

Kenneth J. Waldron / Gary L. Kinzel

SECOND EDITION

KINEMATICS, DYNAMICS, AND DESIGN OF MACHINERY

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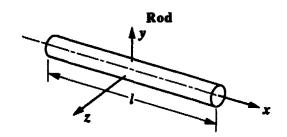
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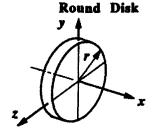
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Table A.1: Mass Moments of Inertia for Common Solid Shapes

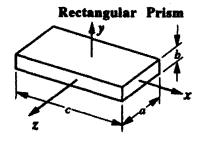


$$I_{y} = I_{z} = \frac{ml^{2}}{12}$$



$$I_x = \frac{mr^2}{2}$$

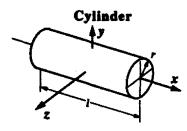
$$I_y = I_z = \frac{mr^2}{4}$$



$$I_x = \frac{m(a^2 + b^2)}{12}$$

$$I_y = \frac{m(a^2 + c^2)}{12}$$

$$I_z = \frac{m(b^2 + c^2)}{12}$$



$$I_x = \frac{mr^2}{2}$$

$$I_y = I_z = \frac{m(3r^2 + l^2)}{12}$$

$$I_x = \frac{m(a^2 + b^2)}{2}$$

$$I_y = I_z = \frac{m(3a^2 + 3b^2 + l^2)}{12}$$

$$I_x = I_y = I_z = \frac{2mr^2}{5}$$

Table A.2: Properties of Areas

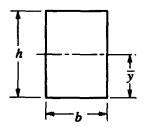
A = Area

= Area moment of inertia

J = Polar area moment of inertia

y = Centroidal distance

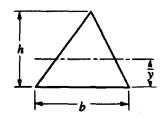
Rectangle



$$A = bh y = \frac{h}{2}$$

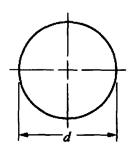
$$I = \frac{bh^3}{12}$$

Triangle



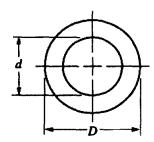
$$A = \frac{bh}{2} \qquad y = \frac{h}{3}$$
$$I = \frac{bh^3}{36}$$

Circle



$$A = \frac{\pi d^2}{4} \qquad J = \frac{\pi d^4}{32}$$
$$I = \frac{\pi d^4}{64} \qquad y = \frac{d}{2}$$

Hollow Cylinder



$$A = \frac{\pi(D^2 - d^2)}{4}$$

$$J = \frac{\pi(D^4 - d^4)}{32}$$

$$I = \frac{\pi(D^4 - d^4)}{64}$$

$$y = \frac{D}{2}$$

KINEMATICS, DYNAMICS, AND DESIGN OF MACHINERY

PREFACE

The second edition of a textbook always provides the opportunity to improve those parts of the material that have been found in the classroom to fall short of the authors' original intent. So it is with this edition. Our intent of providing a teaching tool that features a straightforward presentation of basic principles while having the depth and rigor to serve as a basis for more advanced work has not changed. However, we have been listening to the voices of those who have used this work as students and as teachers. In this new edition we have attempted to address their main themes.

One major change is structural. We have separated the "superchapter" that was Chapter 2 in the first edition into three new chapters with some added explanatory material. This change is intended to ease the student's progress into the more mathematical part of the material. We have also removed the superscript notation for identifying reference frames in situations where it is superfluous because multiple frames are not needed. This means, for example, that the superscript notation is not used in the introductory chapter on graphical analysis techniques that is the new Chapter 2. The superscript notation is retained for those situations in which it is necessary to keep track of multiple moving reference frames, but it is introduced later after students should have gained some confidence in their basic techniques.

The introduction to algebraic solution techniques, which now forms Chapter 5, has been somewhat reorganized to provide a smoother progression into the topic. Once again, the superscript notation for reference frames has been removed whenever it is not needed.

We have added a significant number of new problems and new worked examples in selected locations in the text. The added problems include some open-ended design problems.

Chapter 1 now contains sections on bearings and actuation that provide a stronger link to practical engineering in which mechanisms must not only have the right dimensions, but must move freely over long service lives, and must be driven.

We have expanded the design chapter (Chapter 6) to include two-position double-rocker designs. Both graphical and analytical approaches are given. The section on path generation has also been expanded to include the design of eight-link mechanisms that guide a coupler along a path in curvilinear motion. In addition, the section on crank-rocker designs has been expanded.

We have upgraded the software provided on the CD that accompanies the book. The CD includes a new set of programs based on Matlab's graphical user interface (GUI). These programs are easier to use than the original programs. In the rigid-body guidance programs, we have also included a rectification feature that identifies solutions that have a branch problem. This greatly improves the usability of the programs for design problems. The cam design program also has been greatly improved. It is now possible to optimize cam motion and to create displacement profiles that are made up of several different standard mathematical functions. As is the case with other GUI-based programs, the entire process is interactive.

Although the new programs are easier to use, writing them is beyond the scope of what would normally be expected for students in an introductory level kinematics class. Therefore, the original programs are also included on the CD. These programs are simpler and could be written by many students at the junior and senior levels in mechanical engineering. These can be used directly or modified by the students. We have included comments so that an interested programmer can understand the flow of the programs.

The CD also contains animations of selected mechanisms generated using a solid modeling program (SolidEdge). These illustrate most of the common single-loop mechanisms. In addition, a set of PowerPoint slides is included as a supplement for the lectures. Some of these include step-by-step procedures for several of the figures given in the book. While these do not cover all aspects of the book, they cover most of the topics that would be included in an entry-level class. These can be easily tailored by the user for specific lectures.

These are only the more visible changes. There are lesser improvements throughout.

The book is intended for courses ranging from an introduction to planar linkage kinematics to more advanced courses that include spatial mechanisms. For example, an introductory course might cover Chapters 1, 2, 4, 5, 6, 8, and 13. The gear chapters could also be covered to some extent. A more advanced course might cover Chapters 1, 3, 4, 7, 8, 9, 14, and 15. Again, the gear chapters could be covered or omitted. In most instances, there are programs on the CD to augment the lecture material included in the book.

Although the book is intended mainly as a textbook, we have written it so that it can serve also as a reference book for mechanism kinematics. For example, where appropriate, we have summarized the equations developed in tables for easy access.

We trust that those who have used the work as a teaching tool will find that this new edition better serves their needs and that they will continue to tell us about the strengths and weaknesses they find in it. This is a topic of fundamental importance to mechanical engineering, as it has been since the time of James Watt. We hope that we have contributed positively to the training of students and hence to the practice of this important and rewarding field.

We would like to express our sincere thanks to the colleagues and students who have contributed to the success of the second edition of this book. We especially acknowledge Necip Berme for several of the examples and exercise problems in the book. A number of these are based on class assignments that he has made during the years. We also thank Yueh-Shao Chen, Sung-Lyul Park, and Michael Stevens who wrote the GUI-based programs that are included with this book. And finally, we would like to thank Edward Kinzel who contributed to the new section on design for path generation.

Kenneth J. Waldron Gary L. Kinzel April 2003

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INTRODUCTION

1.1 HISTORIC PERSPECTIVE

A mechanism is a machine composed of rigid members that are jointed together. The members interact with one another by virtue of the joints. The joints are formed by portions of the surfaces of the members joined that contact one another. The geometries of the contacting surface segments determine the properties of each joint.

Mechanisms may be simple or complex. Figure 1.1 shows a walking machine that is composed of dozens of mechanisms that must be coordinated through complex control systems. Other machines may involve only a single mechanism.

The design of mechanisms is a technical area that is unique to mechanical engineering. Its history stretches back to prehistoric times. Artisans such as blacksmiths and carpenters also functioned as the designers of mechanisms. One of the original functions of engineers was the design of mechanisms both for warfare and for peaceful uses. In Renaissance times, we find Leonardo da Vinci depicting a sophisticated variety of mechanisms, mostly for military purposes. Sometime thereafter the distinction between civil engineering and military engineering appeared. The modern era in mechanism design, along with the



FIGURE 1.1 The Adaptive Suspension Vehicle. Each leg is a planar pantograph mechanism hinged to the body about an axis parallel to the longitudinal axis of the vehicle.

history of mechanical engineering as a distinct discipline, can be viewed as starting with James Watt.

That is not to say that the subject has remained static. In fact, there have been dramatic changes in the practice of mechanism design in recent years. Traditionally, machines have been designed to be powered by a single "prime mover," with all functions mechanically coordinated. That tradition certainly predates Watt. Recent developments in computer technology, coupled with improvements in electric motors and other actuators, have made it possible to use a different approach. This is an approach in which machines are powered by multiple actuators coordinated electronically. The resulting machines are simpler, less expensive, more easily maintained, and more reliable. Another major change is in the techniques used in mechanism design. The use of interactive computer graphics has had a dramatic impact on design practice. One of our motivations in producing this book, even when a number of excellent texts are already available in mechanism kinematics, is to provide a treatment that reflects these changes in practice.

1.2 KINEMATICS

Kinematics is the study of position and its time derivatives. Specifically, we are concerned with the positions, velocities, and accelerations of points and with the angular positions, angular velocities, and angular accelerations of solid bodies. Together these entities are sufficient to describe the motions of solid bodies. The position of a body can be defined by the position of a nominated point of that body combined with the angular position of the body. In some circumstances we are also interested in the higher time derivatives of position and angular position.

The subject of kinematics is a study of the geometry of motion. This is an accurate title because kinematics is geometry with the element of time added. The bulk of the subject matter of this book is often referred to as the kinematics of mechanisms. Our objective is to present techniques that can be used to design mechanisms to meet specific motion requirements. That is why the subject matter is approached from a mechanical designer's perspective.

1.3 DESIGN: ANALYSIS AND SYNTHESIS

The material in this book falls into two classifications. The first consists of techniques to determine the positions, velocities, and accelerations of points in the members of mechanisms and the angular positions, velocities, and accelerations of those members. These are kinematic analysis techniques. The second type of material comprises methods for mathematically determining the geometry of a mechanism to produce a desired set of positions and/or velocities or accelerations. These are rational synthesis techniques.

The activity that distinguishes engineering from science is design. Science is the study of what is; engineering is the creation of what is to be. This creative activity is design or, more formally, synthesis. The rational synthesis techniques developed by kinematicians offer a rather direct route to mechanism design that lends itself well to automation using computer graphics workstations. However, these techniques do not represent the only way to design mechanisms and they are relatively restrictive: Rational synthesis techniques exist only for specific types of mechanism design problems, and many practical mechanism design problems do not fit within the available class of solutions. An alternative is to use

informal synthesis. This is a methodology used by engineers to solve design problems in many technical areas, not just in mechanism design. The basic procedure is to "guess" a set of dimensions and then use analysis to check the resulting performance. The dimensions are then adjusted to attempt to match more closely the performance specifications, and the mechanism is analyzed again. The process is repeated until an acceptably close match to the specifications is achieved. Thus, a primary use of the analysis material is also in mechanism design.

From an engineering point of view, it is not possible to treat mechanism design solely in terms of kinematics. The motivation for performing an acceleration analysis is often to enable inertia forces on the links to be calculated, allowing, in turn, computation of the forces transferred between links and the internal forces, or stresses, within the links. Mechanisms must usually drive loads, as well as generate motions. Of course, as soon as we introduce the concept of force, we leave the domain of pure kinematics and enter that of kinetics. Insofar as the largest forces in many mechanisms are inertia forces created by motion, it is convenient to study them within the general framework of kinematic techniques. There is also an important symmetry between the geometry of the force distribution and that of the velocity distribution that is particularly useful when working with spatial mechanisms. Thus, it is entirely appropriate to treat mechanism statics or kinetics within the general geometry of motion framework constructed to study mechanism kinematics. Such a treatment is presented in the later chapters of this book.

1.4 MECHANISMS

Mechanisms are assemblages of rigid members connected together by joints. Mechanisms transfer motion and mechanical work from one or more actuators to one or more "output" members. For the purposes of kinematic design, we idealize a mechanism to a kinematic linkage in which all the members are assumed to be perfectly rigid and are connected by kinematic joints. A kinematic joint is formed by direct contact between the surfaces of two members. One of the earliest codifications of mechanism kinematics was that of Reuleaux (1876), and some of the basic terminology we use originated with him. He called a kinematic joint a "pair." He further divided joints into "lower pairs" and "higher pairs." A lower pair joint is one in which contact between two rigid members occurs at every point of one or more surface segments. A higher pair is one in which contact occurs only at isolated points or along line segments. All other things being equal, a higher pair will produce higher contact stresses than will a lower pair.

Joints are the most important aspect of a mechanism to examine during an analysis. They permit relative motion in some directions while constraining motion in others. The types of motion permitted are related to the number of degrees of freedom (dof) of the joint. The number of degrees of freedom of the joint is equal to the number of independent coordinates needed to specify uniquely the position of one link relative to the other constrained by the joint.

Lower pair joints are necessarily restricted to a relatively small number of geometric types, because the requirement that surface contact be maintained constrains the geometry of the contacting surfaces. It can be shown that there are only six fundamentally different types of lower pair joints, classified by the types of relative motion that they permit. There is, in contrast, an infinite number of possible higher pair geometries. The lower pair joint types are shown in Table 1.1. Some important examples of higher pair joints are shown in Table 1.2.

¹ Reuleaux, F., *The Kinematics of Machinery* (Translated and edited by A. B. W. Kennedy), Dover Publications, Inc., New York, 1963.

TABLE 1.1 Lower Pair Joints

Connectivity (Number of degrees of freedom)	Names	Letter symbol	Typical form	Sketch symbol
1	Revolute Hinge Turning pair	R	STO .	(Planar) (Spatial)
1	Prismatic joint Slider Sliding pair	P		(Planar) (Spatial)
1	Screw joint Helical joint Helical pair	н	$s = h\theta$	(Spatial)
2	Cylindrical joint Cylindrical pair	С		(Spatial)
3	Spherical joint Ball joint Spherical pair	S		(Spatial)
3	Planar joint Planar pair	P _L		(Spatial)

Lower pair joints are frequently used in mechanism design practice. They give good service because wear is spread out over the contact surface and because the narrow clearance between the surfaces provides good conditions for lubrication and a tight constraint on the motion. Changes in the geometric properties of the joint with wear occur slowly for a lower pair. At least as important are the simple geometries of the relative motions that these joints permit.

Higher pair joints that involve pure rolling contact, or that approximate that condition, are also used frequently. In pure rolling contact, the points in one of the two joint surfaces that are actually in contact with the other surface at any instant are at rest relative to that surface. Hence there is no relative sliding of the surfaces and joint friction and wear are minimized. Physically, the limitation of this kind of joint is the stress intensity that the material of the contacting bodies can support. Stresses are necessarily high because of the very small