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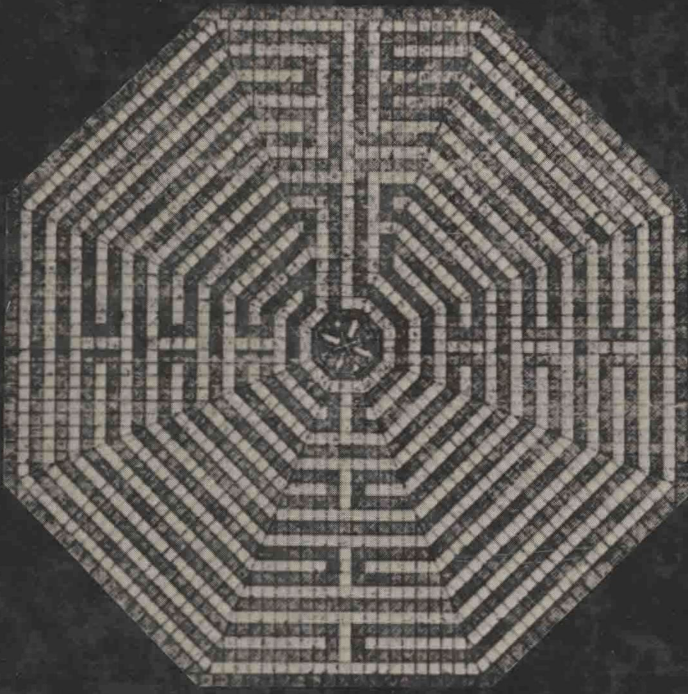

MATHEMATICAL MORPHOLOGY

VOLUME 2: THEORETICAL ADVANCES

EDITED BY **JEAN SERRA**

Image Analysis and Mathematical Morphology

Volume 2: Theoretical Advances

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Image Analysis and Mathematical Morphology

Volume 2: Theoretical Advances

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Preface

This is the second in a series of three books devoted to Mathematical Morphology, and published by Academic Press. The first volume, *Image Analysis and Mathematical Morphology*, which appeared in 1982, dealt mainly with the Euclidean case. The third volume will be devoted to algorithms. This second volume extends the scope of the first.

The manuscript was read by Dr N. Fisher, whose critical comments and corrections considerably improved the style of the original document. I am most grateful to him. I should also like to thank my collegues from the School of Mines of Paris and from other institutes for their constructive advice during the development of the theory presented here. Thanks are due to Mrs L. Pipault, Mrs M. Kreyberg and Miss A. Andriamasinoro for typing this text, and Mr Waroquier for the figures and the drawings. Finally, I am grateful to Academic Press for the quality of their production.

JEAN SERRA

Contents of Volume 1

Notation

1 SETS

λ, ρ	scalars (i.e. positive numbers)
x, y, b, h, etc.	(latin lower case letters) points in $\mathbb{R}^n$ or $\mathbb{Z}^n$, and *also vectors* O_x, O_y, . . .; when one wants to specify that x is a point (geometrical figure) and not a vector, one writes $\{x\}$
$\{x:*\}$	set of points x satisfying property *
X, Y, Z	(latin capital letters) Euclidean or digital sets under study
B	structuring element
Π	test plane generating cross-sections
Ω	set of directions ω, i.e. the unit sphere
$\mathbb{R}^n$, $\mathbb{Z}^n$	Euclidean space, digital space of dimension n
U	umbra
E	arbitrary set
$\mathscr{P}(*)$	set of all subsets of set * (i.e. Boolean lattice)
$\mathscr{P}$	arbitrary complete lattice
$\mathscr{P}'$	lattice of increasing mappings on $\mathscr{P}$
$\mathscr{T}$, $\dot{\mathscr{T}}$	lattice of dilations, erosions, on $\mathscr{P}$
$\mathscr{F}(*)$, $\mathscr{G}(*)$, $\mathscr{K}(*)$	set of all closed, open, compact subsets of *
$\mathscr{U}$, $\mathscr{U}(*)$	set of umbrae, set of umbrae of family *
$\mathscr{C}$	connected class
$\mathscr{C}(*)$	set of convex sets of family *

2 LOGIC AND SET TRANSFORMATIONS

$\exists x$	there exists an x such that
$\forall x$	for all x
w.r.t.	with respect to
$\Rightarrow$	implies
$\Leftrightarrow$; iff	if and only if
$x \in X$; $x \notin X$	point x belongs to set X; point x does not belong to set X
$X = Y$; $X \neq Y$	sets X and Y coincide; set X is different from set Y
$X \subset Y$; $Z \supset B$	X is included in Y; Z contains B
$X \Uparrow Y$	set X hits set Y, i.e. $X \cap Y \neq \varnothing$

X^c	complement of X, i.e. set of point x such that $x \notin X$
$\{X_i\}$	family of sets X_i
C	complement operator
$<, >, \vee, \wedge$ $\prec, \succ, \curlyvee, \curlywedge$	smaller than, longer than, sup, inf in a lattice
λX	homothetic of X with scaling factor λ; $\lambda X = \{x : x/\lambda \in X\}$
X_h	translate of X by vector h; $X_h = \{x : x - h \in X\}$
$\Psi(X)$	set transform of X w.r.t. set transformation Ψ
$\Psi^*(X)$	dual transformation (w.r.t. the complementation), i.e. $\Psi^*(X) = [\Psi(X^c)]^c$
$X \cup Y$	set union, i.e. set of points belonging to X or to Y
$X \cap Y$	set intersection, i.e. set of points belonging to both X and Y
X / Y, $X \setminus Y$	set difference, i.e. set of points belonging to X and not to Y
$\bigcup_{b \in B} X_b$, $\bigcap_{b \in B} X_b$	union, intersection, of all the translates X_b, with $b \in B$
$X \oplus B$, $X \ominus B$	Minkowski addition, substraction
γ, ϕ	generic notation for opening, closing
Γ, $\dot{\Gamma}$	generic notation for dilation, erosion

3 TOPOLOGY

$\overline{X}$, $\mathring{X}$	closure, interior, of set X
∂X	boundary of set X
$X_i \to X$	X_i tends towards X (for the Hit or Miss topology) in $\mathscr{F}$
$\overline{\lim}$, $\underline{\lim}$	upper limit, lower limit
$\lim_{t \to t_0} (*)$	limit of * when t tends towards t_0
inf, sup	lower, upper bound
$X_i \downarrow X$; $X_i \uparrow X$	monotonic sequential convergence of $\{X_i\}$ to X, by upper (resp. lower) values
$d(X, Y)$; $d(x,y)$	distance between X and Y; distance between x and y
u.s.c.;l.s.c.	upper semicontinuous; lower semicontinuous
LCS space	locally compact Hausdorff and separable space
$\mu_n(dx)$	Lebesgue measure in $\mathbb{R}^n$
$P\{*\}$	probability of the event *
$E(X)$	mathematical expectation of X

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Introduction

J. SERRA

Nec ipsa tamen intrant (in memoriam) sed rerum sensarum imagines, illic praesto sunt cogitationi reminiscenti eas. Quae, quomodo fabricatae sint? Quis dicit?

(Saint Augustine, *Conf X-8*)

MORPHOLOGICAL OPTICS

Traditionally, mathematical morphology has been used to describe objects by considering them as subsets of Euclidean space, which results in emphasis on their shapes, their volumes and their textures, as well as on their luminosity and colour at each point. In order to compare bodies, to recognize them, and to uncover their genesis, or to follow their evolution in time—in brief, to reduce them to their essentials—mathematical morphology classifies them into groups of more or less similar entities by putting them through sequences of set transformations.

In recent work, this principle—the use of set transformations for the purpose of description—has proved efficient. The past twenty years have witnessed the construction of a consistent methodology, sometimes mathematically complex, accompanied by the design of morphological processors and their use in a large number of applications.

Yet, what is the value of a hybrid mixture of abstract mathematics and more or less reliable recipes? Does it look like a symbiosis, or rather like a badly wrapped package? Is it necessary to know the ins and outs of the theory of increasing and idempotent mappings acting on the most general lattices, when one wants to control the manufacture of carburettors by means of a morphological processor?

To answer these questions, it seems sensible to set this book, which will be essentially theoretical, in the more general framework of what we shall call *morphological optics*, in terms of the dual aspects—experimental and mathematical—of this method.

Classically, the terms "optics" designates that branch of physics whose

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primary purpose was the study of human vision, but which was finally used to develop theories on the nature and the behaviour of light. Optics is part of physics, since in parallel to each of its mathematical developments, optics is used to invent equipment adapted for the visual field (microscopes, photoelectric cells, lasers, etc.). Moreover, thanks to its mathematical structure, the science of optics has extended beyond the strict domain of human vision (for example, by going from the visible spectrum to consider all electromagnetic radiation) and has expanded the formalism itself. The history of optics in the nineteenth century illustrates this point: Fresnel discovered a series of wave-related phenomena, but he interpreted them in terms of geometry, which is somewhat inadequate when it comes to distinguishing between longitudinal and transverse waves. Some time later, Green invented an appropriate vectorial formalism, which justified Fresnel's results, and laid the groundwork for Maxwell and his famous synthesis between electromagnetism and light waves.

It is noteworthy that the theory of mathematical morphology tends to be organized similarly to optical theory, i.e. with an initial emphasis on vision, the same dialectical distinctions between theory and instrumentation, and the same gradual breaking-up of the approach, moving from the "seen" to the "unseen", via generalization of the mathematical framework.

MORPHOLOGICAL OPTICS DERIVED FROM VISION

In vision, a distinction should be made between geometrical and morphological optics. It is well known that the theories of geometrical optics depend initially on the postulate that a system is identified when it is possible to predict the image of a luminous *point*. The complete field of vision is then deduced by the superposition of elementary transforms. Often, there is an even stronger hypothesis; namely that "the image of the point is itself a point". Obviously, in such a process the convolution and point transforms of Euclidean space (homothetics, rotations, affinities, etc.) play major roles. The usefulness of the linearity property need not be demonstrated: when an image is taken, one sums the images to attenuate the background noise; short-sighted correction lenses deconvolute the retinal image, etc.

Linearity also occurs in acoustics. Indeed, the intensity of sounds, when one leaves aside considerations of phase, combine arithmetically. When several sources emit sound at the same time, the hearing process accommodates all the vibrations, and, to a certain extent, isolates and compares them. If this were not the case then there would be no orchestras! Since preserving the ratios among the sound sources is necessary for an intelligent understanding of the sound scene, all amplifiers (or transmitters) are required to comply with the relative proportions of the source origins, i.e. in mathematical terms, they must be *linear*.

However, visual signals combine differently. Objects in space generally have three dimensions, which are reduced to two dimensions in a photograph or on the retina. In this projection, the luminances of the points located along a line oriented directly away from the viewer are not summed, because most physical objects are not translucent to light rays, in the way that they would be to X-rays, but are opaque. Consequently, any object that is seen hides those that are placed beyond it with respect to the viewer: this self-evident property is a basic one.

In fact, it serves as a starting point for mathematical morphology, since, whenever we wish to describe quantitatively phenomena in this domain, a set-theoretic approach must be used. Stating that A is in front of B is equivalent to asserting that we see the visible contour of B minus, in the set-theoretic sense, that of A. Stating that A hides B is equivalent to saying that the contour of B is included in the contour of A, etc. A morphological description, i.e. a description of their shapes, must primarily use *portions* of space. When transformations are involved, they must apply globally, i.e. they cannot be reduced to simple juxtapositions of point transformations (just like *gestalt* psychology when it deals with human vision).

Now, the set $\mathscr{P}(\mathbb{R}^n)$ of subsets of Euclidean space is equipped with an order relation, called inclusion, such that any family X_i of elements of $\mathscr{P}(\mathbb{R}^n)$ admits a smallest upper bound, called the union, and denoted $\cup X_i$ and a greatest lower bound, called the intersection (the dual of the union), and denoted $\cap X_i$. In the same way that the theory of geometrical optics puts its emphasis on the transformations that commute with addition, morphology naturally stresses the transformations, or mappings $\psi : \mathscr{P}(\mathbb{R}^n) \to \mathscr{P}(\mathbb{R}^n)$ that are related to the basic structures of $\mathscr{P}(\mathbb{R}^n)$. Thus they will be developed:

(i) either from those that preserve inclusion, i.e.

$$X \subset Y \Rightarrow \psi(X) \subset \psi(Y), \qquad X, Y \in \mathscr{P}(\mathbb{R}^n);$$

these transformations are designated as *increasing* transformations; or

(ii) from those that commute with the union, i.e.

$$\psi(\bigcup_i X_i) = \bigcup_i \psi(X_i), \qquad X_i \in \mathscr{P}(\mathbb{R}^n),$$

which are called *dilations* (the dual operation that commutes with the intersection is called an *erosion*).

It will be noted, moreover, that the three classes are not independent: each of the latter two can generate the first class.

IRREVERSIBILITY AND MORPHOLOGICAL DISCOURSE

The parallel that could be drawn with linear methodologies ceases at this point, insofar as there is a major problem. When we say that in geometrical optics we improve a fuzzy picture, making it sharp, we are expressing the point of view of the spectator. The physicist would tend to feel that nothing had been gained, since it is always possible to revert from the sharp to the fuzzy picture: both of them contain exactly the same amount of information. The implied linear process is *reversible*. We are well aware, since Wiener's work, of the emphasis that signal processing lays on the notion of the information content of a message. This high level of interest is all the more justified when one considers questions related to transmission (amplifiers, broadcasting, etc.).

In computer vision, what are we seeking—to transmit information or rather to assimilate it? Reversibility is acceptable when we improve the images that provide the input to a system, as with the case of spectacles for shortsighted people. It is also acceptable to encode images for transfer to processing devices, as is the case when the retinal image is transferred to the visual cortex of the occipital lobe. But beyond this point? The brain does not add a third "eye", which would then look at the visual zones and be observed itself. The chain stops there, and with it the notion of reversibility. Recognition of an object simply means that all the rest has been eliminated from the scene. This is a definitive irreversible operation.

The tool created within the framework of mathematical morphology satisfies this property. The simplest dilation, the union of a set X and its translate X_h can only lose information: from $X \cup X_h$, one cannot backtrack to identify X. The question then arises as to how we can spread out successive losses among a series made up of dozens of transformations, so that the result converges to a single aim? This is the central question of morphology; in various forms and in particular expressions this question occurs frequently.

(i) Since we no longer have the structure of a group, which proved to be useful in the case of geometrical optics (the usual similarities and convolutions), what are the conditions required in order that the composition of two morphological operators remains in the same class as one of them? If the answer is that no such conditions exist then does this mean that the composition takes on a new meaning? If such conditions do exist then how can we interpret the composition? For example, the product of two dilations is yet another dilation, but a dilation followed by an erosion leads to a product that has the characteristics of neither. Thus, as the various possible combinations between dilations and their complementary operations take place,