

FRACTURE MECHANICS



25^{TH VOLUME}

FAZIL ERDOGAN

E D I T O R



STP 1220

Library of Congress

ISSN: 1040-3094

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Printed in Philadelphia, PA
August 1995

Foreword

This publication, *Fracture Mechanics: 25th Volume*, contains papers presented at the 25th National Symposium on Fracture Mechanics: New Trends in Fracture Mechanics, held in Bethlehem, Pennsylvania on 28 June through 1 July 1993. The symposium was sponsored by ASTM Committee E-8 on Fatigue and Fracture. Dr. Fazil Erdogan of Lehigh University in Bethlehem, PA presided as symposium chairman and is the editor of the resulting publication.

Overview

The origin of fracture mechanics as it is understood today goes back to Griffith's work on the fracture of glass in the 1920's. After a rather long lapse, the field was revived in the 40's and 50's. During this period some key contributions to the physical understanding and analytical modeling of the phenomenon of fracture were made by Sneddon, Rivlin, and Thomas and, particularly, by Irwin and his coworkers. Irwin's work was motivated by some well-known and costly failures of ships, aircraft, storage tanks, and other engineering structures due to brittle fracture. Despite the apparent success of the techniques proposed by Irwin in explaining many of these spectacular failures, the acceptance of fracture mechanics as a design and maintenance tool by the engineering community was rather slow in coming. Perhaps not surprisingly, the pioneers in the application of this new technology were the aircraft manufacturers, specifically, the Boeing Company. These early applications demonstrated that the fracture mechanics parameters may be used quite effectively to model fatigue crack growth and fracture instability in some critical aircraft components. As a consequence, fracture mechanics as a field of research started to attract widespread attention in applied mechanics and materials engineering communities and the field started to grow very rapidly in the early 60's. It was then felt that a regular forum was needed to present and discuss new ideas and new applications, and to exchange views on the subject periodically; hence, the idea of a national symposium on fracture mechanics. The first few of these symposia were organized by Professor P. C. Paris and were held at Lehigh University. In 1971, ASTM, through its Committee E-24 on Fracture Testing, recognized the importance of the field and assumed the responsibility for sponsoring and organizing the symposia. With few exceptions, the symposium has been an annual affair. After the reorganization of the committee structure at ASTM, Committee E-8 on Fatigue and Fracture is now providing the necessary guidance for the organization of the symposium. Even though the annual meeting has been called the National Symposium, from the beginning there has always been some participation from abroad. In recent years such participation has been openly encouraged by ASTM. Thus, of the 45 articles published in this volume, 14 are contributed by scientists and engineers from nine foreign countries.

Fracture mechanics is a very young field and is very much in the development stage. The main purpose of the National Symposium on Fracture Mechanics is, therefore, to provide an open forum for the presentation of new ideas, new developments, and new applications. Also, interest in the subject is very broad and is still growing. Partly because of this, in recent years it has become customary to adopt a *theme* for each symposium. The emphasis in selecting the papers for presentation at this symposium was on new trends in fracture mechanics research. The topics covered included fracture mechanics of ceramics, interface fracture, and new testing methods, as well as new developments in inelastic fracture, fatigue, and computational and analytical techniques. Thus, the contributions vary from basic research in micromechanics of interfaces and of transformation toughening in ceramics to the applications of some of the advanced techniques to aging aircraft. Forty-five papers selected for publication in this volume are divided into six categories and are presented in six sections. There are some papers that could be placed in more than one category and, hence, in some cases an arbitrary selection had to be made. Following is a brief description of research presented in each section. The material

presented in this volume is intended to benefit those scientists and engineers involved in doing cutting edge research in various aspects of fracture mechanics as well as the engineers interested in the application of fracture mechanics to actual design problems.

Fracture Mechanics of Ceramics

In such advanced technologies as high speed civil transport and advanced gas turbines, at the operating temperatures contemplated, the use of metal alloys will not be feasible. For example, in large natural gas fired stationary turbines currently in service with 1260°C rotor inlet temperature, the lower heating value plant efficiency has been demonstrated to be around 54%. It is estimated that with cycle innovations and certain changes in design and materials, it is possible to raise the efficiency to 60%. This, however, requires raising the rotor inlet temperature to well over 1400°C which would, in turn, necessitate the development and use of new high-temperature materials and coatings. In this and other high-temperature applications the use of structural ceramics and ceramic matrix composites is, therefore, becoming an absolute necessity. Because of the nature of the material, cracking and spallation in ceramics and ceramic coatings have always been a problem. One needs to understand the factors causing surface cracking and spallation, and develop mechanisms to improve the corresponding material toughness. The papers in this section address various aspects of the fracture phenomenon in structural ceramics. In studying the transformation toughening of ceramics in the neighborhood of a crack tip, the dilational component of the strain field is assumed to play the dominant role. The first paper in this section considers the role of transformation-induced shear strains on the stability of crack growth.

Generally, the critical factors controlling the strength of ceramic components are known to be surface imperfections and residual compressive stresses that result from processing. The second paper in this section investigates these factors by using a new technique to determine the microstrains and an X-ray diffraction technique to determine the residual stresses. In the third paper, the crack extension resistance curve (*R*-curve) behavior of sintered $\text{Al}_2\text{O}_3/\text{ZrO}_2$ ceramic is investigated by using various standard cracked specimens with long cracks and small indentation cracks. The results show specimen dependence of the *R*-curve and the importance of crack-bridging on toughness. In the fourth paper, a generalized three-dimensional Dugdale-Barenblatt cohesive zone model is developed for short-rod specimens and the influence of the specimen size in toughness measurements is investigated. The results demonstrate the applicability of small specimens with inelastic correction to toughness measurements in ceramics. The fifth paper presents some analytical results for an elliptic cavity or a crack problem in piezoelectric ceramics. The material is known to deform under electric voltage and, conversely, generate an electric charge when subjected to mechanical load. These properties make piezoceramics quite attractive for applications in adaptive structures and in a broad variety of instrumentations. The final paper in this section deals with the question regarding the difference between initiation and arrest values of the strain energy release rate in hot-pressed silicon nitride subjected to stable crack growth. A new statistical model is used to study the variabilities in the critical crack length, initiation and arrest values of energy release rates, and the length of dynamic crack increments.

Interface Fracture

Nine papers presented in this section deal with the general problem of fracture mechanics of interfaces and interfacial zones in bonded dissimilar materials and cover a broad range of topics from bicrystals to adhesively bonded T-peel joints. In the first paper, the dependence of interfacial cracking on the nature of loading in Cu and Fe-Si alloy bicrystals and in metal/sapphire bimaterial systems is studied. The directionality of interface cracking and the effect

of loading phase angle are explained by considering the dislocation emission from the crack tip, the interface decohesion, and the asymmetric orientation of the slip planes relative to the interface. The second paper is a theoretical investigation of the effect of remote mode mix as characterized by the elastic far field stress state on the interfacial fracture toughness. A steadily growing plane-strain crack in a ductile material is considered. The material is assumed to be characterized by J_2 flow theory of plasticity with linear hardening. The next two papers present a micromechanical approach for studying the influence of the structure of fiber matrix interfacial zone on the fracture behavior of reinforced composites. In the third paper, epoxy-resin composites containing unidirectional Kevlar and carbon fibers with a thin layer of thermoplastic (polyvinyl alcohol) coating is considered. The results show a 100% improvement in transverse impact fracture toughness compared to uncoated fibers without significant losses in strength and interlaminar toughness. A shear-lag model is used to analyze the problem. In the fourth paper the fibers were coated either by low-modulus/low-glass transition temperature T_g resin, or high-modulus/high T_g resin, or left uncoated, and the influence of this tailored interfacial region with different T_g on the micromechanical behavior, specifically on microcracking of the composite, is investigated.

The following four papers present a variety of analytical results in bonded dissimilar materials including the effect of material orthotropy, the singular behavior of the stress state at the intersections of free boundaries and interfaces, relative properties of the layers on the strain energy release rate for interface cracks under thermal loading, three-dimensional effects, and the effect of specimen geometry and loading conditions. The last paper in this section discusses the results of finite element solution of a symmetric T-peel joint consisting of aluminum adherents and epoxy adhesive of finite thickness or weld-bonded adherents.

New Techniques

This section presents some new experimental methods and application of the known standard techniques in novel applications. A nonintrusive method of measuring surface and subsurface stresses is outlined in the first paper. The method is based on the piezo-spectroscopic effect in optical fluorescence and provides high accuracy stress measurements with spatial resolution of the order of one micron. The method is demonstrated by measuring the residual stresses in sapphire fiber-reinforced γ -TiAl or α -Ti matrices. In the second paper, the microindentation cracking technique is used to demonstrate the feasibility of fabricating a superconducting Josephson junction. Various other applications of microindentation cracking are also discussed in the paper. The third paper outlines a novel technique of crack healing by releasing certain chemicals that are carried in the hollow fibers into the cracks in the matrix. The next paper presents the results for crack kinking in concrete specimens under dynamic mixed-mode loading obtained from transient moiré interferometry and dynamic finite element solution. Similarly, the last paper uses the method of dynamic photoelasticity to study a modified Charpy specimen for measuring lower-bound toughness.

Inelastic Fracture

Nine papers presented in this section are concerned with the type of fracture problems which may generally be classified under inelastic fracture. In the first paper, the analytical details of a two-parameter description of crack tip stress field in elastic-plastic materials are given. The finite element results for a single-edge cracked specimen under tension are discussed in detail. The stress field is characterized in terms of the J integral and the amplitude of the second term of the elasto-plastic asymptotic stress field. The two-term asymptotic stress field is shown to provide an accurate description of the spatial and temporal variation of the crack-tip stresses in creeping solids. The next paper in this section presents a hierarchical scheme for modeling

the ductile damage in solids. The third paper discusses the results of dynamic tensile and instrumented impact tests. The paper also shows the comparison of the computational results obtained by simulating the problem by using two different strain-rate dependent micromechanical models. The results seem to indicate that the deformation and fracture behavior of Charpy-V and single edge notched bending specimens can be evaluated by using the parameters determined from dynamic tensile and static fracture tests. In the fourth paper, the critical crack tip opening angle (CTOA) is incorporated into a two-dimensional finite element method and used as the fracture criterion to study ductile fracture propagation in thin sheets of aluminum alloys. Rather good agreement is obtained between the experimental and calculated results concerning the load line displacement and crack extension as function of applied load.

The next three papers in this section are concerned with the application of the J integral to ductile fracture of structural components of various geometries and crack configurations. The eighth paper also discusses the application of the J integral to fracture of ice. The last paper presents the results of a probabilistic study of cleavage fracture of ferritic steels in the transition region.

Fatigue and Fracture

Eight papers appearing in this section deal with some fundamental work and practical applications of fatigue crack growth and fracture in engineering materials. The first paper considers the subcritical growth of "shear cracks" in large grain 7029 aluminum alloy under mixed mode cyclic loading. It is shown that the resolved shear stress intensity factor calculated in terms of K_I , K_{II} , and K_{III} is a highly effective correlation parameter for the fatigue crack growth rate where the cracking is due primarily to shear decohesion. The second paper revisits the concept of the crack closure problem in fatigue and discusses its role and significance. The experimental results presented seem to indicate that the effect of oxides and asperities on the crack closure is greater than that of plasticity and, generally, the importance of crack closure in fatigue studies is overemphasized. The third paper presents systematic results for constant and variable amplitude fatigue crack growth results in aluminum-lithium alloys 2090-T8E41 and 8090-T8771 and investigates the effect of roughness induced crack closure.

The fourth and fifth papers discuss the results of fatigue crack growth and fracture experiments in some large-scale structural elements. In the fourth paper, 8 m long cellular box beams simulating a double-hull ship were tested under four-point bending. An interesting feature of the results was that cracks 1500 mm or more in total length still grew in a stable manner showing remarkable fracture resistance of boxes made of high-strength and high-toughness steels. In the fifth paper, the effect of welding discontinuities on the variability of fatigue life is studied. The next two papers deal with the application of some advanced fracture mechanics techniques to aging aircraft. The last paper in this section discusses a novel technique used for dynamic rip-arrest characterization of heat-treated low-alloy steels.

Computational and Analytical Techniques

The first two papers in this section present some analytical results for various flaw interaction problems for a half plane. The remaining papers deal with the application two- and three-dimensional finite element methods to various fracture mechanics problems.

Fazil Erdogan

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Fracture Mechanics of Ceramics

ANALYSIS OF THE EFFECT OF TRANSFORMATION-INDUCED SHEAR STRAINS ON STABILITY AND CRACK GROWTH IN ZIRCONIA-CONTAINING CERAMICS

REFERENCE: Stam, G. T. M. and van der Giessen, E., "Analysis of the Effect of Transformation-Induced Shear Strains on Stability and Crack Growth in Zirconia-Containing Ceramics," Fracture Mechanics: 25th Volume, ASTM STP 1220, F. Erdogan, Ed., American Society for Testing and Materials, Philadelphia, 1995.

ABSTRACT: Mode I crack growth is studied in ceramics which undergo a stress-induced martensitic phase transformation that leads to a substantial increase of toughness. The transformation involves both dilatant and shear transformation strains. Full crack growth analyses are carried out numerically and it is demonstrated that the transformation shear strains are a major contribution to toughening.

KEYWORDS: crack growth, transformation toughening, ceramics, martensitic, numerical analysis

INTRODUCTION

It is well-known that the fracture toughness of zirconia (ZrO_2)-containing materials can be greatly enhanced by a stress-induced crystal transformation. This transformation involves a phase transformation of tetragonal particles of the composite to a monoclinic structure, $t \rightarrow m$, leading to a volume increase of about 4.5%. The amount of transformation shear strain has been subject to debate for many years ([1], [2]), as in the composite the unconstrained shear strain of 16% is observed to be reduced by twinning. The early model due to McMeeking and Evans [3] and Budiansky et al. [4] did not take into account the average shear component, thus leading to a simple yet pioneering model that accounted for the transformation induced dilatation. This model significantly enlarged the understanding of transformation toughening but there was unsatisfactory quantitative agreement with experimental toughening results. Since then, the multi-axial stress-strain behaviour has been studied to enlarge the understanding of the shear component of the transformation. Hydraulic compression experiments in a pressure vessel have been carried out ([5], [6]), where the

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hydrostatic and deviatoric (or differential) driving forces could be controlled separately so as to investigate the interplay between dilatation and shear effects. Typical stress-strain curves for CeO_2 -TZP materials under different test conditions are shown in Fig. 1a. The results show shear and dilatation effects of comparable magnitude. Furthermore, experiments at different temperatures revealed that the martensitic transformations are thermoelastic in nature, and give rise to pseudoelastic and shape memory effects. Based on this work a new micromechanics based model was developed by Sun et al. [7]; this is the model that we adopt here to account for the shear effects and their influence on toughening during crack growth.

TRANSFORMATION MODEL

Throughout the present work, the composite is idealized as a homogenized continuum, where the constitutive element consists of a large number of transformable inclusions (referred to with I) embedded coherently in an elastic matrix (referred to with M). Microscopic quantities (inside the continuum element) are referred to with small characters and macroscopic quantities are referred to with capital characters. The macroscopic quantities can be found by taking the volume average $\langle \rangle$ of the microscopic quantities of the element. For instance, the microscopic stress and strain in the element are referred to as σ_{ij} and ϵ_{ij} , and by assuming a volume fraction f of transformable inclusions it is easy to show that the relation between microscopic and macroscopic stresses is

$$\Sigma_{ij} = \langle \sigma_{ij} \rangle_V = \frac{1}{V} \int_V \sigma_{ij} dV = f \langle \sigma_{ij} \rangle_{V_I} + (1-f) \langle \sigma_{ij} \rangle_{V_M} \quad (1)$$

where the volume of the element, matrix and inclusions is given by V , V_M and V_I , respectively. The condition for the $t \rightarrow m$ transformation to occur is given by

$$F_+(\Sigma_{ij}, f, \theta, \langle \epsilon_{ij}^{ps} \rangle_{V_I}) = \frac{2}{3} A \sigma_e^M + 3 \sigma_m^M \epsilon^{pd} - C_0(\theta, f) = 0 \quad (2)$$

similar to a yield surface in metal plasticity. With reference to Table 1, σ_e^M is the von Mises stress in the matrix and σ_m^M is the hydrostatic stress in the matrix, which can be derived from the macroscopic stresses Σ_{ij} by accounting for the influence of the transformation strains (see TABLE 1). The strains associated with the $t \rightarrow m$ transformation are envisioned as 'plastic' strains; the constant lattice volume dilatation ϵ^{pd} is about 1.5%. The average shear strain of the transformed fraction of the continuum element $\langle \epsilon_{ij}^{ps} \rangle_{V_I}$ is much less than the stress-free lattice shear strain of 16% because of the twinning effect in the particles due to the constraint of the surrounding elastic matrix. Experimental work ([8], [9]) has suggested the assumption that the incremental average shear strain is related to the current stress situation through $\langle \epsilon_{ij}^{ps} \rangle_{dV_I} = A \sigma_{ij}^M / \sigma_e^M$, where the material parameter $A = 3h_0 \epsilon^{pd}$ governs the influence of shear. The function $C_0(\theta, f)$ in Eq (2) depends, among others, on the dissipation due to interface friction D_0 , the contribution of energy change due to differences in surface energy A_0 and the free chemical energy $\Delta G_{t \rightarrow m}(\theta)$, which is temperature (θ) dependent. In this paper we consider a constant temperature, thus in the following the value of θ will no longer play a role. The hardening is governed by the parameter α .

TABLE 1--Summary of key variables and relationships.

$\sigma_e^M = \sqrt{(3/2) s_{ij}^M s_{ij}^M}$	$s_{ij}^M = S_{ij} - f B_1 \langle \epsilon_{ij}^{ps} \rangle_{V_I}$	$\sigma_m^M = \Sigma_m - f B_2 \epsilon^{pd}$
$S_{ij} = \Sigma_{ij} - \Sigma_m \delta_{ij}$	$\Sigma_m = \frac{1}{3} \Sigma_{ij} \delta_{ij}$	$E_m = \frac{1}{3} E_{ij} \delta_{ij}$
$C_0(\theta, f) = D_0 + A_0 + \Delta G_{t \rightarrow m}(\theta) - \frac{1}{3} B_1 A^2 - \frac{3}{2} B_2 (\epsilon^{pd})^2 + \alpha B_0 (\epsilon^{pd})^2 f$		
$\langle \epsilon_{ij}^{ps} \rangle_{d V_I} = A \frac{s_{ij}^M}{\sigma_e^M}$	$B_0 = \frac{4G(1+\nu)}{1-\nu} + \frac{Gh_0^2(28-20\nu)}{5(1-\nu)}$	
$B = E/[3(1-2\nu)]$	$B_1 = [2G(5\nu-7)]/[15(1-\nu)]$	
$G = E/[2(1+\nu)]$	$B_2 = B(4\nu-2)/[1-\nu]$	

As the matrix stresses are not only dependent on the macroscopic stress, but also on the amount of transformation strains, it is clear that the model is history and path dependent. Sensitivity to non-proportional stressing histories is due to the influence of the direction of the principle stresses on the direction of the transformation shear strains.

If we define the total strain in the usual way by summation of the elastic and plastic parts, $E_{ij} = E_{ij}^e + E_{ij}^p$, where the plastic part is further decomposed into dilatant and deviatoric components, $E_{ij}^p = E_{ij}^{pd} + E_{ij}^{ps}$, then the relation between macroscopic stress and strain-rates is given by

$$\dot{E}_{ij} = \frac{1}{2G} \dot{S}_{ij} + \frac{1}{3B} \dot{\Sigma}_m \delta_{ij} + \dot{f} (\epsilon^{pd} \delta_{ij} + \langle \epsilon_{ij}^{ps} \rangle_{d V_I}) \quad (3)$$

or in inverse form,

$$\dot{\Sigma}_{ij} = 2G(\dot{E}_{ij} - \dot{E}_m \delta_{ij}) + 3B \dot{E}_m \delta_{ij} - \dot{f} (3B \epsilon^{pd} \delta_{ij} + 2G \langle \epsilon_{ij}^{ps} \rangle_{d V_I}) \quad (4)$$

Finally, Sun et al. [7] give the following expression for the rate of change of the fraction of tetragonal phase during the transformation,

$$\dot{f} = \frac{\langle \epsilon_{ij}^{ps} \rangle_{d V_I} \dot{S}_{ij} + 3 \epsilon^{pd} \dot{\Sigma}_m}{\frac{2}{3} B_1 A^2 + (3 B_2 + \alpha B_0) (\epsilon^{pd})^2} \quad \text{when } F_+ = 0 \quad \text{and } \dot{f} > 0. \quad (5)$$

When all tetragonal material has transformed, $f = f^m$, the material behaves purely elastic again with the original moduli. Further details may be found in [7] and [10].

GOVERNING PARAMETERS

The constitutive model which has been briefly introduced in the previous paragraph contains a variety of material parameters. However, for the purpose of mechanical behaviour it is not necessary to know the value of each individual parameter. For instance for the transformation condition Eq (2) it is sufficient to know the value of $C_0(f=0)$ instead of all the terms where it is comprised of (TABLE 1). When we introduce $\Sigma_c = C_0/(3\epsilon^{pd})$, the

deformation response of the material is characterized by Youngs' modulus E , Poissons ratio ν , the maximum transformable phase f^m , and ϵ^{pd} , Σ_c , α and h_0 .

For the TZP-material described in [8], the elastic properties are given as $E = 190000$ MPa and $\nu = 0.3$. Since the body is completely tetragonal, we have $f^m = 1.0$, and the constant lattice volume dilatation is always assumed to be $\epsilon^{pd} = 0.015$. The remaining parameters Σ_c , α and h_0 will be estimated from results of a hydraulic compression test [8]. The experimental stress-strain curve is reproduced in Fig. 1a, from which Fig. 1b

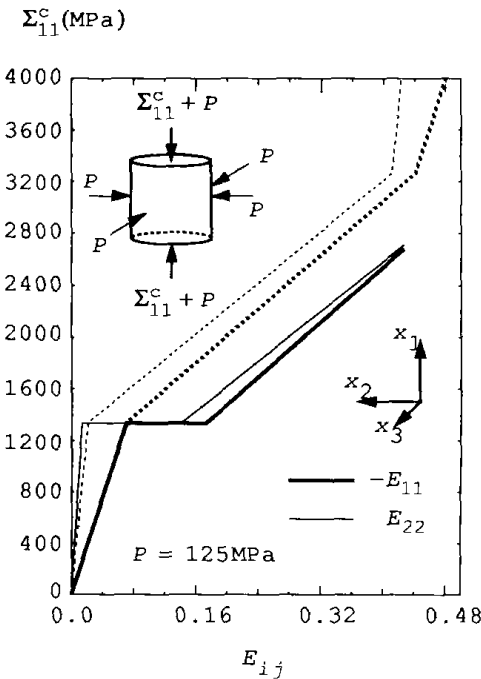


Fig. 1a--Stress-strain curves for CeO₂-TZP obtained by triaxial compression at room temperature. The dashed lines give the response of the constitutive model.

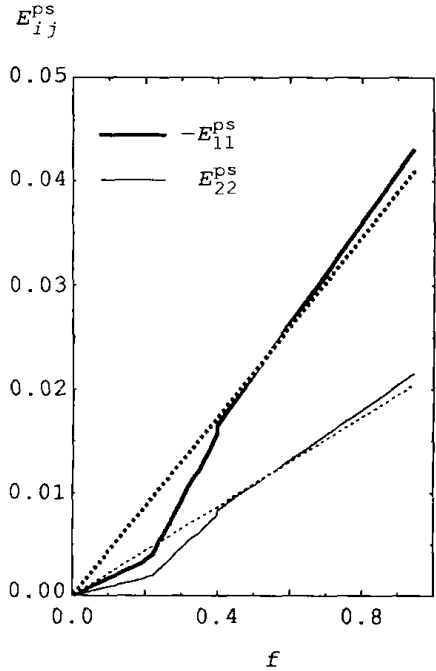


Fig. 1b--Representation of the amount of shear transformation to the transformed fraction. The dashed lines show the fitted curve of the constitutive model.

can be derived, showing the shear transformation strains in the x_1 and x_2 direction as a function of the transformed fraction. In the experiment the loading is proportional and as a consequence the shear transformation strains in the continuum model are given by $E_{ij}^{ps} = 3fh_0\epsilon^{pd}(S_{ij}/\Sigma_e)$. With this relation the value of h_0 can be estimated by curve fitting to the experimentally determined curves of Fig. 1. The dashed lines show the theoretical response for $h_0 = 1.4$ in the axial (x_1) and radial (x_2 and x_3) direction. The critical transformation stress Σ_c can be determined using the transformation condition Eq (2) prior to any transformation, i.e.

$$F_+(\Sigma_{ij}, \langle \epsilon_{ij}^{ps} \rangle_{V_1}) = \frac{2}{3}h_0\Sigma_e + \Sigma_m - \Sigma_c = 0 \quad . \tag{6}$$