Lecture Notes in Mathematics

Habib Ammari (Ed.)

Mathematical Modeling in Biomedical Imaging I

1983

Mathematical Biosciences Subseries

Electrical and Ultrasound Tomographies, Anomaly Detection, and Brain Imaging

Springer

Habib Ammari (Ed.)

Mathematical Modeling in Biomedical Imaging I

Electrical and Ultrasound Tomographies, Anomaly Detection, and Brain Imaging

Springer

Editor
Habib Ammari
Laboratoire Ondes et Acoustique
ESPCI, 10 Rue Vauquelin
75231 Paris, Cedex 05
France
habib.ammari@espci.fr

and

Centre de Mathématiques Appliquées
Ecole Polytechnique
91128, Palaiseau, Cedex
France
habib.ammari@polytechnique.fr

ISBN: 978-3-642-03443-5 e-ISBN: 978-3-642-03444-2
DOI: 10.1007/978-3-642-03444-2
Springer Heidelberg Dordrecht London NewYork

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2009932673

Mathematics Subject Classification (2000): 92C55, 31A25, 31A10, 35B25, 35J55, 35J65, 34A55

Cover design: SPi Publisher Services

Printed on acid-free paper

springer.com

Preface

Mathematical sciences are contributing more and more to advances in life science research, a trend that will grow in the future.

Realizing that the mathematical sciences can be critical to many areas of biomedical imaging, we organized a three-day minicourse on mathematical modelling in biomedical imaging at the Institute Henri Poincaré in Paris in March 2007. Prominent mathematicians and biomedical researchers were paired to review the state-of-the-art in the subject area and to share mathematical insights regarding future research directions in this growing discipline.

The speakers gave presentations on hot topics including electromagnetic brain activity, time-reversal techniques, elasticity imaging, infrared thermal tomography, acoustic radiation force imaging, electrical impedance and magnetic resonance electrical impedance tomographies. Indeed, they contributed to this volume with original chapters to give a wider audience the benefit of their talks and their thoughts on the field.

This volume is devoted to providing an exposition of the promising analytical and numerical techniques for solving important biomedical imaging problems and to piquing interest in some of the most challenging issues. We hope that it will stimulate much needed progress in the directions that were described during the course. The biomedical imaging problems addressed in this volume trigger the investigation of interesting and difficult problems in various branches of mathematics including partial differential equations, harmonic analysis, complex analysis, numerical analysis, optimization, image analysis, and signal theory.

The partial support offered by the ANR project EchoScan (AN-06-Blan-0089) is acknowledged. We also thank the staff at the Institute Henri Poincaré.

Paris
March 2009

Habib Ammari

List of Contributors

Habib Ammari Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, habib.ammari@espci.fr

Sylvain Baillet Cognitive Neuroscience and Brain Imaging Laboratory, Université Pierre and Marie Curie-Paris 6, CNRS UPR 640 – LENA, Paris, France, sylvain.baillet@chups.jussieu.fr

George Dassios Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK, G.Dassios@damtp.cam.ac.uk

Mathias Fink Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, mathias.fink@espci.fr

Hyeonbae Kang Department of Mathematics, Inha University, Incheon 402-751, Korea, hbkang@inha.ac.kr

Julien Lefèvre Cognitive Neuroscience and Brain Imaging Laboratory, Université Pierre and Marie Curie-Paris 6, CNRS UPR 640 – LENA, Paris, France, julien.lefevre@chups.jussieu.fr

Jin Keun Seo Department of Mathematics, Yonsei University, 262 Seongsanno, Seodaemun, Seoul 120-749, Korea, seoj@yonsei.ac.kr

Mickael Tanter Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, mickael.tanter@espci.fr

Eung Je Woo College of Electronics and Information, Kyung Hee University, Seoul 130-701, Korea, ejwoo@khu.ac.kr

Introduction

Medical imaging modalities such as computerized tomography using X-ray and magnetic resonance imaging have been well established providing three-dimensional high-resolution images of anatomical structures inside the human body. Computer-based mathematical methods have played an essential role for their image reconstructions. However, since each imaging modality has its own limitations, there have been much research efforts to expand our ability to see through the human body in different ways. Lately, biomedical imaging research has been dealing with new imaging techniques to provide knowledge of physiologic functions and pathological conditions in addition to structural information.

Electrical impedance tomography, ultrasound imaging, and electrical and magnetic source imaging are three of such attempts for functional imaging and monitoring of physiological events.

The aim of this book is to review the most recent advances in the mathematical and numerical modelling of these three emerging modalities. Although they use different physical principles for signal generation and detection, the underlying mathematics are quite similar. We put a specific emphasis on the mathematical concepts and tools for image reconstruction. Other promising modalities such as photo-acoustic imaging and fluorescence microscopy as well as those in nuclear medicine will be discussed in a forthcoming volume.

Electrical impedance tomography uses low-frequency electrical current to probe a body; the method is sensitive to changes in electrical conductivity. By injecting known amounts of current and measuring the resulting electrical potential field at points on the boundary of the body, it is possible to invert such data to determine the conductivity or resistivity of the region of the body probed by the currents. This method can also be used in principle to image changes in dielectric constant at higher frequencies. However, the aspect of the method that is most fully developed to date is the imaging of conductivity. Potential applications of electrical impedance tomography include determination of cardiac output, monitoring for pulmonary edema, and screening for breast cancer.

Electrical source imaging is an emerging technique for reconstructing brain electrical activity from electrical potentials measured away from the brain. The concept of electrical source imaging is to improve on electroencephalography by determining the locations of sources of current in the body from measurements of voltages.

Ion currents arising in the neurons of the brain produce magnetic fields outside the body that can be measured by arrays of superconducting quantum interference device detectors placed near the chest; the recording of these magnetic fields is known as magnetoencephalography. Magnetic source imaging is the reconstruction of the current sources in the brain from these recorded magnetic fields. These fields result from the synchronous activity of tens or hundreds of thousands of neurons.

Both magnetic source imaging and electrical source imaging seek to determine the location, orientation, and magnitude of current sources within the body.

Ultrasound imaging is a noninvasive, easily portable, and relatively inexpensive diagnostic modality which finds extensive use in the clinic. The major clinical applications of ultrasound include many aspects of obstetrics and gynecology involving the assessment of fetal health, intra-abdominal imaging of the liver, kidney, and the detection of compromised blood flow in veins and arteries.

Operating typically at frequencies between 1 and 10 MHz, ultrasound imaging produces images via the backscattering of mechanical energy from interfaces between tissues and small structures within tissue. It has high spatial resolution, particularly at high frequencies, and involves no ionizing radiation. The weakness of the technique include the relatively poor soft-tissue contrast and the fact that gas and bone impede the passage of ultrasound waves, meaning that certain organs can not easily be imaged.

As we said before, in this book not only the basic mathematical principles of these three emerging modalities are reviewed but also the most recent developments to improve them are reported. We emphasize the mathematical concepts and tools for image reconstruction. Our main focuses are, on one side, on promising anomaly detection techniques in electrical impedance tomography and in elastic imaging using the method of small-volume expansions and in ultrasound imaging using time-reversal techniques, and on the other side, on emerging multi-physics or hybrid imaging approaches such as the magnetic resonance electrical impedance, impediography, and magnetic resonance elastography.

The book is organized as follows. Chapter 1 is devoted to electrical impedance tomography and magnetic resonance electrical impedance tomography. It focuses on robust reconstructions of conductivity images under practical environments having various technical limitations of data collection equipments and fundamental limitations originating from its inherent ill-posed nature. The mathematical formulation of the magnetic resonance electrical impedance tomography and multi-frequencies electrical impedance tomography are rigorously described. Efficient image reconstruction algorithms are provided and their limitations are discussed.

Chapter 2 outlines the basic physical principles of time-reversal techniques and their applications in ultrasound imaging. It gives a good introduction to this very interesting subject.

Chapter 3 covers the method of small-volume expansions. A remarkable feature of the method of small-volume expansions is that it allows a stable and accurate reconstruction of the location and of geometric features of the anomalies, even for moderately noisy data. Based on this method robust and efficient algorithms for imaging small thermal conductivity, electromagnetic, and elastic anomalies are provided. Emerging multi-physics or hybrid imaging approaches, namely impediography, magneto-acoustic imaging, and magnetic resonance elastography are also discussed. In these techniques, different physical types of radiation are combined into one tomographic process to alleviate deficiencies of each separate type of waves, while combining their strengths. Finally, a mathematical formulation of the concept of time reversing waves is provided and its use in imaging is described.

Chapter 4 deals with electrical and magnetic source imaging reconstruction methods for focal brain activity. Mathematical formulations and uniqueness and non-uniqueness results for the inversion source problems are given. The basic mathematical model is described by the Biot–Savart law of magnetism, which makes the mathematical difficulties for solving the inverse source problem very similar to those in magnetic resonance electrical impedance tomography discussed in Chap. 1.

Chapter 5 considers time-resolved imaging of brain activity. It discusses optical flow techniques in order to infer on the stability of brain activity.

Contents

Chapter 1
Multi-frequency Electrical Impedance Tomography and Magnetic Resonance Electrical Impedance Tomography

Jin Keun Seo and Eung Je Woo

1.1 Introduction

Medical imaging modalities such as computerized tomography (CT) using X-ray and magnetic resonance imaging (MRI) have been well established providing three-dimensional high-resolution images of anatomical structures inside the human body and computer-based mathematical methods have played an essential role for their image reconstructions. However, since each imaging modality has its own limitations, there have been much research efforts to expand our ability to see through the human body in different ways. Lately, biomedical imaging research has been dealing with new imaging techniques to provide knowledge of physiologic functions and pathological conditions in addition to structural information. Electrical impedance tomography (EIT) is one of such attempts for functional imaging and monitoring of physiological events.

EIT is based on numerous experimental findings that different biological tissues inside the human body have different electrical properties of conductivity and permittivity. Viewing the human body as a mixture of distributed resistors and capacitors, we can evaluate its internal electrical properties by injecting a sinusoidal current between a pair of surface electrodes and measuring voltage drops at different positions on the surface. EIT is based on this bioimpedance measurement technique using multiple surface electrodes as many as 8 to 256. See Figs. 1.1a and 1.2. In EIT, we inject linearly independent patterns of sinusoidal currents through all or chosen pairs of electrodes and measure induced boundary voltages on all or selected electrodes.

J.K. Seo (✉)
Department of Mathematics, Yonsei University, 262 Seongsanno, Seodaemun, Seoul 120-749, Korea
e-mail: seoj@yonsei.ac.kr

E. Je Woo
College of Electronics and Information, Kyung Hee University, Seoul 130-701, Korea
e-mail: ejwoo@khu.ac.kr

H. Ammari, *Mathematical Modeling in Biomedical Imaging I*, Lecture Notes in Mathematics 1983, DOI 10.1007/978-3-642-03444-2_1,

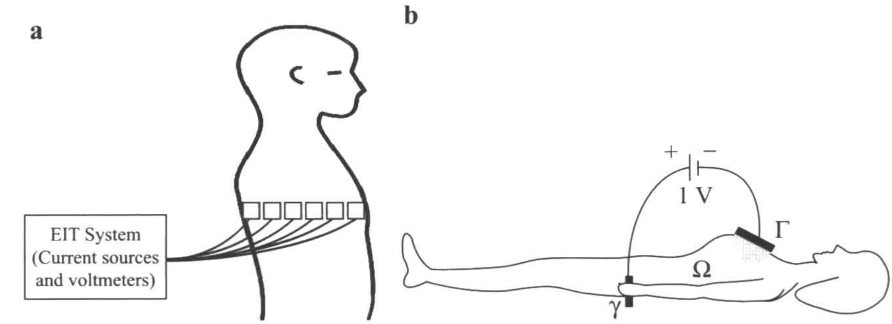

Fig. 1.1 (**a**) EIT system and (**b**) TAS system

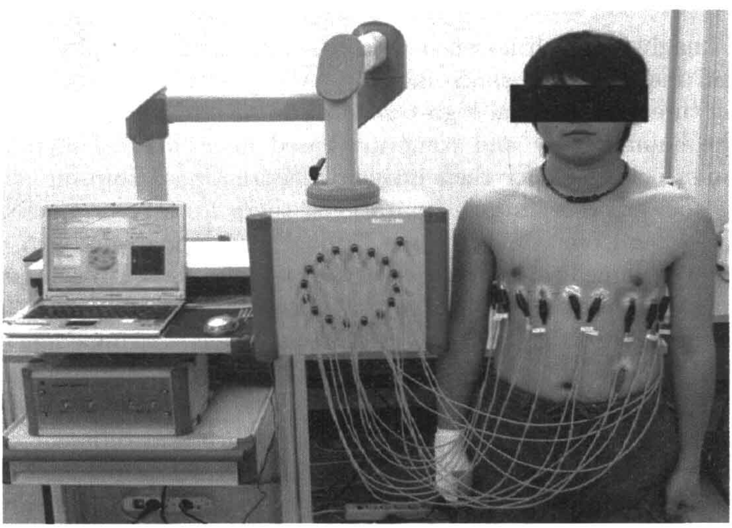

Fig. 1.2 EIT system at Impedance Imaging Research Center (IIRC) in Korea

The measured boundary current–voltage data set is used to reconstruct cross-sectional images of the internal conductivity and/or permittivity distribution. The basic idea of the impedance imaging was introduced by Henderson and Webster in 1978 [13], and the first clinical application of a medical EIT system was described by Barber and Brown [7]. Since then, EIT has received considerable attention and several review papers described numerous aspects of the EIT technique [8, 10, 14, 36, 49, 62]. To support the theoretical basis of the EIT system, mathematical theories such as uniqueness and stability were developed [2, 6, 16, 19, 25, 29, 38, 39, 48, 52, 57–59, 61] since Calderón's pioneering contribution in 1980 [9].

Most EIT imaging methods use a forward model of an imaging object with a presumed conductivity and permittivity distributions. Injecting the same currents into the model, boundary voltages are computed to numerically simulate measured data. Using differences between measured and computed (or referenced) current-to-voltage data, we produce EIT images through a misfit minimization process. However, the inverse problem in EIT has suffered from its ill-posed characteristic due to the inherent insensitivity of boundary measurements to any changes of interior conductivity and permittivity values.

In practice, it is very difficult to construct an accurate forward model of the imaging object due to technical difficulties in capturing the boundary shape and electrode positions with a reasonable accuracy and cost. Therefore, there always exist uncertainties in these geometrical data needed for the model and this causes systematic errors between measured and computed voltages without considering mismatch in the true and model conductivity and permittivity distributions. The ill-posedness of EIT together with these systematic artifacts related with inaccurate boundary geometry and electrode positions make it difficult to reconstruct accurate images with a high spatial resolution in clinical environments. Primarily due to the poor spatial resolution and accuracy of EIT images, its practical applicability has been limited in clinical applications. Taking account of these restrictions, it is desirable for EIT to find clinical applications where its portability and high temporal resolution to monitor changes in electrical properties are significant merits.

Magnetic resonance electrical impedance tomography (MREIT) was motivated to deal with the well-known severe ill-posedness of the image reconstruction problem in EIT. In MREIT, we inject current I (Neumann data) into an object Ω through a pair of surface electrodes to produce internal current density $\mathbf{J} = (J_x, J_y, J_z)$ and magnetic flux density $\mathbf{B} = (B_x, B_y, B_z)$ in Ω. The distribution of the induced magnetic flux density $\mathbf{B}$ is governed by the Ampere law $\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$ where μ_0 is the magnetic permeability of the free space. Let z be the direction of the main magnetic field of an MRI scanner. Then, the B_z data can be measured by using an MRI scanner as illustrated in Fig. 1.3. MREIT takes advantage of the MRI scanner as a tool to capture the z-component B_z of the induced magnetic flux density in Ω. Conductivity imaging in MREIT is based on the relationship between the injection current I and the measured B_z data which conveys the information about any local change of the conductivity σ via the Biot–Savart law:

$$B_z(x, y, z) = \frac{\mu_0}{4\pi}\int_{\Omega} \frac{\sigma(\mathbf{r})\left[(x-x')\frac{\partial u}{\partial y}(\mathbf{r}') - (y-y')\frac{\partial u}{\partial x}(\mathbf{r}')\right]}{|\mathbf{r}-\mathbf{r}'|^3}\, d\mathbf{r}', \ \mathbf{r} = (x, y, z) \in \Omega$$

where u is the induced electrical potential due to the injection current. This supplementary use of the internal B_z data enables MREIT to bypass the ill-posedness problem in EIT.

The technique to measure the internal magnetic flux density $\mathbf{B}$ using an MRI scanner was originally developed for magnetic resonance current density imaging (MRCDI) in late 1980s [17]. In MRCDI, we have to rotate the

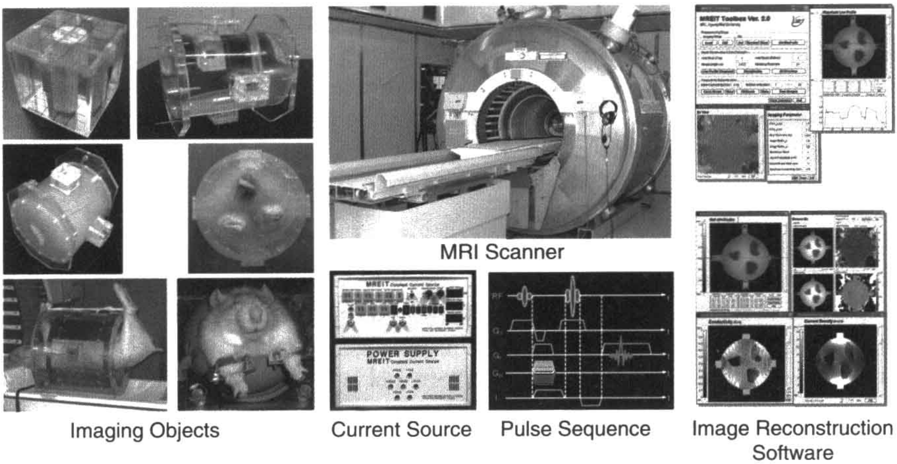

Fig. 1.3 MREIT system at Impedance Imaging Research Center (IIRC) in Korea and its image reconstruction software

object in three orthogonal directions to obtain all three components of **B** since the MRI system can measure only the component of **B** that is parallel to the direction of its main magnetic field (usually denoted as the z-direction). Once we get all three components of **B**, we can visualize the internal current density distribution **J** via the Ampere law $\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$ [50, 51]. This MR-CDI provided a strong motivation of MREIT which combines EIT and MRI techniques [15, 30, 63, 65].

In order for MREIT to be practical, it is obvious that we must be able to produce images of the internal conductivity distribution without rotating the object. This means that the z-component B_z of **B** is the only available data. The first constructive B_z-based MREIT algorithm called the harmonic B_z algorithm was proposed by Seo et al. in 2001 [56]. It is based on the following curl of the Ampere law:

$$\frac{1}{\mu_0}\Delta B_z = \left\langle \nabla\sigma \,,\, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \nabla u \right\rangle \tag{1.1}$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product and Δ is the Laplacian. After the harmonic B_z-algorithm, various image reconstruction algorithms based on the B_z-based MREIT model have been developed [41, 42, 44, 45, 54, 55]. Recent published numerical simulations and phantom experiments show that conductivity images with a high spatial resolution are achievable as long as the measured B_z data has an enough signal-to-noise ratio (SNR).

Although imaging techniques in MREIT have been advanced rapidly, rigorous mathematical theories of the numerical algorithms such as stability and convergence have not been supported yet. Theoretical as well as experimental studies in MREIT are essential for the progress of the technique. In this lecture note, we explains recent results on the convergence behavior and numerical stability of the harmonic B_z algorithm based on a mathematical model replicating the actual MREIT system [34, 35]. Before clinical applications of MREIT, it is necessary to study how errors in the raw data are propagated to the final result of the conductivity imaging. Hence, it is highly necessary to set up an actual mathematical model for MREIT describing the accurate relationship among input current, B_z data and conductivity distribution. For the real MREIT model, boundary conditions are different from conventional styles in PDE and great care is required in using non-standard boundary conditions. The disadvantages of MREIT over EIT may include the lack of portability, potentially long imaging time and requirement of an expensive MRI scanner. Hence, EIT still has various advantages over MREIT although we should not expect EIT to compete with MREIT in terms of spatial resolution.

Lately, a frequency-difference electrical impedance tomography (fdEIT) has been proposed to deal with technical difficulties of the conventional static EIT imaging caused by unknown boundary geometry, uncertainty in electrode position and other systematic measurement artifacts [21, 33, 43]. Conductivity (σ) and permittivity (ϵ) spectra of numerous biological tissues show frequency-dependent changes indicating that we can view a complex conductivity ($\sigma + i\epsilon$) distribution inside an imaging object as a function of frequency. In fdEIT, we inject currents with at least two different frequencies and use the difference between induced boundary voltages at different frequencies to eliminate unknown common modelling errors. To test its feasibility, we consider anomaly detection problems where an explicit representation formula for the potential is available. The formula provides a clear connection between its Cauchy data and the anomaly [3, 29]. As an example of such an anomaly detection problem, let us consider the breast cancer detection problem. In this case, the inverse problem is reduced to detect a suspicious abnormality (instead of imaging) underneath the breast skin from measured boundary data. Figure 1.1b depicts trans-admittance scanner (TAS) which is a device for breast cancer diagnosis. Most of anomaly detection methods used a difference between measured data and reference data in the absence of anomaly. However, in practice, the reference data is not available and its computation is not possible since the inhomogeneous complex conductivity of the background is unknown. To deal with this problem, multi-frequency TAS system has been proposed where a frequency difference of measured data sets at a certain moment is used for anomaly detection [43].

This lecture note focuses on robust reconstructions of conductivity images under practical environments having various technical limitations of data collection equipments and fundamental limitations originating from its inherent nature. We describe the mathematical formulation of MREIT and multi-frequency EIT in clinical environments, image reconstruction algorithms, measurement techniques and examples of images.

1.2 Electrical Impedance Tomography

1.2.1 Inverse Problem in RC-Circuit

The human body can be viewed as a mixture of distributed resistors and capacitors and a circuit model containing resistors and capacitors can be used to explain the one-dimensional EIT problem. Let us begin with considering a simple RC-circuit. Electrical impedance, denoted by Z, is a measure of the total opposition of a circuit to a time-varying electrical current flow. It comprises resistance and reactance taking account of the effects from resistors and capacitors, respectively.

Consider a linear circuit containing a resistor, capacitor and sinusoidally time-varying current source connected in series. If the current source in the circuit is given by $I(t) = I_0 \cos(\omega t)$ where I_0 is the amplitude and ω is the angular frequency, then the resulting voltage $V(t)$ is also sinusoidal with the same angular frequency ω. The relation between $I(t)$ and $V(t)$ is governed by

$$RI(t) + \frac{1}{C}\int I(t)dt = V(t) \tag{1.2}$$

where R is the resistance and C is the capacitance. The voltage can be expressed as

$$V(t) := RI_0 \cos(\omega t) + \frac{I_0}{\omega C}\sin(\omega t) = V_0 \cos(\omega t - \phi)$$

where $V_0 = \sqrt{(RI_0)^2 + \left(\frac{I_0}{\omega C}\right)^2}$ is the amplitude and ϕ is the phase angle such that $\tan\phi = \frac{1}{\omega RC}$ and $0 \le \phi \le \frac{\pi}{2}$. In order to see the interrelation among the impedance $Z := R + \frac{1}{i\omega C}$, voltage and current, it is convenient to express sinusoidally time-varying functions $I(t)$ and $V(t)$ in terms of time-independent phasors $\tilde{I}$ and $\tilde{V}$ such as

$$I(t) = \Re\{\tilde{I}e^{i\omega t}\} \quad \text{and} \quad V(t) = \Re\{\tilde{V}e^{i\omega t}\}$$

where $\tilde{I} = I_0$ and $\tilde{V} = V_0 e^{-i\phi}$. The phasor $\tilde{V}$, corresponding to the time function $V(t)$, contains the amplitude $|\tilde{V}| = V_0$ and phase $\arg(\tilde{V}) = \phi$. With the use of phasors $\tilde{I}$ and $\tilde{V}$, (1.2) can be expressed as

$$\left[R + \frac{1}{i\omega C}\right]\tilde{I}e^{i\omega t} = \tilde{V}e^{i\omega t}$$

or simply

$$\left[R + \frac{1}{i\omega C}\right] I_0 = V_0 e^{-i\phi}. \tag{1.3}$$