DYNAMIC FRACTURE MECHANICS
VOLUME 1: STATIONARY CRACKS
Revised Edition

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PREFACE

An analysis of brittle fracture in power engineering, mechanical and civil engineering, transport, and other branches of engineering shows that fracture is caused in most cases by the growth of crack-like defects in regions of stress concentration. It is practically impossible to prevent the appearance of cracks since a material always contains different types of microscopic cracks irrespective of the preliminary processing to which is it subjected. However, one can learn to design structures in such a way that the growth of these defects should be a steady-state process that can be monitored even under extreme operating conditions (dynamic loading, low and high temperatures, corrosive media, etc.). Hence, the designing of constructions taking into account the viability of damaged parts requires the development of computational methods for predicting the crack growth right up to the attainment of the limiting state. Such computations are made by using a new branch of engineering concerning the strength of materials, viz., fracture mechanics. By applying the criteria of fracture mechanics we can judge the strength, reliability, and life of structures and work out effective nondestructive methods of control, thus helping in the prevention of accidents which may have serious economic and social repercussions. Moreover, fracture mechanics is used for investigation of technical processes involving controlled fracture, for example, in exploration of mineral deposits, drilling of wells, and cutting of metals.

Investigations in the field of fracture mechanics, which was established as a separate branch of mechanics of deformable bodies back in the fifties, require: the construction of fracture models; the creation of analytical and numerical methods for solving problems of bodies with stationary and running cracks in the theory of elasticity, plasticity, and viscoelasticity, and for nonlinear media; as well as the development of experimental techniques. Fracture mechanics is closely related to the physics of fracture phenomena since the processes occurring on microscopic and macroscopic scales are linked inseparably.

The efforts of scientists and engineers working in various fields led to the formulation of basic concepts of fracture mechanics, correct statements of mathematical problems, and the development of methods of their solution by the end of the sixties. It should be emphasized that these investigations were carried out by specialists from many countries and a significant contribution was made by Soviet scientists in establishing the basic principles of fracture mechanics. This was made possible by the traditionally high level of the Soviet school of mathematical theory of elasticity.

Thus, for example, M. Ya. Leonov and V. V. Panasyuk proposed the δ_k-model of fracture, which is far more universal than the well-known model developed by Dugdale. A large number of problems on the stress distribution in cracked bodies under complicating factors (complex configuration of cracks and bodies, bifurcation of cracks, material inhomogeneities, nonlinear and dynamic effects, etc.) were solved by V. M. Aleksandrov, G. I. Barenblatt, N. M. Borodachov, G. P. Cherepanov, L. A. Fil'shtinskii, D. V. Grilitskii, A. A. Kaminskii, B. V. Kostrov, E. M. Morozov, V. I. Mossakovskii, L. V. Nikitin, V. V. Panasyuk, V. Z. Parton, and others. On the atomic and molecular scales, the fracture of metals, polymers, and glasses was studied by the physicists G. M. Bartenev, N. S. Enikolopov, V. A. Kargin, A. N. Orlov, P. A. Rebinder, G. L. Slonimskii, V. I. Vladimirov, S. N. Zhurkov, and others.

Significant contributions have been made by L. M. Kachanov, V. V. Panasyuk, and Yu. N. Rabotnov in developing methods for calculating the fracture toughness of bodies subjected to static, dynamic, and cyclic loading; by V. V. Bolotin in calculating the safety margin of constructions subjected to random loading; by A. A. Kaminskii in calculating the crack propagation in viscoelastic bodies; by B. A. Kudryavtsev and V. Z. Parton in formulation of fracture criteria for piezoelectric materials; by E. M. Morozov and G. P. Nikishkov in developing numerical methods in linear and nonlinear fracture mechanics; by G. S. Pisarenko, V. T. Troshchenko, A. Ya. Krasovskii, S. Ya. Yarema, and others in investigating the effect of low temperature on crack propagation and fracture toughness under cyclic and static loading, in formulation of the law of fatigue growth of cracks and the mechanism of crack propagation in brittle plastic materials, in investigating the effect of loading conditions on the form of fracture, cyclic thermal fracture, delayed fracture, and hydrogen brittleness; by V. S. Ivanova, N. A. Makhutov, E. M. Morozov, V. V. Novozhilov, L. I. Slepyan, and G. S. Vasil'chenko in formulating new fracture criteria; and by V. M. Finkel in development of methods of crack arrest.

Several monographs published in the USSR provide a complete description of the basic principles of fracture mechanics. These include works by V. V. Panasyuk [109], V. Z. Parton and E. M. Morozov [129, 130], G. P. Cherepanov [25], L. I. Slepyan [155], and others.

In spite of the brilliant achievements in the field of fracture mechanics and its numerous applications, the formulation and solution of the dynamic problems of this theory remained unknown until recently on account of their extremely complicated nature. Only the latest elegant analytic solutions of certain model problems and the development of new effective numerical methods have helped in surmounting this obstacle. The number of publications devoted to problems of dynamic fracture mechanics is continuously rising and several hundred papers on the subject are published annually in journals like International Journal of Fracture and Engineering Fracture Mechanics. In order to understand the growing interest towards investigations in dynamic fracture mechanics, it is necessary to grasp the essence of the subject and its interaction with the quasistatic fracture mechanics. Indeed, the process of fracture is characterized (at least in its final stage) by a rapid propagation of the arterial crack or a set of branched cracks and is therefore an essentially dynamic process.

A large number of problems still remain unsolved in the description of this process on microscopic and macroscopic levels. Hence, when we state that fracture mechanics is an essential tool for computing the strength of bodies and structures, we mean the quasistatic fracture mechanics which determines whether or not an arterial crack is stable. Indeed, the quasistatic mechanics of brittle fracture, which is based on the idealized model of a sharp arterial crack and the concept of the stress intensity factor at its tip, has been developed quite extensively; however, it provides only the first approximation to the description of fracture and can simply indicate whether or not a catastrophic growth of the crack sets in.

Obviously, the field of dynamic fracture mechanics is much more extensive than that of quasistatic fracture mechanics. While quasistatic fracture mechanics deals only with the formulation of criteria for a transient crack propagation, dynamic fracture mechanics requires the formulation of a large number of criteria dealing with the start, arrest, propagation, bending, and branching of cracks. In the idealized model mentioned above, this leads to the emergence of a whole range of critical stress intensity factors: the starting stress intensity factor, which depends on the loading rate; the arrest stress intensity factor; the branching stress intensity factor; and, finally, the critical stress intensity factor, which depends on the rate of propagation of the crack. Some of the experimental results can be explained satisfactorily by this model, while some others lead to contradictions with the theoretical results. However, the published experimental results themselves are in contradiction with one another. This is probably due to the fact that many experiments are incorrect because they neglect the interaction of waves scattered at the boundary with the crack tip, or because the rate of crack propagation and the stress intensity factors were not measured quite precisely.

Thus, dynamic fracture mechanics occupies a special place in the mechanics of a deformable body. In the first place, a large number of questions still remain to be answered and this branch of science is still in its formative stage. New results sometimes necessitate a reconsideration of even its basic concepts. Secondly, this field has no equal in the diversity of analytic, numerical, and experimental methods employed in it.

In the present two-volume course we have endeavored to describe the state-of-the-art in dynamic fracture mechanics, the basic methods, and achievements. Hence, together with the results of investigations carried out by the authors in this field, the book also contains important results of investigations carried out by other scientists, including those in the USA, who have made a significant contribution to the development of dynamic fracture mechanics.

A part of Vol. 1 was published as a monograph in the USSR under the title "Dynamic Fracture Mechanics" in 1985, although the work on the manuscript was completed in 1983. This two-volume course has been prepared specially for publication in the USA and contains important results obtained after 1983 in the USSR and abroad. The first volume contains a description of the basic principles of dynamic fracture mechanics, and analytic and numerical methods for determining the stress intensity factors in two- and three-dimensional bodies with stationary rectilinear, curvilinear, plane, and penny-shaped cracks subjected to harmonic or impact loading. The second volume contains results of investigations of the laws of crack propagation at constant and varying rates in elastic bodies, taking into account the influence of the elastic lattice. Numerical and experimental methods of determining stress intensity factors in bodies with transient cracks are also described in Vol. 2.

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V. Z. Parton
V. G. Boriskovsky

INTRODUCTION

Two methods stand out among the approaches describing the process of fracture. In the first method, the process is characterized by the behavior of an arterial crack, while the second one considers the growth and propagation of a set of microscopic defects. The first approach dominates in the scientific literature mainly because it provides a satisfactory stability criterion and a simple computational apparatus under quasistatic loading. The second approach has not been developed extensively so far and has not led to satisfactory computational methods. However, the development of a set of microscopic defects and the growth of an arterial crack are interconnected not only at the stage of origination of a macroscopic crack, but also during its propagation. Macroscopic cracks ensure a high localized concentration of stresses and their behavior begins to depend on the growth of microscopic defects appearing in this case.

In the most prevalent idealized model, the growth of a rectilinear crack is usually considered in an elastic body. The unbounded stresses appearing at the crack tip in this case are characterized by the stress intensity factors, and the fracture is assumed to occur at the very tip of the crack. Moreover, it is assumed that the energy required to create a unit new surface is a constant of the material. Proceeding from this consideration, we can calculate the elastodynamic stress field at the crack tip and formulate the criterion for crack propagation, i.e., the energy balance equation. The basic aspects of this idealized model can be written in the following form:

1. The stress fields at the crack tip are described with the help of stress intensity factors.
2. The criteria of start, arrest, and propagation of a crack are derived from the condition that the fracture energy per unit area of new surface is constant.

The adequacy of this model can be determined from an analysis of the experimental data on stress intensity factors and a comparison of the conditions of start, propagation, and arrest of a crack with theoretical conditions. The following situation arises in this case. In the first place, the criteria of start, propagation, and arrest of a crack are found to be "disconnected," which is proven by the existence of various critical stress intensity factors describing the start, propagation, and arrest of a crack. Secondly, a one-to-one correspondence is established between the various stress intensity factors and crack velocity by using the energy balance equation and assuming that the process is independent of the rate and past history of loading. However, this correspondence is not always confirmed by experiment [67, 68]. Finally, the idealized model does not provide an automatic explanation for the branching of cracks.

In spite of the contradictions concerning the criteria for start, propagation, and arrest of cracks, it must be borne in mind that in accordance with the first basic concept of the idealized model, the stress fields are described with the help of the stress intensity factors. Hence, it is interesting to compare the analytic and experimental values of these coefficients. It must be realized that such a comparison is quite difficult to make. Analytic solutions are obtained for infinite regions, while experiments are carried out on small samples. Hence, a comparison of the results is possible only until the beginning of interaction of the waves scattered at the boundary with the crack tip, i.e., only for a very short time. Moreover, experiments are performed on plates where wave dispersion and departure of the stress state from the two-dimensional case is observed (at least at the crack origin). However, the authors of works [68, 140–143] were able to carry out a direct comparison for plates made of a purely brittle material Homalite-100 in which a stationary crack was set into motion by the application of an impact load at the crack faces. It was shown that as far as the correspondence of theoretical and experimental stress fields is concerned, the idealized model can be assumed to be quite satisfactory except for some isolated cases of transient processes.

It is usually assumed that the departure of the behavior of a crack from that predicted by the idealized model is due to the existence of various nonlinear effects. A more accurate and realistic substantiation of this disparity was given by W. G. Knauss and K. Ravi-Chandar [68, 140–143], who used the concept of leading microscopic cracks. With the help of a visual and microscopic analysis of fracture surfaces, they showed that the process of dynamic fracture takes place in a certain finite region in front of the crack tip, like in the case of viscous bodies upon quasistatic fracture. In the first place, the authors paid attention to the inhomogeneity and nonsmoothness of the fracture surfaces in which three zones could be singled out: a "mirror" zone, a "mist" zone, and a "hackle" zone. In the "mirror' zone, the fracture surface is very smooth and completely reflects light; in the mist region, where the stresses are more pronounced, the fracture surface becomes rougher, and is very coarse in the hackle region. It was shown that initially a single crack propagates in the mirror zone, and its behavior does not differ significantly from the quasistatic growth. In the mist zone there is a simultaneous uniform propagation of a set of cracks. In the hackle region the crack propagation follows the same physical pattern but the size of the microscopic fracture region increases. Thus, crack propagation under high stresses is controlled by the growth (transformation) of microscopic cavities in the microscopic cracks, followed by their union and mutual interaction.

With the help of these concepts it is possible to provide an acceptable qualitative description of branching of cracks as a continuous process of evolution of leading microscopic cracks, to explain the dependence of the initiating stress intensity factor on the loading rate, and several other facts. However, at present it is not possible to make exact quantitative computations for the interaction between a macroscopic crack and a set of microscopic cracks. These microscopic cracks have a complex and predominantly three-dimensional statistical distribution and "perceive" the existence of other microscopic cracks not instantaneously, but only through the propagation of stress waves.

Thus, it is quite clear that the idealized fracture model has several drawbacks which should be taken into consideration in actual practice of engineering application of the dynamic fracture mechanics. On the other hand, this is practically the only model which can be used to describe the propagation of the fracture front at a macroscopic level. On the basis of what has been stated above, it can be assumed that although the idealized model is unsuitable for the derivation of fracture criteria (i.e., criteria for start, propagation, arrest, bending, and branching), it is quite acceptable in such cases where the basic properties of the fracture process (crack propagation rate, conditions of start and arrest, etc.) are known from experiment, and one is required to make computations for the stress state or to simulate the crack growth. Thus, the mixed analytical-experimental and numerical-experimental approaches acquire a special significance in dynamic fracture mechanics.

As a rule, all the results described in this book have been obtained with the help of the idealized model (the only exception to this is the propagation of a crack in a lattice and the construction of the unified fracture model considered in Vol. II). We shall give a general formulation of the dynamic fracture mechanics problems to be solved. It should be noted that a consideration of inertia effects is essential in view of the dynamic nature of loading and crack propagation (this may occur separately or simultaneously). Thus, it is quite possible that a crack is stationary (i.e., its rate of propagation is equal to zero), but a dynamic load is applied to it. If this load assumes the critical value, the crack begins to propagate (irrespective of whether this happened as a result of dynamic or static loading). The law of propagation of the crack (i.e., the dependence of its propagation rate on time) is derived from several energy relations. Subsequently, the crack may be arrested as a result of redistribution of stresses and again become stationary.

It can be concluded from what has been stated above that the analysis of the stress state in a body with a stationary crack subjected to a dynamic loading acquires a special significance in fracture mechanics since it allows a deeper understanding of the processes preceding brittle fracture. Moreover, it is sufficient to consider two types of loading, viz., impact and harmonic (obviously, these two types of loading are basic and all other existing types of loading can be reduced to these two). In the case of impact loading, the right-hand side of the system of equations of motion is expressed in the form of a Heaviside function and specific initial conditions must be given. It is required to determine the dependence of the basic characteristics of the fracture process, viz., the stress intensity factors, on time. In the case of harmonic loading, the steady-state condition is usually considered, when all quantities have a harmonic dependence on time with the same frequency. In this case it is required to determine the dependence of the stress intensity factors on crack length, amplitude, and frequency of the applied load.

While formulating the dynamic problems of fracture mechanics for propagating cracks it is assumed that the trajectory of motion of the crack is rectilinear and its rate of propagation is constant or an arbitrary function of time. Loading may be static, harmonic, or impact-type.

The following factors can be used for further classification of dynamic problems of fracture mechanics for stationary as well as nonstationary cracks: 1) initial crack length (finite or infinite); 2) type of the body (plane, three-dimensional, strip, plate, or shell); 3) type of deformation.

On the basis of this classification, we can isolate the following types of problems:

1. To determine the dependence of stress intensity factors on frequency for a stationary crack subjected to a harmonic loading.
2. To determine the dependence of stress intensity factors on time for a stationary crack subjected to impact loading.
3. To determine the time dependence of stress intensity factors and the rate of propagation of a nonstationary crack.
4. To determine the law of motion of a nonstationary crack.

Dynamic fracture mechanics also includes various types of branching problems and the determination of trajectory of moving cracks.

The solution of these problems requires the application of numerical as well as analytic methods in combination (sometimes) with the experimental results. As a rule, analytic methods are used while considering plane configurations like an infinite plane with a semi-infinite crack, a plane with a finite crack, strips with a finite or a semi-infinite crack, as well as a medium with a penny-shaped crack. Analytic solutions of problems of dynamic fracture mechanics involving opening modes, and inplane and antiplane shear modes lead to very important qualitative conclusions about the processes preceding brittle fracture in the case of dynamic loading, and about the propagation of the fracture front. However, while solving specific problems of dynamic fracture mechanics in actual practice one sometimes has to determine the stress intensity factors in finite cracked bodies, including plates and shells. As a rule, this is accomplished with the help of various numerical methods and by constructing algorithms for solving these problems.

NOMENCLATURE

The following notation is used in the text unless stated otherwise:

x, y, z	Cartesian coordinates;
r, θ, z	cylindrical coordinates;
ρ, θ, z	elliptic coordinates;
t	time;
t_A	time of crack arrest;
$u_x, u_y, u_z = w$	components of displacement vector $\{u\}$ in Cartesian coordinate system;
u_r, u_θ, u_z	the same components in cylindrical coordinate system;
u_ρ, u_θ, u_z	the same components in elliptic coordinate system;
$\epsilon_{ij}, \sigma_{ij}$ $(i, j = x, y, z$ or $i, j = 1, 2, 3)$	strain and stress tensor components in Cartesian coordinate system;
$\epsilon_{ij}, \sigma_{ij}$ $(i, j = r, \theta, z)$	the same components in cylindrical coordinate system;
$\epsilon_{ij}, \sigma_{ij}$ $(i, j = \rho, \theta, z)$	the same components in elliptic coordinate system;
$\Sigma_i^\alpha(\alpha = I, II, III,$	functions of angular distribution of stresses

$i = xx, yy, xz, yz)$	in the vicinity of a crack tip;
λ, μ	Lamé constants;
$\nu = \lambda/(2(\lambda + \mu))$	Poisson's ratio;
$E = \mu(3\lambda + 2\mu)/(\lambda + \mu)$	Young's modulus;
$\varkappa = \nu, E' = E/(1 - \nu^2)$	for plane deformation;
$\varkappa = \nu/(1 + \nu), E' = E$	for plane stress state;
c_1, c_2, c_R	velocities of dilatational, shear and Rayleigh waves respectively;
ρ_0	density of material;
$n_*^2 = c_2/c_1, c_1^2, = (\lambda + 2\mu)/\rho_0, c_2^2 = \mu/\rho_0$	
K_I, K_{II}, K_{III}	stress intensity factors for opening mode, inplane shear and antiplane shear modes, respectively;
$K_{IS}, K_{IIS}, K_{IIIS}$	static stress intensity factors;
K_c, K_{Ic}	fracture toughness (critical stress intensity factor) in static approximation;
K_D, K_{ID}	fracture toughness in dynamic approximation;
l	crack length (half-length);
v	velocity of crack propagation;
U	total deformation energy;
T	kinetic energy;
W	strain energy density;
G	energy release rate;
G_c	critical energy release rate;
$\{\mathbf{J}\} = \{J_1, J_2\}$	J-integral;
2γ	surface energy of fracture per unit area of new surface;
Γ	boundary of the body;
n_i ($i = x, y, z$ or $i = 1, 2, 3$)	components of the unit normal vector $\{\mathbf{n}\}$ directed outward from the contour;
T_i ($i = x, y, z$ or $i = 1, 2, 3$)	components of the vector $\{\mathbf{T}\}$ denoting tension at the boundary of a body;
F_i ($i = x, y, z$ or $i = 1, 2, 3$)	components of the vector $\{\mathbf{F}\}$ of mass forces;
$q^{(1)}, q^{(2)}, q^{(3)}$	opening, inplane shear, and antiplane shear loads, respectively
ω	loading frequency;
$\alpha_{1,2} = \omega/c_{1,2}, \alpha_R = \omega/c_R;$ $m_{1,1} = \omega^2 l^2/4c_{1,2}^2$	wave numbers;
ϑ	angle of wave incidence;
$\{\tilde{\mathbf{p}}\}$	vector in the direction of wave propagation;
φ, ψ	wave potentials;
$\varphi^{(i)}, \psi^{(i)}$	incident wave potentials;

$\varphi^{(s)}$, $\psi^{(s)}$	scattered wave potentials;
s	Fourier transform parameter;
$\beta_{1,2}^2 = s^2\alpha_{1,2}^2$	
$H(t) = \{0$ for $t < 0$; 1 for $t \geq 0\}$	Heaviside function;
p, p_1, p_2	Laplace transform parameters;
$\gamma_{1,2}^2 = s^2 + (p/c_{1,2})^2$, $\epsilon_{1,2} = v/c_{1,2}$, $\delta_{1,2}^2 = 1 - \epsilon_{1,2}^2$,	
$R_*(\epsilon_1,\epsilon_2) = -(\epsilon_2^2 - 2)^2 + 4(1 - \epsilon_1^2)^{1/2}(1 - \epsilon_2^2)^{1/2}$	Rayleigh function;
[M]	mass matrix;
[K]	stiffness matrix;
{x}	nodal displacement vector;
{f}	nodal loading vector;
$\delta_{ij} = \{1$for $i = j$; 0 for $i \neq j\}$	Kronecker's delta;
J_n	n-th order Bessel function of the first kind for a real argument;
I_n, K_n	modified n-th order Bessel functions of the first and second kind, respectively, for an imaginary argument.

CONTENTS