STUDY GUIDE

SINGLE VARIABLE

SECOND EDITION

CALCULUS

CONCEPTS AND CONTEXTS

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Study Guide for Stewart's

SINGLE VARIABLE

SECOND EDITION

CALCULUS

CONCEPTS AND CONTEXTS

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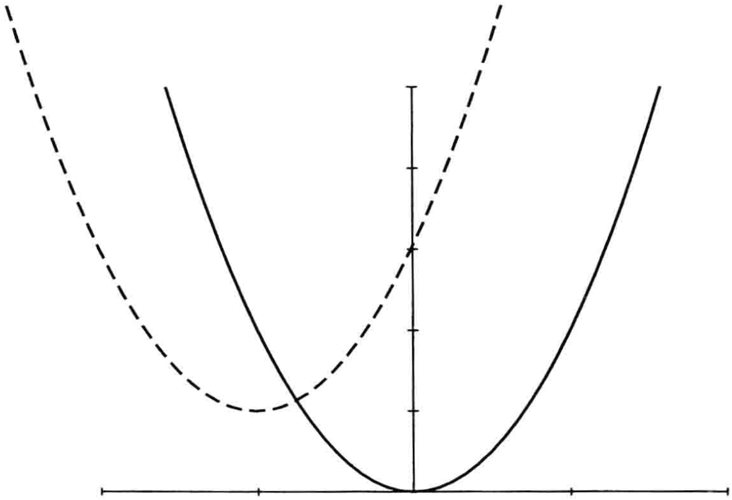
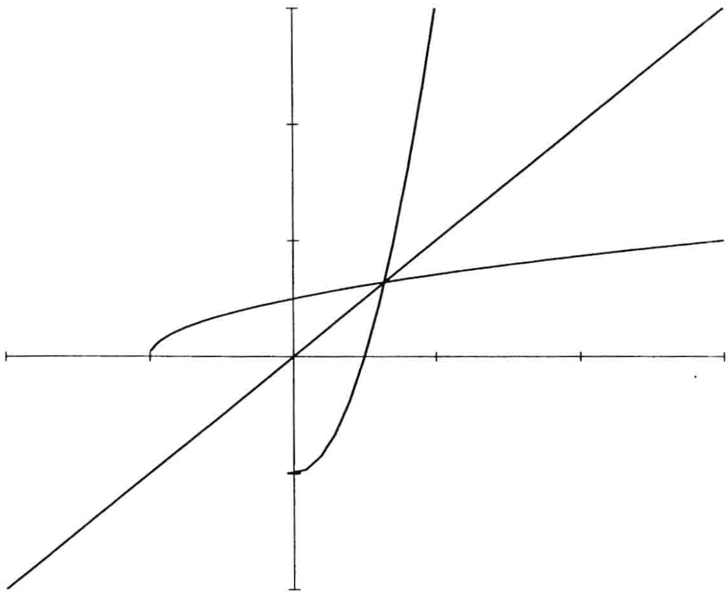
Chapter 1 - Functions and Models





Section 1.1 –Four Ways to Represent a Function

Key Concepts:

- Representing functions verbally, by a table of values, by a graph, or by an explicit formula
- Functions that are defined piecewise
- Increasing or decreasing functions

Skills to Master:

- Use the different representations of functions to solve real world problems.
- Understand and be able to determine domain and range of specific functions.
- Sketch graphs of piecewise defined functions.

Discussion:

This section reviews the concept of a function and introduces you to some ideas that will be important as you continue learning more about Calculus. Work as many problems in this section as possible to remind yourself of how to represent functions. Make sure that you understand the definition of increasing and decreasing functions. By the end of the course you will be able to use the techniques of Calculus to determine where many functions are increasing and decreasing and where these functions achieve maximum or minimum values.

Key Concept: Representing functions verbally, by a table of values, by a graph, or by an explicit formula

TEXT pages 14-15.

Study the examples in this section that illustrate how a function can be represented in different ways. A function can be illustrated verbally by a description in words, numerically by a table of values, visually by a graph and algebraically by an explicit formula. Most functions that you are familiar with such as polynomials, trigonometric functions, exponential functions and logarithmic functions are represented by formulas. However, functions that are not given by explicit formulas arise in many situations. Section 1.2 in the text will show in more detail how to deal with such functions.

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Key Concept: Functions that are defined piecewise

A piecewise defined function $f(x)$ is a function that is given by different formulas for different values of x. For example, in some applications, a function may be given by one formula when x is positive and by a different formula when x is negative. Study the examples given of piecewise defined functions and make sure that you know how to graph such functions.

Key Concept: Increasing or decreasing functions

A function is *increasing* on an interval I if

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I.$$

A function is *decreasing* on an interval I if

$$f(x_1) > f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I.$$

Study the examples and make sure that you understand what if means to say that functions are increasing or decreasing.

SkillMaster 1.1: Use the different representations of functions to solve real world problems.

In a real world problem that arise in the sciences or in economics, a function is often first described verbally or is given by a table of values or a graph. Understanding the different ways that a function can be represented will allow you to work with functions that come up in these ways. Approximating such functions by other functions given by an explicit formula is one of the main ways to work with such functions.

page 24.

SkillMaster 1.2: Understand and be able to determine domain and range of specific functions.

page 12.

The *domain* of a function $f(x)$ is the set of values x for which the function is defined. The *range* of a function $f(x)$ is the set of values taken on by the function. If a function is graphed, the domain consists of all x values for which a vertical line drawn through the point $(x, 0)$ intersects the graph. The range consists of all y values for which a horizontal line drawn through $(0, y)$ intersects the graph. Make sure that you know how to find the domain and range of functions that are given verbally, by data, by a graph or by a formula.

SkillMaster 1.3: Sketch graphs of piecewise defined functions.

To sketch a graph of a piecewise defined function given by a number of different formulas, you need to separately graph each the function represented by each formula for the appropriate values of x. For example, the function $f(x) = |x|$ can also be given as the piecewise defined function

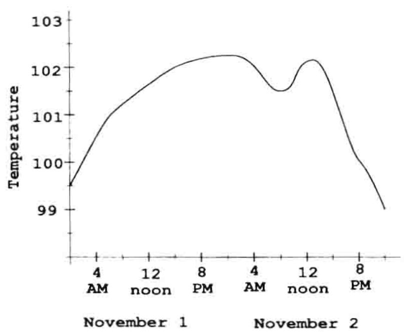
$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

The graph of this piecewise defined function consists of parts of the lines $y = x$ and $y = -x$.

Worked Examples

For each of the following examples, first try to find the solution without looking at the middle or right columns. Cover the middle and right columns with a piece of paper. If you need a hint, uncover the middle column. If you need to see the worked solution, uncover the right column.

Example	Tip	Solution
SkillMaster 1.1.		
The graph describes Androcles' temperature over a period of two days when he was ill. At what time and date did his temperature first reach 102°? What was his temperature at 8:00 am on November 2? When did Andy begin to feel healthy again (interpret this as the time his temperature first fell below 100°)?	Use a paper edge to see where a line parallel to the x-axis first crosses the graph. Then drop a perpendicular line to the x-axis to find the time.	Andy's temperature first reached 102 ° at approximately 4:00 P.M. on November 1. At 8:00 am on November 2 Andy's temperature began to rise again after falling a bit. His temperature at that time was 101.5° His temperature fell to 100° for the first time at about 8:00 P.M. on November 2.



• Lisa is about to have a baby. She has charted her weight over 40 week period. She weighs 140 pounds before her pregnancy and at the end of each four week period she writes down her weight.

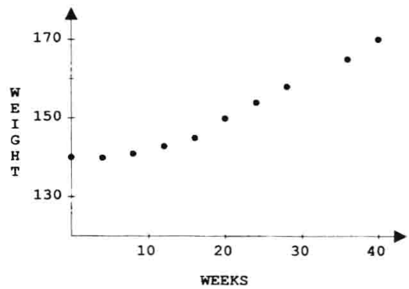
Week	0	4	8	12
Weight	140	140	141	143

Week	16	20	24	28
Weight	145	150	154	158

Week	32	36	40
Weight	160	165	170

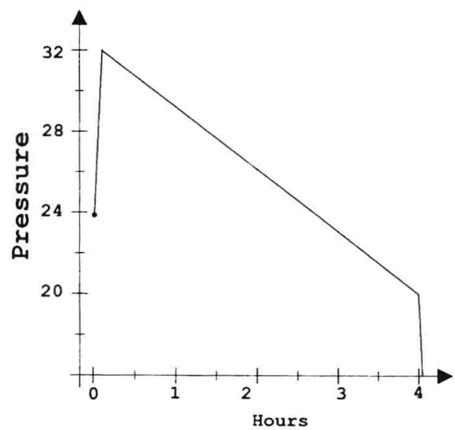
Graph Lisa's weight as a function of time. Use the graph to estimate Lisa's weight at 17 weeks.

Use the time in weeks as the x-axis and the use the weight in pounds for the y-axis. Plot the points according to the information in the table.



Lisa's weight at 17 weeks was about 146.25 lb.

• Androcles is taking a trip to Arizona. One of his tires has a slow leak and is low. Andy is in too much of a hurry to have it fixed so he inflates it from 24 lb/in^2 to the recommended 32 lb/in^2. As he drives the tire deflates and its pressure gradually lowers from 32 lb/in^2 to 20 lb/in^2 over the next 4 hours of driving time. At this time the tire goes over a nail and has a blow-out. Sketch a graph of the tire pressure over this time interval.



• A metal can has height h and radius r. The side of the can is rectangular when unwrapped so can be produced cheaper than the top and bottom which are circular and have waste when cut out of sheet metal. The cost of the top is 14 dollars per square unit and the cost of the sides is 10 dollars per square unit. Write a function that gives the cost of the can as a function of h and r.

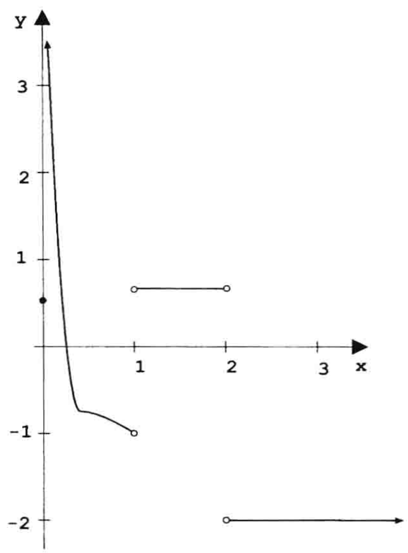
The side when unwrapped is a rectangle with height h and length $2\pi r$, the circumference of the circle. Its area is the product of the side lengths $2\pi rh$. The areas of the top and bottom together are two times the area of the circle or $2\pi r^2$.

The cost of the side is $14(2\pi rh)$ and the cost of the top and bottom together is $10(2\pi r^2)$. the total cost is

$$C = 28\pi rh + 20\pi r^2.$$

SkillMaster 1.2.

Find the domain and the range of the function shown in the graph.



Which values of x give you a y value? These are the points on the x-axis for which a vertical line intersects the graph. These are the points in the domain. It may be easier to see certain points that are not in the domain. The range is the set of possible y values.

The domain is $[0,1) \cup (1,2) \cup (2,\infty)$. Note that the 1 and 2 are excluded from the domain because a vertical line through these points does not intersect the graph. The range is $\{-2\} \cup (-1,\infty)$.

• Find the domain and range of the function. Sketch a graph of the function using your calculator.

$$f(x) = \frac{1}{\sqrt{x^2 - 4}}$$

The function is NOT defined when the denominator is 0 or when the expression inside the square root is negative.

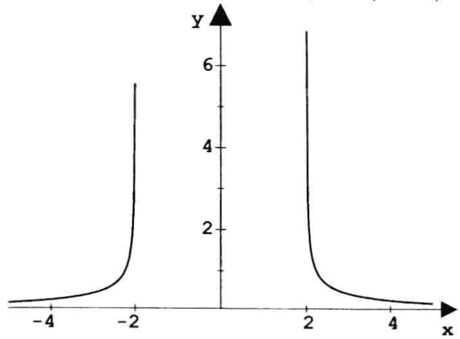
The denominator cannot be 0 so

$$\begin{aligned} \sqrt{x^2 - 4} &\neq 0 \\ x^2 - 4 &\neq 0 \\ x^2 &\neq 4 \\ x &\neq \pm 2. \end{aligned}$$

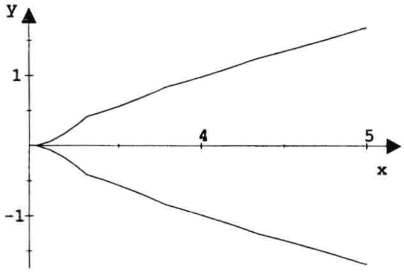
The expression inside the square root cannot be negative so

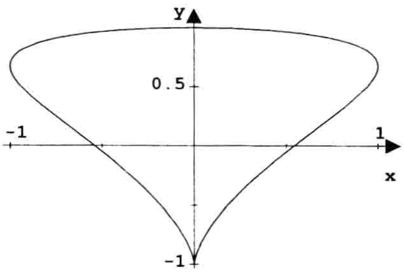
$$\begin{aligned} x^2 - 4 &> 0 \\ (x-2)(x+2) &> 0 \\ x &> 2 \text{ or } x < -2. \end{aligned}$$

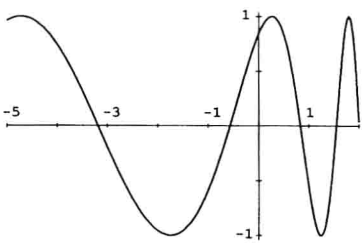
Putting these together gives us that the domain is all points x not in [-2,2]. The domain is $(-\infty, -2) \cup (2, \infty)$. From the graph we see that the range is $(0, \infty)$.



• Which of the following graphs is a function?







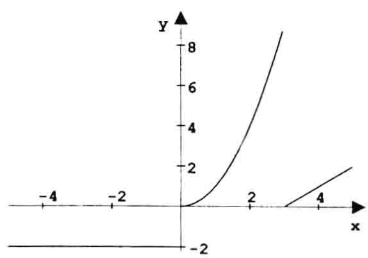
The vertical line test tells us that a graph represents a function if and only if each vertical line passes through at most one point on the graph.

The first two are not functions, while the third is a function.

SkillMaster 1.3.

• Sketch the graph of the following piecewise defined function.

$$f(x) = \begin{cases} -2 & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 3 \\ x - 3 & \text{if } 3 < x \end{cases}$$



Section 1.2 – Mathematical Models

Key Concepts:

- Review of polynomial, rational, algebraic and trigonometric functions
- Mathematical models used to represent physical situations
- Finding a curve that best fits collected data

Skills to Master:

- Recognize different specific types of functions.
- Create scatter plots and select an appropriate model.
- Use a graphing calculator or CAS to fit a model to the data.
- Use a model to estimate and predict other values.

Discussion:

This section introduces the process of using mathematical models to fit real-world situations and using curve fitting to find curves that best fit certain data points. These activities are used over and over again in the sciences and in other areas that use mathematics. Pay attention to the techniques in this section. They will be used later in the text. If any of the functions discussed in this section seem unfamiliar, you should spend time now reviewing the necessary material. Ask your instructor for some additional references if you need them. The material in this section is crucial to the later sections in the text.

Key Concept: Review of polynomial, rational, algebraic and trigonometric functions

You should already be familiar with polynomial, root and trigonometric functions