. Exercises in group ① are keyed to appropriate sections of the chapter. Exercises in group ② consist of a mixed group of exercises that are not keyed to the sections. This helps students practice identifying problems by type.

**Chapter Tests** Sample tests, of a length comparable to that of actual classroom tests, help students prepare for examinations. Answers to every test question are given at the back of the book.

### *New Content Highlights*

- Interval notation is introduced in the fifth edition and is used throughout the book for expressing solutions of inequalities in one variable.
- Complex numbers are now introduced in Chapter 5, after roots and radicals are presented. Although imaginary number solutions of quadratic equations are included in Chapter 6, an instructor may delete the coverage of imaginary numbers since the exercise sets in Chapter 6 are divided into separate groups with real number and imaginary number solutions.
- Chapter 6 has been reorganized to make the presentation clearer and to improve the flow of ideas. The chapter now opens with material on completing the square, followed by a section on the quadratic formula and a new section on the discriminant and the sum and product of the roots. Then equations that are quadratic in form are discussed. The chapter closes with second-degree formulas and applications and a section on nonlinear inequalities.

- The introduction to the parabola in Chapter 9 is now more intuitive, with point-by-point plotting and a discussion of intercepts, symmetry, and the general formula for the vertex.
- In Chapter 10 the concepts of relation and function are introduced with new examples from everyday life. More figures are used to provide a better understanding.

*Supplements* An extensive supplemental package is available for *Intermediate Algebra*, fifth edition, including testing materials, solutions, and electronic media support.

The **Instructor's Guide** features six ready-to-duplicate tests for each chapter, including two multiple-choice tests. In addition, two forms of a final examination are provided, as well as a pretest that can be used for diagnostic purposes. More than 4000 additional test items, keyed to each objective in the text, are also included. Answers are provided for all tests and test items. A list of the challenging exercises in the textbook is given, to help in making assignments.

An **Instructor's Answer Manual** provides answers to all exercises in the text, for quick reference in the classroom if desired.

A **Computer-Assisted Testing System (CATS)**, for both Apple II and IBM computers, provides more than 7500 questions, organized by objectives, and is available free to departments that adopt the book. CATS features an editing capability that allows instructors to add their own problems or edit existing problems. The system enables instructors to test by chapter, section, or objective, or by groups of chapters, sections, or objectives. Instructors can elect to generate "instant" tests, create multiple forms of tests, or design item-specific tests of their own choosing.

A set of **computer-assisted tutorials**, based on a mastery learning approach, has been developed for this text. Available for Apple II and IBM computers, the package not only gives additional worked-out examples and practice exercises but also requires students to demonstrate mastery of the skills associated with a particular concept or objective.

A set of twelve **videotapes**, one per chapter, is available at low cost to users of the book. The tapes follow the outline of each chapter—section-by-section and objective-by-objective.

A set of twelve **audiotapes**, covering the material in each section in the textbook, is available at no charge to users of the book. Students who need help with a particular topic or those who have missed class will find these tapes helpful.

A **Study Guide**, in semiprogrammed format, provides additional practice problems and reinforcement for the students. A self-test is given at the end of each chapter as well.

A **Student's Solutions Manual** contains solutions to every other odd-numbered exercise in the textbook plus solutions to all the exercises in the chapter tests.

An **Instructor's Solutions Manual** includes solutions to all even-numbered exercises and solutions to the odd-numbered exercises not included in the *Student's Solutions Manual*.

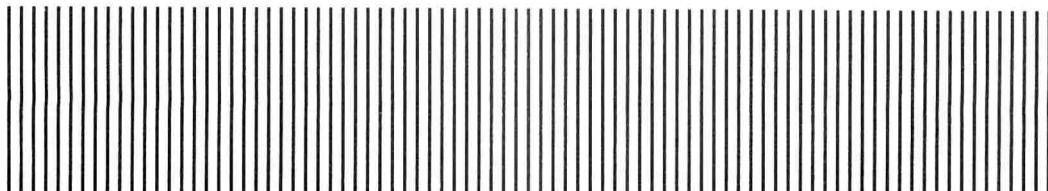
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# Intermediate Algebra • 5th





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# Contents

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- 1** *The Real Numbers* 1
  - 1.1 Basic Terms* 1
  - 1.2 Equality and Inequality* 9
  - 1.3 Properties of the Real Numbers* 14
  - 1.4 Operations on Real Numbers* 20
  - Chapter 1 Summary* 31
  - Chapter 1 Review Exercises* 33
  - Chapter 1 Test* 35



- 2** *Linear Equations and Inequalities* 36
  - 2.1 Linear Equations in One Variable* 36
  - 2.2 Formulas* 45
  - 2.3 Applications* 51
  - 2.4 Linear Inequalities in One Variable* 61
  - 2.5 Absolute Value Equations* 70
  - 2.6 Compound Inequalities* 74
  - 2.7 Absolute Value Inequalities* 80
  - Supplementary Exercises on Linear Equations and Inequalities* 85
  - Chapter 2 Summary* 86
  - Chapter 2 Review Exercises* 88
  - Chapter 2 Test* 92



### 3 *Exponents and Polynomials* 94

- 3.1 *Integer Exponents* 94
- 3.2 *Further Properties of Exponents* 103
- 3.3 *Polynomials* 108
- 3.4 *Multiplication of Polynomials* 115
- 3.5 *Greatest Common Factors; Factoring by Grouping* 122
- 3.6 *Factoring Trinomials* 128
- 3.7 *Special Factoring* 134
- 3.8 *General Methods of Factoring* 138
- 3.9 *Solving Equations by Factoring* 142
- Chapter 3 *Summary* 146
- Chapter 3 *Review Exercises* 147
- Chapter 3 *Test* 150



### 4 *Rational Expressions* 152

- 4.1 *Basics of Rational Expressions* 152
- 4.2 *Multiplication and Division of Rational Expressions* 159
- 4.3 *Addition and Subtraction of Rational Expressions* 164
- 4.4 *Complex Fractions* 170
- 4.5 *Dividing Polynomials* 175
- 4.6 *Synthetic Division* 180
- 4.7 *Equations with Rational Expressions* 184
- Supplementary Exercises on Rational Expressions and Equations* 189
- 4.8 *Applications* 191
- Chapter 4 *Summary* 199
- Chapter 4 *Review Exercises* 200
- Chapter 4 *Test* 203



## 5 *Rational Exponents and Radicals* 205

- 5.1 *Rational Exponents* 205
- 5.2 *Radicals* 211
- 5.3 *Simplifying Radicals* 216
- 5.4 *Adding and Subtracting Radical Expressions* 223
- 5.5 *Multiplying and Dividing Radical Expressions* 228
- 5.6 *Equations with Radicals* 232
- 5.7 *Complex Numbers* 237
- Chapter 5 *Summary* 244
- Chapter 5 *Review Exercises* 245
- Chapter 5 *Test* 247



## 6 *Quadratic Equations and Inequalities* 248

- 6.1 *Solving Quadratic Equations by Completing the Square* 248
- 6.2 *The Quadratic Formula* 254
- 6.3 *The Discriminant and the Sum and Product of the Solutions* 260
- 6.4 *Equations Quadratic in Form* 267
- 6.5 *Formulas and Applications* 274
- 6.6 *Nonlinear Inequalities* 280
- Chapter 6 *Summary* 286
- Chapter 6 *Review Exercises* 287
- Chapter 6 *Test* 289



## 7 *The Straight Line* 291

- 7.1 *The Rectangular Coordinate System* 291
- 7.2 *The Slope of a Line* 299
- 7.3 *Linear Equations* 306
- 7.4 *Linear Inequalities* 313
- 7.5 *Variation* 317
- Chapter 7 *Summary* 323
- Chapter 7 *Review Exercises* 325
- Chapter 7 *Test* 326



## 8 *Systems of Linear Equations* 328

- 8.1 *Linear Systems of Equations in Two Variables* 328
- 8.2 *Applications of Linear Systems of Equations* 338
- 8.3 *Linear Systems of Equations in Three Variables* 345
- 8.4 *Determinants* 353
- 8.5 *Solution of Linear Systems of Equations by Determinants—Cramer's Rule* 358
- 8.6 *Solution of Linear Systems of Equations by Matrix Methods* 363
- Chapter 8 *Summary* 370
- Chapter 8 *Review Exercises* 371
- Chapter 8 *Test* 373



## 9 *Conic Sections* 374

- 9.1 *The Parabola* 375
- 9.2 *The Circle and the Ellipse* 386
- 9.3 *The Hyperbola* 392
- 9.4 *Nonlinear Systems of Equations* 398
- 9.5 *Second-Degree Inequalities (Optional)* 404
- Chapter 9 *Summary* 408
- Chapter 9 *Review Exercises* 409
- Chapter 9 *Test* 411



## 10 *Functions* 412

- 10.1 *Definition of Function* 412
- 10.2 *Functional Notation* 422
- 10.3 *Types of Functions* 427
- 10.4 *Inverse Functions* 434
- Chapter 10 *Summary* 439
- Chapter 10 *Review Exercises* 440
- Chapter 10 *Test* 442



## 11 *Exponential and Logarithmic Functions* 444

*11.1 Exponential Functions* 444

*11.2 Logarithms* 450

*11.3 Properties of Logarithms* 455

*11.4 Calculating with Logarithms* 461

*11.5 Logarithmic and Exponential Equations* 467

*11.6 Natural Logarithms; Change of Base* 472

*Chapter 11 Summary* 478

*Chapter 11 Review Exercises* 478

*Chapter 11 Test* 480



## 12 *Sequences and Series* 481

*12.1 Sequences* 481

*12.2 Series* 484

*12.3 Arithmetic Sequences* 488

*12.4 Geometric Sequences* 494

*12.5 The Binomial Theorem* 502

*Chapter 12 Summary* 506

*Chapter 12 Review Exercises* 507

*Chapter 12 Test* 509

## *Appendices* 511

*Appendix A Formulas from Geometry* 511

*Appendix B Other Formulas; Formulas from Consumer Mathematics* 513

*Appendix C Symbols Used in This Book* 514

*Appendix D The Metric System* 515



*Tables* 516

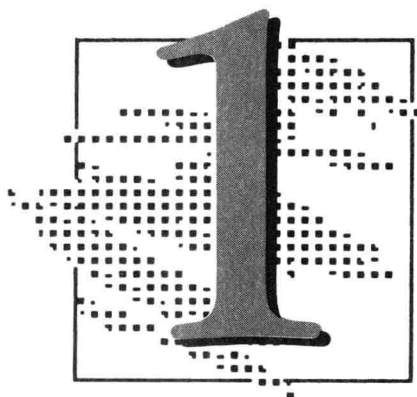
***Table 1*** *Selected Powers of Numbers* 516

***Table 2*** *Powers and Roots* 517

***Table 3*** *Common Logarithms* 518

*Answers to Selected Exercises* 520

*Index* 567



## ***The Real Numbers***

### ***1.1 Basic Terms***

#### *Objectives*

- 1** Write sets.
- 2** Decide if one set is a subset of another.
- 3** Use number lines.
- 4** Find additive inverses.
- 5** Use absolute value.
- 6** Know the common sets of numbers.

Algebra depends on symbols, and many of the symbols used in this book are introduced in this first chapter. A summary of all the symbols used is given in Appendix C.

**1** A basic term used in algebra is **set**, a collection of objects. The objects in a set are the **elements** or **members** of the set. In algebra, the elements in a set are usually numbers, with braces,  $\{ \quad \}$ , used to enclose the elements. For example, 2 is an element of the set  $\{1, 2, 3\}$ .

A set can be defined either by listing or by describing its elements. For example,

$$S = \{\text{Oregon, Ohio, Oklahoma}\}$$

defines the set  $S$  by *listing* its elements. The same set might be *described* by saying that set  $S$  is the set of all states in the United States whose names begin with the letter "O."

Set  $S$  above has a countable or **finite** number of elements. Some sets contain an uncountable or **infinite** number of elements, such as

$$N = \{1, 2, 3, 4, 5, 6, \dots\},$$

where the three dots show that the list continues in the same pattern. Set  $N$  is called the set of **natural numbers**, or **counting numbers**. A set containing no elements, such as the set of natural numbers less than 1, is called the **empty set**, or **null set**, written  $\emptyset$ .

To write the fact that 2 is an element of the set  $\{0, 1, 2, 3\}$ , use the symbol  $\in$ :

$$2 \in \{0, 1, 2, 3\}.$$

The number 2 is also an element of set  $N$  above, or

$$2 \in N.$$

To show that 0 is *not* an element of set  $N$ , draw a slash through the symbol  $\in$ :

$$0 \notin N.$$

Two sets are **equal** if they contain exactly the same elements. For example,

$$\{1, 2\} = \{2, 1\},$$

because the sets contain the same elements. (The order doesn't matter.) Also,  $\{1, 2\} \neq \{0, 1, 2\}$  ( $\neq$  means "is not equal to") since one set contains the element 0 while the other does not.

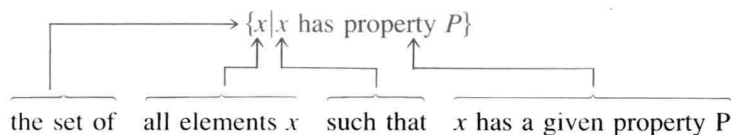
In algebra, letters called **variables** are often used to represent numbers. Variables also can be used to define sets of numbers. For example,

$$\{x | x \text{ is a natural number between 3 and 15}\}$$

(read "the set of all elements  $x$  such that  $x$  is a natural number between 3 and 15") defines the set

$$\{4, 5, 6, 7, \dots, 14\}.$$

The notation  $\{x | x \text{ is a natural number between 3 and 15}\}$  is an example of **set-builder notation**.



**Example 1** List the elements in each set.

- (a)  $\{x | x \text{ is a natural number less than 4}\}$

The natural numbers less than 4 are 1, 2, and 3. The given set is

$$\{1, 2, 3\}.$$

- (b)  $\{y | y \text{ is one of the first five even natural numbers}\} = \{2, 4, 6, 8, 10\}$

(c)  $\{z | z \text{ is a natural number at least } 7\}$

The set of natural numbers at least 7 is an infinite set; write it with three dots as  $\{7, 8, 9, 10, \dots\}$ . ✚

**2** Set  $A$  is a **subset** of set  $B$  if every element of  $A$  is also an element of  $B$ . The symbol  $\subseteq$  is used for subset, so  $A \subseteq B$  says that  $A$  is a subset of  $B$ . The symbol  $\not\subseteq$  indicates “is not a subset of.” A sketch of a set  $A$  that is a subset of a set  $B$  is shown in Figure 1.1.

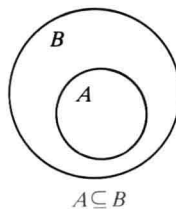


Figure 1.1

*Example 2* Decide whether each statement is *true* or *false*.

(a)  $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$

Every element of the set  $\{1, 2, 3\}$  is also an element of the set  $\{1, 2, 3, 4, 5\}$ . Because of this,  $\{1, 2, 3\}$  is a subset of  $\{1, 2, 3, 4, 5\}$  and the given statement is true.

(b)  $\{0, 1, 2\} \subseteq \{1, 2, 3, 4, 5\}$

The number 0 is an element of  $\{0, 1, 2\}$  but not of  $\{1, 2, 3, 4, 5\}$ , so that

$$\{0, 1, 2\} \not\subseteq \{1, 2, 3, 4, 5\}$$

The statement is false.

(c)  $\{0, 1, 2\} \subseteq \{0, 1, 2\}$

Every element of the first set is an element of the second set, so the given statement is true. ✚

Do not confuse the symbol  $\in$ , “is an element of,” with the symbol  $\subseteq$ , “is a subset of.” The symbol  $\in$  is used only between an element and a set, while  $\subseteq$  is used only between two sets. For example,

$$5 \in \{3, 4, 5, 7, 9\},$$

but

$$\{5\} \subseteq \{3, 4, 5, 7, 9\}.$$

By the definition above, set  $A$  is a subset of set  $B$  if every element of  $A$  is also an element of  $B$ . Since every element of a set  $A$  is an element of the same set  $A$  itself, every set is a subset of itself. (See Example 2(c) above.) Rephrasing the definition of a subset, set  $A$  is a subset of set  $B$  if there are no elements of  $A$  that are not in  $B$ . Since there are no elements in the empty set, there are none that are not in set  $B$ . For this reason, the empty set is a subset of every set.