Paperson General Topology and Related Category Theory and Topological Algebra

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PAPERS ON GENERAL TOPOLOGY AND RELATED CATEGORY THEORY AND TOPOLOGICAL ALGEBRA

and Aaron R. Todd

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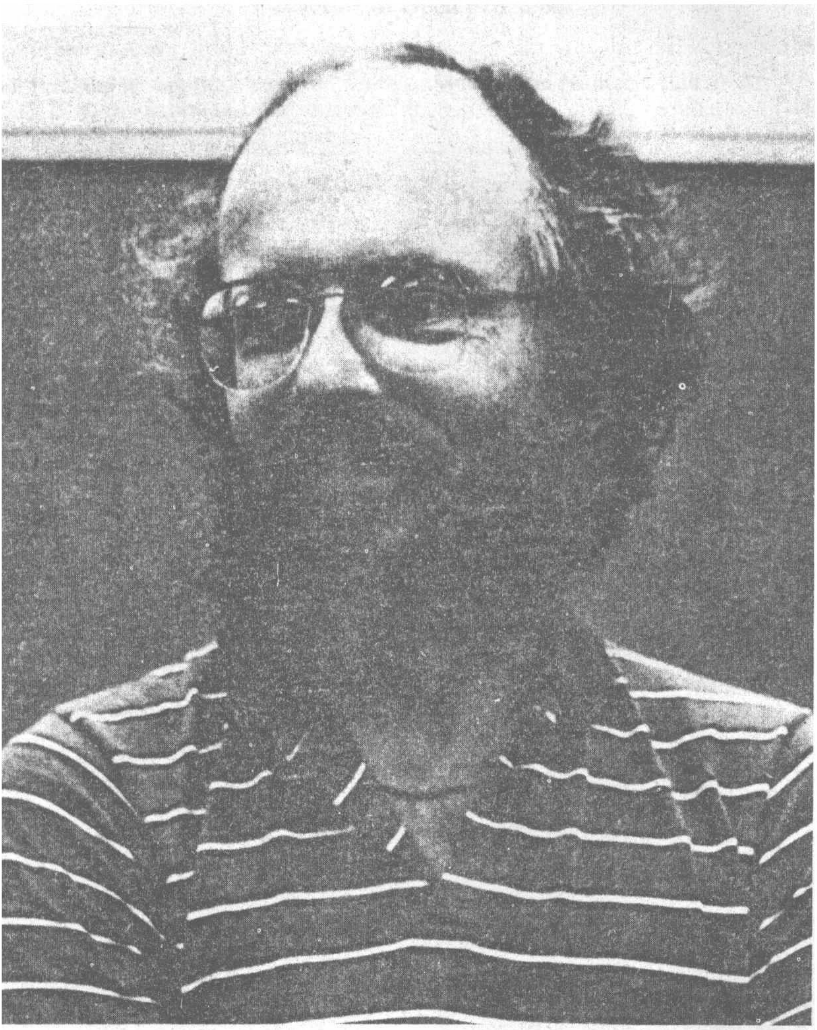
Dedicated to the memory of

ERIC KAREL VAN DOUWEN

(1946–1987)



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Preface

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The papers in this volume are based largely on talks given at the Second and Third Conferences on Limits at the City College of New York–CUNY on July 2–3, 1985, and June 12–13, 1987. The 1985 conference was organized by the Seminar on General Topology and Topological Algebra of the Department of Mathematics of the City College of New York, whereas the 1987 conference was sponsored jointly by this Seminar and the Section of Mathematics of the New York Academy of Sciences. Eighteen of the twenty-five speakers at these two meetings have contributed papers to this publication.

This volume is dedicated to the memory of Eric van Douwen (1946–1987), who was an invited speaker at the July 1985 conference. His loss has been keenly felt by all of us. We were pleased to learn, though, that Jerry Vaughan was willing to contribute a paper based on results that Eric and he had obtained independently and had discussed at the 1985 conference. The paper, "Some Subspaces of Ordinals with Normal Products", appears with both their names.

In contrast to the previous meetings, the June 1987 meeting included a short open problems session. We know of work that grew out of that session and we felt that it would be useful to include several open problems in this volume as well.

The conferences have continued: A fourth was held at Wesleyan University on June 16–18, 1988, and a fifth is planned at The College of Staten Island–CUNY for June 15–17, 1989.

Organizing the meetings and editing this volume has been an exhilarating experience, but we could not have done it without a great deal of help and support from many others. Particular thanks go to: the Division of Science of the City College of New York–CUNY for support of the first three meetings; the New York Academy of Sciences for the publication of this volume; and Efua Tonge for her help at these meetings. In addition, we are grateful to the conference participants and contributors

to this volume for the cooperative and productive atmosphere surrounding the meetings and the editorial process. Last, but not least, special thanks are due to the referees for their careful reviews of submitted articles. Although we are not naming them, their constructive criticisms and willingness to review papers within our deadline constraints have been very much appreciated by all of us on the editorial board.

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PAPERS ON GENERAL TOPOLOGY AND RELATED CATEGORY THEORY AND TOPOLOGICAL ALGEBRA[a]

Editors

RALPH KOPPERMAN, PRABUDH MISRA, JACK REICHMAN, and AARON R. TODD

CONTENTS

[a]The papers in this volume were presented at two conferences, namely, the Second and Third Conferences on Limits at the City College of New York—CUNY on July 2–3, 1985, and June 12–13, 1987.

A Collection of Open Problems

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Orientation of Orbits in Flows[a]

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INTRODUCTION

The basic facts needed in the sequel are presented in this section. For the notions and results not discussed here, the reader is referred to reference 6 (also see references 2, 4, 7, and 8). All spaces under consideration are Hausdorff.

$\mathbb{R}$ denotes the space of the real numbers. A flow on a topological space X is a continuous mapping $\pi: X \times \mathbb{R} \to X$ such that the following statements hold for all $x \in X$ and for all $s, t \in \mathbb{R}$:

(i) $\pi(x, 0) = x$, and
(ii) $\pi(\pi(x, s), t) = \pi(x, s + t)$.

For each $x \in X$, the mapping $\pi_x: \mathbb{R} \to X$ is defined by $\pi_x(t) = \pi(x, t)$. It is called the motion through x. The motion π_x is continuous, of course, and its image $\{\pi_x(t) \mid t \in \mathbb{R}\}$ is called the orbit of x. The orbit of x is denoted by $\Gamma(x)$.

A point x in a flow is said to be moving if, for some s and t in $\mathbb{R}$, $\pi(x, s) \neq \pi(x, t)$, that is, if it is not a rest point. If a moving point x is periodic, its orbit $\Gamma(x)$ is homeomorphic to the circle S^1. It is important to observe in this case that the motion $\pi_x: \mathbb{R} \to S^1$ is a so-called covering map. (See reference 3 or 5 for information regarding covering maps.) If the moving point x is nonperiodic, its motion π_x is a continuous bijection. In this case, the orbit $\Gamma(x)$ may look quite complicated. For example, consider the irrational flow on the torus $T^2 = S^1 \times S^1$, which for an irrational number α is defined by

$$\pi(z, w, t) = (z \exp 2\pi i t, w \exp 2\pi i \alpha t),$$

where $(z, w) \in T^2$ and $t \in \mathbb{R}$. For each point $y = (z, w) \in T^2$, the orbit $\Gamma(y)$ is a dense subset of T^2. The motion π_y is neither a topological embedding nor a covering map.

The flows $\pi: X \times \mathbb{R} \to X$ and $\rho: Y \times \mathbb{R} \to Y$ are said to be topologically equivalent if there is a homeomorphism $h: X \to Y$ that maps each orbit of π onto an orbit of the flow ρ and that preserves the orientation of orbits. This definition is somewhat superfluous. The point is that a homeomorphism $h: X \to Y$ that maps orbits of π onto

[a]Dedicated to Yukihiro Kodama on his sixtieth birthday.

orbits of the flow ρ automatically either preserves or reverses the orientation of each orbit. We will prove this result according to the following outline. First, we consider the case where the point x is moving, but nonperiodic. We consider the map $f = \rho_{h(x)}^{-1} \circ h \circ \pi_x$: $\mathbb{R} \rightarrow \mathbb{R}$. As $h(x)$ also is nonperiodic, the mapping f is well defined. It can easily be seen to be bijective. However, a quite intricate argument is needed to show that f is a homeomorphism. Following this, we discuss the case where x is a periodic and moving point. Here, the orbit $\Gamma(x)$ is homeomorphic to S^1 and, as we have seen, the motion π_x: $\mathbb{R} \rightarrow \Gamma(x)$ is a covering map. As $h(x)$ also is periodic and moving, the map $\rho_{h(x)}$: $\mathbb{R} \rightarrow \Gamma(h(x))$ is a covering map too. From the general theory of covering maps, it follows that the map $h|\Gamma(x)$: $\Gamma(x) \rightarrow \Gamma(h(x))$ has a lift f: $\mathbb{R} \rightarrow \mathbb{R}$, which is a homeomorphism. The homeomorphism h is said to be orientation preserving if the homeomorphism f, defined above, is increasing for each moving point.

As previously stated, the proof that f is a homeomorphism is quite complicated in the nonperiodic case. In references 1 and 6, use of Sierpinski's theorem—stating that a continuum cannot be partitioned nontrivially into countably many closed sets—is made. In reference 9, the local product structure of a flow at its moving points is invoked. The proof of THEOREM 1 (to be presented later) is, in contrast with these proofs, a quite straightforward application of the Baire category theorem. Thus, the methods of proof developed here can be used to obtain new proofs of some results in reference 9. This is illustrated in THEOREM 2.

ORIENTATION

We first define the parametrization of orbits of moving points. We distinguish between periodic and nonperiodic motions.

DEFINITION 1. Suppose π: $X \times \mathbb{R} \rightarrow X$ is a flow. Let x be a moving point. If the point x is periodic, then any covering map $\mathbb{R} \rightarrow \Gamma(x)$ is called a parametrization of $\Gamma(x)$. If the point x is nonperiodic, then any continuous bijective map $\mathbb{R} \rightarrow \Gamma(x)$ is called a parametrization of $\Gamma(x)$.

EXAMPLE 1. Let x be a moving point in a flow π. The motion π_x as well as the mappings $t \rightsquigarrow \pi_x(2t)$ and $t \rightsquigarrow \pi_x(-t)$ are parametrizations of $\Gamma(x)$.

Parametrizations are locally bijective. The most important property of parametrizations—the so-called covering path property—is expressed by THEOREM 1.

THEOREM 1. Suppose x is a moving point in a flow. Let p: $\mathbb{R} \rightarrow \Gamma(x)$ be any parametrization of $\Gamma(x)$. Suppose j: $[0, 1] \rightarrow \Gamma(x)$ is a continuous map. Then, for each $y \in \mathbb{R}$ such that $p(y) = j(0)$, there exists a unique continuous map $\tilde{j}$: $[0, 1] \rightarrow \mathbb{R}$ such that $\tilde{j}(0) = y$ and $p \circ \tilde{j} = j$.

The map $\tilde{j}$ is called a lifting or a covering path of j.

In the nonperiodic case, $p^{-1}(j(0))$ consists of one point only and the conclusion of THEOREM 1 can be strengthened as follows: there exists a unique map $\tilde{j}$: $[0, 1] \rightarrow \mathbb{R}$ such that $p \circ \tilde{j} = j$. In passing, also note that the dumbbell and the figure eight are curves that lack the arc lifting property. Thus, for this result, we need the hypothesis that $\Gamma(x)$ comes from a flow.

The proof of THEOREM 1 will be presented in the next section.

Now, using THEOREM 1, we shall show that the parametrizations of an orbit fall

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into two classes. This will enable us to define orientations. The following is a preparatory lemma.

LEMMA 1. Suppose that x is a moving point in a flow. Suppose that both p_1 and p_2 are parametrizations of $\Gamma(x)$. Then, for each a_1 and a_2 such that $p_1(a_1) = p_2(a_2)$, there is a unique homeomorphism $h: \mathbb{R} \to \mathbb{R}$ such that $p_2 = p_1 \circ h$ and $h(a_2) = a_1$.

Proof: Let $t \in \mathbb{R}$. By THEOREM 1, the map $p_2 | [a_2, t]$, which is the mapping p_2 restricted to $[a_2, t]$, has a unique lifting f_t such that $p_2 = p_1 \circ f_t$ and $f_t(a_2) = a_1$. Now, $h(t)$ is defined by $h(t) = f_t(t)$. It is easily seen that h is well defined and that $p_2 = p_1 \circ h$. As p_1 and p_2 are locally bijective, h is bijective. As h maps intervals onto intervals, it is a homeomorphism.

DEFINITION 2. Let x be a moving point in a flow. Two parametrizations p_1 and p_2 of $\Gamma(x)$ are called equivalent if there is an increasing homeomorphism: $\mathbb{R} \to \mathbb{R}$ such that $p_2 = p_1 \circ h$.

It is not hard to see that the relation, defined in DEFINITION 2, is an equivalence relation.

EXAMPLE 2. Let x be a moving point in a flow. The motion π_x and the parametrization p_1, defined by $p_1(t) = \pi_x(2t)$, are equivalent. With $h(t) = 2t$, we have $p_1 = \pi_x \circ h$. However, the motion π_x and the parametrization p_2, defined by $p_2(t) = \pi_x(-t)$, are not equivalent. This can be seen as follows: if h is a homeomorphism such that $p_2 = \pi_x \circ h$, then $\pi_x(-t) = \pi_x(h(t))$ for all $t \in \mathbb{R}$. In the case where x is nonperiodic, π_x is bijective and it follows that $h(t) = -t$ for all $t \in \mathbb{R}$. In the case where x is periodic from the covering property of π_x, it follows that $h(t) = -t + C$ for some $C \in \mathbb{R}$. In both cases, h is decreasing.

The following proposition is now clear.

PROPOSITION 1. Suppose x is a moving point in a flow. There are then two equivalence classes of parametrizations.

DEFINITION 3. Let x be a moving point in a flow. An equivalence class of parametrizations is called an orientation. A parametrization that is equivalent to the motion π_x is called a forward parametrization of $\Gamma(x)$. The equivalence class of forward parametrizations of $\Gamma(x)$ is called the positive orientation of $\Gamma(x)$.

The proof of the following lemma is obvious.

LEMMA 2. Suppose that $\pi: X \times \mathbb{R} \to X$ and $\rho: Y \times \mathbb{R} \to Y$ are flows. Suppose $h: X \to Y$ is a homeomorphism that maps orbits of π onto orbits of ρ. Suppose x is a moving point of X. For all parametrizations p_1 and p_2 of x, the maps $h \circ p_1$ and $h \circ p_2$ are parametrizations of $h(x)$. Moreover, p_1 and p_2 are equivalent if and only if $h \circ p_1$ and $h \circ p_2$ are equivalent.

From LEMMA 2, it follows that h induces a mapping between the orientations of x and $h(x)$. In this way, we can give the following equivalent definition of orientation-preserving homeomorphism.

DEFINITION 4. Suppose that $\pi: X \times \mathbb{R} \to X$ and $\rho: Y \times \mathbb{R} \to Y$ are flows. Suppose $h: X \to Y$ is a homeomorphism that maps orbits onto orbits. Then, h is said to be orientation preserving if the positive orientation of $\Gamma(x)$ for each moving point x is mapped to the positive orientation of $\Gamma(h(x))$. A topological equivalence of π to ρ is a homeomorphism $h: X \to Y$ that maps orbits onto orbits and preserves orientation.

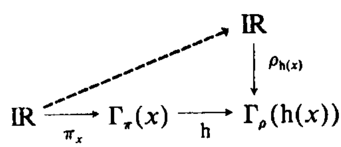
To conclude this section, we give an alternative proof of a result of Ura (cf reference 9). We formulate the result as follows.

THEOREM 2. Suppose h is a topological equivalence of the flow π on X to the flow ρ on Y. Suppose $x \in X$ is a moving point. Then, there exists a unique homeomorphism τ_x: $\mathbb{R} \to \mathbb{R}$ such that

$$\mathrm{h} \circ \pi_x = \rho_{\mathrm{h}(x)} \circ \tau_x.$$

The homeomorphism τ_x is increasing and $\tau_x(0) = 0$.

Proof: We consider the following diagram:



Both π_x and $\rho_{\mathrm{h}(x)}$ are forward parametrizations. As h preserves the orientation, $\mathrm{h} \circ \pi_x$ is a forward parametrization as well. Because $\mathrm{h} \circ \pi_x(0) = \mathrm{h}(x) = \rho_{\mathrm{h}(x)}(0)$, then (according to LEMMA 1) there is a unique homeomorphism τ_x: $\mathbb{R} \to \mathbb{R}$ such that $\mathrm{h} \circ \pi_x = \rho_{\mathrm{h}(x)} \circ \tau_x$ and $\tau_x(0) = 0$. As $\rho_{\mathrm{h}(x)}$ and $\mathrm{h} \circ \pi_x$ are equivalent, τ_x is increasing.

PROOF OF THEOREM 1

In proving THEOREM 1, we need to distinguish between periodic and nonperiodic motions. There are three parts: the periodic case, the case of $\mathrm{p} = \pi_x$, and the general nonperiodic case.

The Periodic Case

In this case, THEOREM 1 expresses the covering path property of covering maps (see reference 3 or 5). However, before embarking upon the nonperiodic case, we first have to prove a lemma.

LEMMA 3. Suppose x is a moving point in a flow π. Let p be any parametrization of $\Gamma(x)$. If C is a nonempty and compact subset of $\Gamma(x)$, then there exists a nonempty and relatively open subset U of C such that $\mathrm{p}^{-1}(\mathrm{U})$ is bounded.

Proof: This is a straightforward application of Baire's category theorem. Write $\mathrm{H}_n = \mathrm{p}([-n, n]) \cap \mathrm{C}$, where $n = 1, 2, \ldots$. As $\bigcup\{\mathrm{H}_n | n = 1, 2, \ldots\} = \mathrm{C}$, there is a nonempty and relatively open subset U of C such that $\mathrm{U} \subset \mathrm{H}_n$ for some n.

As a corollary, we have the following well-known theorem.

THEOREM 3. Let x be a point in a flow π. If the orbit $\Gamma(x)$ is compact, then x is periodic.

Proof: We assume that x is moving, but not periodic, and derive a contradiction. By LEMMA 3, there is a nonempty open subset U of $\Gamma(x)$ such that $\mathrm{V} = \pi_x^{-1}(\mathrm{U})$ is bounded. For any $t \in \mathbb{R}$, we have $\pi^t(\mathrm{U}) = \pi^t(\pi_x(\mathrm{V})) = \pi_x(\mathrm{V} + t)$; hence, we can see that $\{\pi^t(U) \mid t \in \mathbb{R}\}$ is a cover of $\Gamma(x)$. By compactness, there is a finite subcover. As π_x is bijective, it follows that $\mathbb{R}$ is covered by finitely many sets of the form $V + t$, $t \in \mathbb{R}$. However, if this is so, then V cannot be bounded.

The Nonperiodic Case of $p = \pi_x$

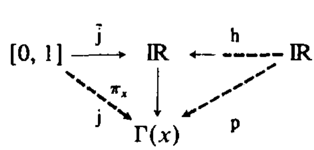
As $\pi_x: \mathbb{R} \to \Gamma(x)$ is bijective, $\tilde{j}$ necessarily is of the form $\tilde{j}: \pi_x^{-1} \circ j$. This proves the unicity of $\tilde{j}$. However, the problem here is to prove that $\tilde{j}$ is continuous. This amounts to showing that $\tilde{j}([0, 1])$ is a bounded set because π_x restricted to $\text{cl}_{\mathbb{R}}\ \tilde{j}([0, 1])$ is a topological embedding in that case. Let $\mathcal{V}$ denote the collection of all open subsets V of $[0, 1]$ such that $\tilde{j}(V)$ is a bounded subset of $\mathbb{R}$. The fact that $\mathcal{V}$ is nonempty may be seen as follows. By LEMMA 3, there is a nonempty and relatively open subset U of $j([0, 1])$ such that $\pi_x^{-1}(U)$ is bounded. By continuity of j, there is a nonempty and open V in $[0, 1]$ that by j is mapped into U. It follows that $\tilde{j}(V)$ is bounded and that $\mathcal{V}$ is nonempty. We write $W = \bigcup \mathcal{V}$ and $D = [0, 1] \setminus W$. Because $\tilde{j}|V$ is continuous for each $V \in \mathcal{V}$ and $W = \bigcup \mathcal{V}$, the map $\tilde{j}|W$ is continuous.

Now, we will show that if C is any interval open in W, then $C \in \mathcal{V}$. As $\tilde{j}|W$ is continuous, $\tilde{j}(C)$ is an interval in $\mathbb{R}$. We shall show that $\tilde{j}(C)$ is bounded. Assume that $\tilde{j}(C)$ is not bounded from above. Then, $\pi_x(\tilde{j}(C)) \subset j([0, 1])$. Because $j([0, 1])$ is compact, the set $\Omega(x)$ of positive limit points is nonempty and $\Omega(x) \subset j([0, 1]) \subset \Gamma(x)$. As $\Omega(x)$ is invariant, we have $\Gamma(x) \subset \Omega(x)$. Hence, $\Gamma(x) = j([0, 1])$ and so $\Gamma(x)$ is compact. By THEOREM 3, x is periodic, which is a contradiction.

We also take a close look at the set D. From the definition of $\mathcal{V}$ and LEMMA 3, it follows that D is nowhere dense. Furthermore, no point of D is isolated. This can be proved as follows. We assume that $z \in D$ is isolated and that there are adjacent intervals C_1 and C_2 that are contained in W. Then, for $i = 1, 2$, we have $j(z) \in j(\text{cl}\ C_i) \subset \text{cl}\ j(C_i) = \text{cl}\ \pi_x(\tilde{j}(C_i)) = \pi_x(\text{cl}\ \tilde{j}(C_i))$. The last equality holds because $\text{cl}\ \tilde{j}(C_i)$ is compact ($i = 1, 2$). It follows that $\tilde{j}(z) \in \text{cl}\ \tilde{j}(C_i)$, $i = 1, 2$, and that $C_1 \cup \{z\} \cup C_2 \in \mathcal{V}$. Thus, $z \notin D$, which is a contradiction. Then, to complete the second part of the theorem, we show that $D = \emptyset$. Assuming that this is not true, we can apply LEMMA 3 once more to find an open interval C_3 in $[0, 1]$ such that $C_3 \cap D \neq \emptyset$ and $\tilde{j}(C_3 \cap D)$ is bounded. Let F be the smallest closed interval containing $\tilde{j}(C_3 \cap D)$ and let E be the smallest closed interval in $[0, 1]$ such that $C_3 \cap D \subset E$. As D has no isolated points, E is nondegenerate. Now, each component C_4 of $E \setminus D$ is an interval contained in W. It follows that $\tilde{j}(C_4)$ is bounded. As both endpoints of C_4 are in D, $\tilde{j}(C_4) \subset F$. Thus $\tilde{j}(E) \subset F$ and $\text{int}\ E \in \mathcal{V}$. However, $\text{int}\ E \cap D \neq \emptyset$, which is a contradiction.

The General Nonperiodic Case

Let p be any parametrization. We consider the diagram



The map h is of the form $\pi_x^{-1} \circ$ p. We shall show that h is a homeomorphism. As h is bijective, it is sufficient to show that h maps intervals to intervals. Let $[a, b]$ be any interval in $\mathbb{R}$. From the second part of the proof, it follows that p$|[a, b]$ has a lift that necessarily equals h$|[a, b]$. It follows that $h^{-1} \circ \tilde{j}$ is a lift for j.

LOCAL FLOWS

In this section, we indicate how the results of the preceding sections can be adapted for local dynamical systems or local flows (see reference 9). A local flow (called a partial flow in reference 6) is a continuous mapping π: D $\rightarrow X$, where D is an open subset of $X \times \mathbb{R}$ of the form D $= \bigcup \{\{x\} \times J(x) \mid x \in X\}$ with, for every $x \in X$, $J(x)$ being an open interval in $\mathbb{R}$ containing 0. The mapping π satisfies conditions (i) and (ii) stated in the INTRODUCTION plus a maximality condition.

Now, suppose π: D $\rightarrow X$ is a local flow. Let $x \in X$. In case $J(x) = \mathbb{R}$, the definitions of the parametrization and the orientation of $\Gamma(x)$ are as listed in the second section. It is to be observed that orbits of periodic points are included in this case.

DEFINITION. Suppose π: D $\rightarrow X$ is a local flow. Suppose x is a nonperiodic point. Any continuous bijective map $J(x) \rightarrow \Gamma(x)$ is called a *parametrization* of $\Gamma(x)$.

There is the following supplement of THEOREM 1.

THEOREM 4. Suppose x is a nonperiodic point in a local flow. Let p: $J(x) \rightarrow \Gamma(x)$ be any parametrization of $\Gamma(x)$. Suppose j: $[0, 1] \rightarrow \Gamma(x)$ is a continuous map. Then, there exists a unique and continuous map $\tilde{j}$: $[0, 1] \rightarrow J(x)$ such that p $\circ \tilde{j} =$ j.

Therefore, only minor adaptations are required in the second section. In LEMMA 1, DEFINITION 2, and THEOREM 2, $\mathbb{R}$ must be replaced by $J(x)$ throughout. In LEMMA 2 and DEFINITION 4, $X \times \mathbb{R}$ must be replaced by D and $Y \times \mathbb{R}$ must be replaced by E. In the proof of THEOREM 2, $\mathbb{R}$ must be replaced by $J(x)$ and $J(h(x))$.

In the previous section, the following adaptations also have to be made. The last line of the conclusion of LEMMA 3 must be replaced by the line, "such that $p^{-1}(U)$ has compact closure in $J(x)$" (instead of "$p^{-1}(U)$ is bounded"). At the beginning of the proof of THEOREM 3, the following observation must be inserted: "As $\Gamma(x)$ is compact, the positive and negative limit sets are nonempty and $J(x) = \mathbb{R}$."

Finally, in the second and third part of the proof of THEOREM 1 (the case of p $= \pi_x$ and the general nonperiodic case), $\mathbb{R}$ must be replaced by $J(x)$ throughout and "bounded" must be interpreted as having a compact closure in $J(x)$.

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On the Inequality $L(X) \leq 2^{s(X)}$ and Related Topics

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A discrete subspace of a topological space X is a subset D of X such that all points of D are isolated in its relative (subspace) topology. In other words, for each $z \in D$, there exists an open set $U_z \subset X$ such that $U_z \cap D = \{z\}$. For example, the set of the integers forms a discrete subspace of the real line R in its usual (interval) topology.

It is a reasonable guess that any large topological space with a "fair number" of open sets must have relatively large discrete subspaces. Questions of this nature are best formulated and answered in the context of (topological) cardinal functions. In the present case, the following cardinal function is required.

DEFINITION 1. The spread $s(X)$ of a topological space X is defined as the supremum of cardinalities of discrete subspaces of X:

$$s(X) = \sup\{|D|:\ D \subset X,\ D \text{ is discrete}\}.$$

The given definition generates a question: must there exist a discrete subspace D_0 of X such that $|D_0| = s(X)$? This natural question has turned out to have a deep answer (cf. references 25 and 35).

For the real line R, we have $|R| = 2^{\omega}$ and $s(R) = \omega$; hence, $|R| = 2^{s(R)}$. In order to see that $s(R) = \omega$, let us assume that $D = \{x_i : i \in I\}$ is an uncountable discrete subspace of R. For each $x_i \in D$, there then exists a positive integer n_i such that $D \cap I_i = \{x_i\}$, with

$$I_i = \left(x_i - \frac{1}{n_i},\ x_i + \frac{1}{n_i}\right).$$

Because there exist only countably many integers and I is uncountable, there must exist an uncountable subset J of I and an integer m such that $n_j = m$ for all $j \in J$. It follows that the intervals

$$I_j = \left(x_j - \frac{1}{2m},\ x_j + \frac{1}{2m}\right)$$

must be pairwise disjoint; hence, there exist uncountably many rational numbers, which is a contradiction. (This proof is an application of the so-called "pigeonhole principle".)

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