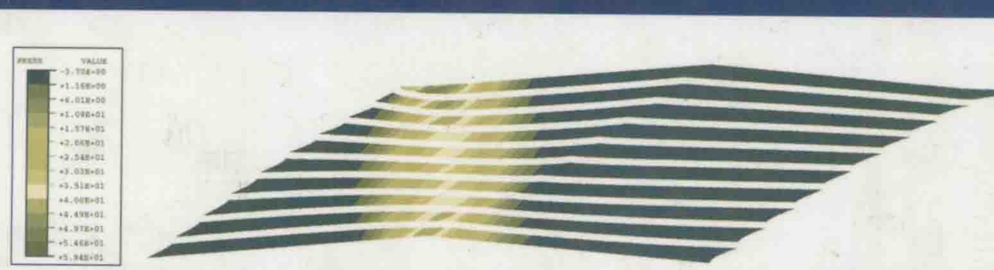
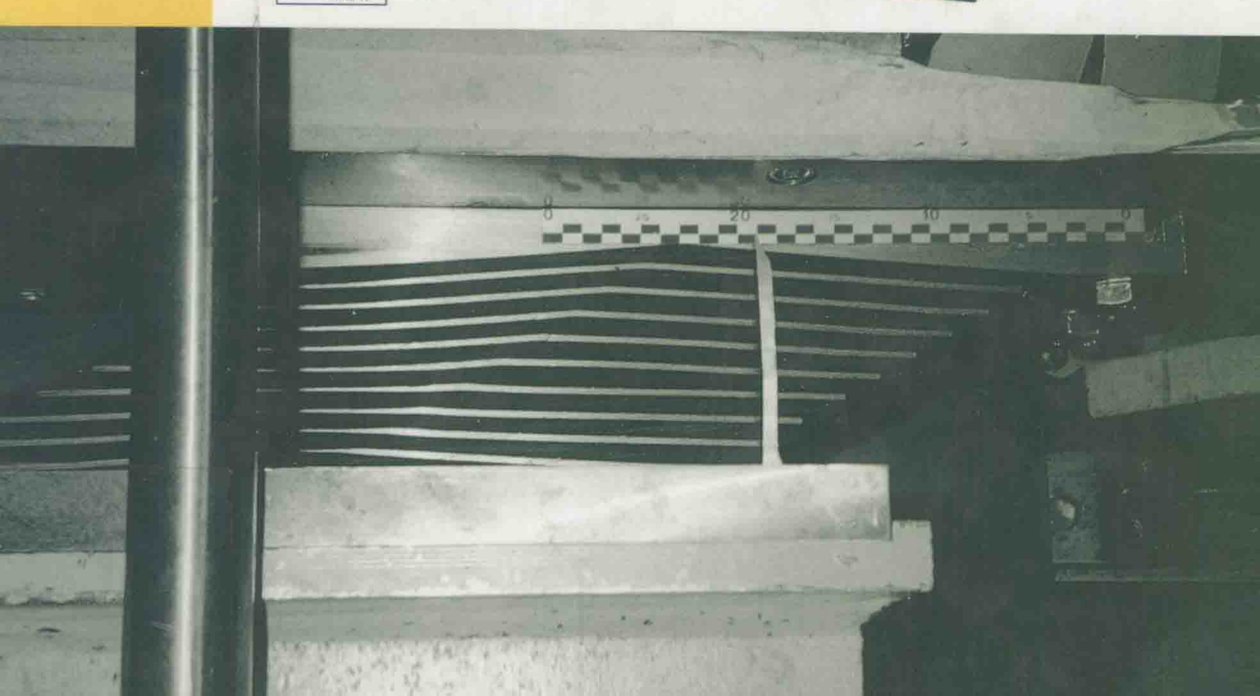
Constitutive Models for Rubber

Al Dorfmann & Alan Muhr Editors





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CONSTITUTIVE MODELS FOR RUBBER

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Foreword

The extraordinary stress strain behaviour of rubber has presented an opportunity for inventive engineers and a challenge for scientists since the mid-nineteenth century, and continues to do so today. Major branches of theory, such as the statistical theory of rubber elasticity and finite strain elasticity theory, have been spawned by the properties of rubber. Until recently, however, the theoretical framework for large deformations found little application among rubber engineers because the mathematics rapidly becomes intractable for all but the simplest components. The advent of affordable and powerful computers has changed all this, and brought the challenge of rubber to new sets of people – software engineers and desk top, as opposed to empirical, designers.

The development of the statistical theory of rubber elasticity in the 1940s, of finite strain elasticity theory in the 1950s, and of convenient forms for the strain energy function in the 1970s, all focused on modelling the elastic characteristics of rubber. Although much literature has appeared in recent years following this theme, the Physics of Rubber Elasticity by L.R.G. Treloar (3rd Edition, Clarendon Press, Oxford, 1975) and the proceedings of a Discussion on Rubber Elasticity (Proc. Roy. Soc. London, 1976, A351, No. 1666, 295-406) remain very valuable reviews.

The treatment of rubber as a 'hyperelastic' material – that is, a material modelled by a strain-energy function for finite strains – was implemented into finite strain finite element analysis in the 1980s and is now widely available in commercial software packages.

However, only a few engineering elastomers – such as unfilled natural rubber and some grades of polyurethane – really conform to the 'hyperelastic' ideal. Most other engineering elastomers incorporate 'reinforcing' fillers, needed to confer adequate strength properties and also improving processing characteristics and enabling adjustment of hardness over a wide range. The stress-strain characteristics of such filled elastomers depart significantly from elasticity. While ways of thinking about these departures – such as the 'dynamic static-to-ratio' of rubber springs – may have satisfied a previous generation of design engineers, there is now an opportunity to apply more sophisticated models.

One major current challenge is thus to model these aspects of the inelastic behaviour that are relevant to engineers, and to do this in such a way that the models are implementable in finite element analysis.

Although potentially the involvement of representatives of several disciplines should facilitate progress, this is only the case if they talk to each other. In practice, software engineers might rely on the literature and on desk-top designers as sources of information about rubber, and fail to achieve as good a balance of understanding as they could if they listened also to experimental rubber scientists and empirical designers. Applied mathematicians might develop phenomenological models which address issues of secondary interest to designers, or which misrepresent important aspects of the experimentally observed behaviour. Experimentalists might develop models without reference to the existing framework of continuum mechanics, resulting in internal inconsistencies and difficulty in implementation in software packages. The First European Conference on Constitutive Models for Rubber sprang from the idea of providing a forum for multi-disciplinary discussion, seeking to bring the fragmented strands of recent research together.

Within the UK a start has been made in this direction – through a workshop on Deformation Modelling for Solid Polymers (Oxford University, 1997) and a seminar on Finite Element Analysis of Elastomers (Institution of Mechanical Engineers, London, 1997). The proceedings of the latter are available as a publication of the same name (Professional Engineering Publications, London, 1999). Similarly, in Germany a workshop on Finite Element Analysis – Basics and Future Trends was organised by the Deutsche Institut für Kautschuk Technology (Hanover, 1998). The interest in these essentially national meetings suggested that further cross-fertilisation should be stimulated by providing a European forum for discussion.

The contributions to this Proceedings cover a wide range of subjects. Consistent with the analysis given above, relatively few authors chose to present hyperelastic models for rubber; however, readers interested in this topic will find ample references to earlier work. Several contributions address inelastic effects associated with filled elastomers – such as Mullins' effect and quasi-static hysteresis. For others – most obviously in processing uncured rubber – the interest is in modelling viscoelasticity. In addition to stress-strain behaviour, work is presented on frictional contact and on mechanical failure. Looking at the applications side, computational techniques are addressed and applied to a diverse range of components, including tyres, earthquake isolation bearings and intervertebral discs. Overall, the authors have achieved progress in a wide range of areas – including experimental results, theory, and practical utility. They raise many questions as well, as one might expect from the first forum of this kind.

We would like to thank our colleagues on the Scientific Committee (R.W.Ogden, Chairman; D.Besdo, R.de Borst, K.N.G.Fuller, H.A.Mang, H.Menderez, G.Meschke and H.Rothert) and all the authors who have worked with us to produce this book.

Al Dorfmann
Alan H. Muhr
Vienna/Hertford, June 1999

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Advanced FE analysis of elastomeric automobile components under realistic loading conditions

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ABSTRACT:

In this paper the frequently used constitutive models in the simulation of rubber components will be discussed. Both the simple models, often used in industrial applications, with extensions to visco-elasticity and the more advanced quasi-static models will be reviewed. Attention will also be paid to the techniques for curve fitting of the material parameters for a particular constitutive model. It is shown that errors in parameter determination can easily be made if insufficient experimental data is available. These errors can partially be avoided if good curve fitting tools are available and if they can be used prior to the analysis. The material models will be applied to analysis of simple components under realistic loading conditions. It is demonstrated that both visco-elasticity and inertia effects play a key role in obtaining realistic simulations.

1 INTRODUCTION

Recent advances in FEM technology has resulted in industrial application of simulation tools in the design of elastomeric components. In the first decades of research in mathematical modelling of the material behavior of elastomers and numerical techniques to handle the nearly in-compressible material behavior, the industrial application of simulation tools was still limited. With the availability of simple to use numerical procedures for handling the contact problem, many manufacturers of rubber components such as seals, tires, motor mounts, sport materials have recognized the potential of numerical analysis in the design of rubber components. Currently car manufacturers often demand the results of a numerical simulation when a new design is presented.

For most applications, still a quasi-static deformation analysis is sufficient. The study of the interaction of the rubber with other deforming parts is nowadays a standard application through simple and easy to use contact algorithms, which include self-contact, friction and thermal contact. Recent advances have made it possible to include dynamic effects in the contact algorithm through implicit or explicit transient analysis procedures, which enables studies of the effects in for instance shock absorbers. An other class of problems where dynamic effects have to be included are steady state vibrations subjected to non-linear prestressed structures. Here it usually is sufficient to analyse the behaviour at a particular excitation frequency (or ranges of frequencies), taking into account the appropriate stiffness and damping of the material at that frequency.

Even in a quasi-static analysis the identification of the material parameters to be used for a particular material model based on results of tests on simple test specimen can still be cumbersome. Often curve fitting techniques are required which can show good predictions for particular loading (e.g. a tensile test), but will behave badly when the material is subjected to an equi-biaxial test. Good and easy to use tools to predict and verify the material behavior, prior to the analysis, will avoid failures in the numerical simulation of realistic components.

It is recognized that the material behavior is visco-elastic which displays itself through for instance relaxation of the stresses after closure of the seal, resulting in potential leakage conditions. Also shock absorbers have a stiffness behaviour which can be different depending on the rate of compression.

Another application requiring visco-elasticity models are cyclically loaded structural components in which the material stiffness and the damping are frequency dependent. In addition, the energy dissipation due to the visco-elasticity produces heat. This local heat production will increase the temperature that, in combination with the temperature dependency of the material properties requires a coupled thermo-mechanical analysis.

Recently progress has been made in the study of the acoustic behavior of rubber seals. The medium surrounding the seal is subjected to cyclic pressure variations and the damping characteristics of the seal as well as the potential of exitating the rubber seal in its eigen frequency needs to be analysed. This requires a coupled analysis in which both the pressure in the medium and the deformation of the seal are determined.

2 MATERIAL CHARACTERISATION

Several decades of research in accurate constitutive models for the description of both in-compressible and compressible elastomeric behaviour have resulted in potential accurate models for the description of the material behaviour under arbitrary multi-axial loading conditions. Application of the models to a particular rubber is difficult due various reasons:

- For a specific rubber often insufficient experimental material data is available to determine the parameters for a particular model.
- Frequently only the result of tensile or a compressive tests is available. These results are then used to determine the material parameters in the constitutive model. Depending on the results of the curve fitting process of these parameters the various constitutive models exhibit a behaviour of other homogeneous stress states which can only be judged globally on correctness or verified if results of these tests are available.
- A large class of the available constitutive models are verified or valid for the description of the quasi-static behaviour only.

Realistic simulations of industrial structural components require however constitutive models which include a dependence of:

- temperature dependence on the material properties
- large strain visco-elastic effects
- frequency dependent stiffness and damping

In the first decade of application of numerical simulation techniques to structural components, often these afore mentioned effects have been neglected, mainly since even with these simplification the analysis was already complex enough and a solution could not always be guaranteed.

Progress in simulation techniques, in particular the availability of robust and easy to use techniques for contact analysis, as well as robust finite element technology for incompressible behaviour has proven that numerical simulation of structural components subjected to realistic loading conditions is feasible. This in turn has resulted in the following questions:

- Which model should be used if only limited experimental data is available?
- How accurate is my material model?
- Can one include visco-elastic effects?
- Can one include thermal effects?
- Can one describe damage effects?

2.1 Simple models used in industry

For particular rubbers used in industry often only the Shore hardness A or a linear shear modulus is available. For limited strain ranges the material is often linear in shear and the classical simple models such as the Neo-Hookean or Mooney-Rivlin model behave linear in shear.

A simple logarithmic model to relate the shear modulus G to the Shore A hardness H is described by Batterman/Kohler (1982)

$$G = 0.086 \bullet 1.045^{H} \qquad (1)$$

Modifications of this relation or tabular data relating the shore hardness to the modulus of elasticity are described by Lindley (1966), Crawford (1985), Gent (1994) and Gobel (1969).

The material models are often formulated by an elastic energy definition e.g.:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \qquad (2)$$

where I_1 and I_2 are the first and second strain invariant, C_{10} and C_{01} are material parameters.

The parameters for the above described most frequently used simple material models can then be obtained from:

Neo Hookean: $G = 2C_{10}$ (3)

Mooney Rivlin: $G = 2(C_{10} + C_{01})$ (4)

where: $C_{01} = 0.2\ ...0.25C_{10}$ (5)

In spite of the known limitations to describe particular stress states, several analysts claim to obtain good results using these models for various structural components with local values of the strains up to about 200%.

These models also allow a simple definition of the quasi-static temperature dependency. It suffices to define G or C_{10} and C_{01} as a function of the temperature. Here often tabular data are used. A consequence of temperature change however is often that visco-elastic effects become more dominant and can no longer be neglected.

Non-linear visco-elastic models are often based on modified forms of the general Schapery model. This model allows great flexibility in modelling, but since experimental data to describe all possible effects is limited, often the model lacks application. A much more simple model is given by Simo. In this model the elastic energy is modelled by a N term Prony series expansion similar to a linear visco-elastic model. In the linear visco-elastic model the shear modulus is approximated by:

$$G(t) = G^{\infty} + \sum_{n=1}^{N} G^{n} \exp\left(-\frac{t}{\tau_n}\right) \tag{6}$$

The elastic energy as function of the strain is then given as

$$W(E, t) = W^{\infty} + \sum_{n=1}^{N} W^{n} \exp\left(-\frac{t}{\tau_n}\right) \tag{7}$$

where W^{∞} represents the long term elastic energy (as described by one of the forms mentioned above) and W^{n} an energy contribution which is added to long term elastic energy and corresponds to a time constant τ_n. The model allows different forms of the elastic energy function for each term, but often for simplicity the terms are assumed to have the same shape and differ only by a scalar multiplier.

Temperature effects have a strong influence on the visco-elasticity. This can in most cases accounted for by the so-called thermo-rheologically simple behaviour assumption, in which a shift of the relaxation time τ_n is obtained depending on the local values of the temperature. The most frequently used shift function is the Willams-Landel and Ferry equation.

The visco-elastic model mentioned above allows a transient analysis with arbitrary large deformations. For the study of small amplitude vibrations in non-linear pre-stressed components often a simplification can be made based on the linear elastic storage and loss modulus. The storage modulus $G'(\omega)$ and loss modulus $G''(\omega)$ as function of the frequency ω are obtained from experiments and can either be used directly in tabular data form or through an approximation in a Prony series in the frequency range.

$$G'(\omega) = G^{\infty} + \sum_{n=1}^{N} G^{n} \frac{\omega^2 \tau_n^2}{1 + \omega^2 \tau_n^2}$$
$$G''(\omega) = \sum_{n=1}^{N} G^{n} \frac{\omega \tau_n}{1 + \omega^2 \tau_n^2} \tag{8}$$

The viscoelastic behaviour results in heat production. The energy dissipation per load cycle in a cyclic test with strain amplitude is defined by:

$$\dot{\theta} = G''(\omega)\gamma_0^2/2$$

where γ_0 is the local value of the cyclic strain. It is clear that these simple models do not always provide the correct solution. If sufficient experimental data on simple test-specimens with a homogenous stress state is available, the models can be refined.

In most models the material behavior is assumed to be incompressible or nearly incompressible. This means that the ratio of bulk and shear modulus is approximately $K/G \approx 10000$. Hence for compressible foam materials other models have to be used. Visco-elasticity for the volumetric behaviour is usually neglected.

2.2 More advanced models and the fitting of parameters

Years of research on accurate constitutive modelling has resulted in the availability of a number of models describing the elastic energy as a function of the deformation. An excellent review of these models can be found in e.g. Treloar (1975). The models are either strain invariant or stretch ratio based and have found their way into finite element codes. The general purpose finite element program MARC provides besides the Neo Hookean and Mooney-Rivlin model the following strain invariant models: Signorini, Yeoh, Gent, and other combinations of the general 3rd order models, the stretch ratio based Ogden model and its variant allowing non-linear volumetric behaviour for e.g. foam materials, and the micromechanics based (Generalized) Arruda-Boyce model. In addition simple program modifications allow the analyst to define special models, examples of which are e.g. the Kilian (1981) model and the micromechanis based tube model. (Heinrich et al. 1988).

In general the Ogden model has become more popular if large strains have to be considered in the structural component. The tensile stress-strain curve typically has a stress stiffening part near the limiting stretch and the simple modesl fail to capture this stress increase. The fitting of the parameters is purely empirical and often more than one experiment is required. This has led to the development of micromechanics based models such as the Arruda-Boyce which claim to give a good prediction for other stress states purely on the result of a tensile test.

Usually a tensile or compressive test is used as basic test and the parameters in the constitutive model are determined such that the best possible fit is obtained with the results of the tensile test. Common problems with this approach are:

- What is the behaviour of the model for strains larger than used in fitting the experimental result?
- What is the behaviour for other homogeneous stress states, such as simple shear, pure shear, equibiaxial and volumetric?
- If results of other stress states are availble how should one perform the fit.

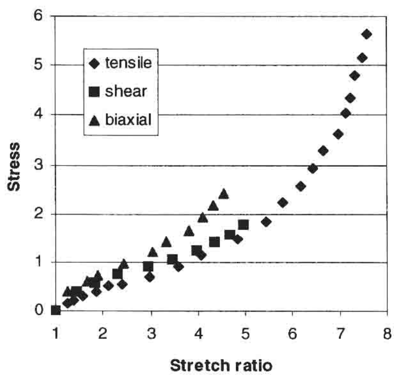

Figure 1 Experimental data from Treloar (1975)

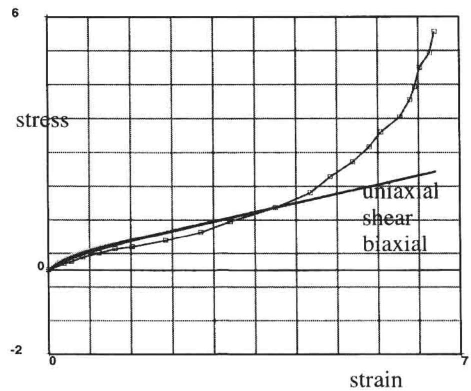

Figure 2 Neo Hookean fit

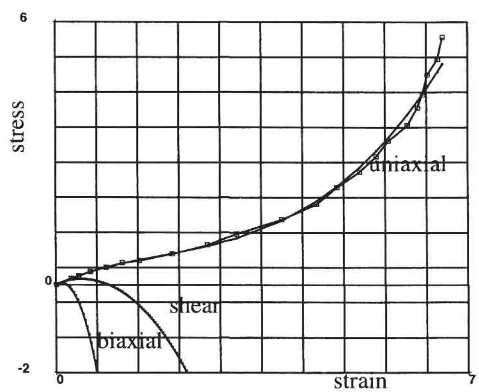

Figure 3 Second Order Model fit

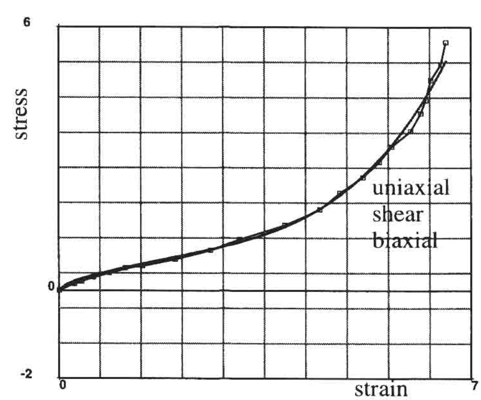

Figure 4 Three Term Ogden model fit

The most limiting factor in the accuracy of either of these models is still the determination of the material parameters. Undetected errors in the determination of these parameters, result in errors in the simulation of the behaviour of the structural component as well. As an example consider the test data given for a tensile, pure shear and a equi-biaxial test as described by Treloar. The experimental data points are shown in Figure 1.

The results of the uniaxial test have been used in the parameter determination for each model. With these parameters the behaviour for other stress states can simply be predicted and compared with the experimental data of Figure 1.

The best possible fit with a number of models is shown in Figure 2-7.

Close inspection of the figures reveals:

- All models fit the uniaxial curve as close as possible. The Neo-Hookean (and the Mooney Rivlin) model fails to predict correct values of the stesses at higher strain levels.
- The higher order models can easily show a so-called material instability. In particular for the stress states other than the uniaxial one this is often the case. By adding additional constraints to the curve fitting that the constants remain positive this effect-can partly be removed.
- Large deviations in predicting the correct stress value for other stress states can be obtained. This can be avoided by performing the curve fit based on all data simultaneously. Often for new materials these experimental data are however not available.
- Micro-mechanics based models claim to give a good prediction for other stress states with curve fitting based on uniaxial data, but it certainly is recommended to verify whether this is true for new materials.
- It is generally recommended to evaluate the behaviour for other stress states than the uniaxial one prior to any finite element analysis. This curve fitting/prediction process has to be part of the pre-processing capabilities.

If compressible foam materials have to be described it is hardly impossible to perform the curve fitting based on the results of the uniaxial test only. Specific volumetric data is recommended, for instance by measuring the effective cross-sectional